The theory of probability:

The theory of probability is a representation of its concepts in formal terms that is, in terms that can be considered separately from their meaning. These formal terms are manipulated by the rules of mathematics and logic, and any results are interpreted or translated back into the problem domain.

There have been at least two successful attempts to formalize probability, namely the Kolmogorov formulation and the Cox formulation. In Kolmogorov's formulation, sets are interpreted as events and probability itself as a measure on a class of sets. In Cox's theorem, probability is taken as a primitive (that is, not further analyzed) and the emphasis is on constructing a consistent assignment of probability values to propositions. In both cases, the laws of probability are the same, except for technical details.

There are other methods for quantifying uncertainty, such as the Dumpster–Shafer theory or possibility theory, but those are essentially different and not compatible with the laws of probability as usually understood.

Sample space:

In probability theory, the sample space of an experiment or random trial is the set of all possible outcomes or results of that experiment. A sample space is usually denoted using set notation, and the possible ordered outcomes are listed as elements in the set. It is common to refer to a sample space by the labels S, Ω, or U (for "universal set").

For example, if the experiment is tossing a coin, the sample space is typically the set $\{head, tail\}$. For tossing two coins, the corresponding sample space would be $\{(head, head), (head, tail), (tail, head), (tail, tail)\}$, commonly written $\{HH, HT, TH, TT\}$. If the sample space is unordered, it becomes $\{\{head, head\}, \{head, tail\}, \{tail, tail\}\}$.

For tossing a single six-sided die, the typical sample space is $\{1, 2, 3, 4, 5, 6\}$ (in which the result of interest is the number of pips facing up).

A well-defined sample space is one of three basic elements in a probabilistic model (a probability space); the other two are a well-defined set of possible events (a sigma-algebra) and a probability assigned to each event (a probability measure function)

$$P(event) = \frac{number\ of\ outcom\ in\ event}{number\ of\ outcom\ in\ sample\ space}$$

The Event:

In probability theory, an event is a set of outcomes of an experiment (a subset of the sample space) to which a probability is assigned. A single outcome may be an element of many different events,[2] and different events in an experiment are usually not equally likely, since they may include very different groups of outcomes. An event defines a complementary event, namely the complementary set (the event not occurring), and together these define a Bernoulli trial: did the event occur or not?

Typically, when the sample space is finite, any subset of the sample space is an event (i.e. all elements of the power set of the sample space are defined as events). However, this approach does not work well in cases where the sample space is uncountable infinite. So, when defining a probability space it is possible, and often necessary, to exclude certain subsets of the sample space from being events.

Since all events are sets, they are usually written assets (e.g. {1, 2, 3}), and represented graphically using Venn diagrams. In the situation where each outcome in the sample space Ω is equally likely, the probability P of an event A is the following formula: $P = \frac{|A|}{|\Omega|}\left(Alternatively: Pr = \frac{|A|}{|\Omega|}\right)$

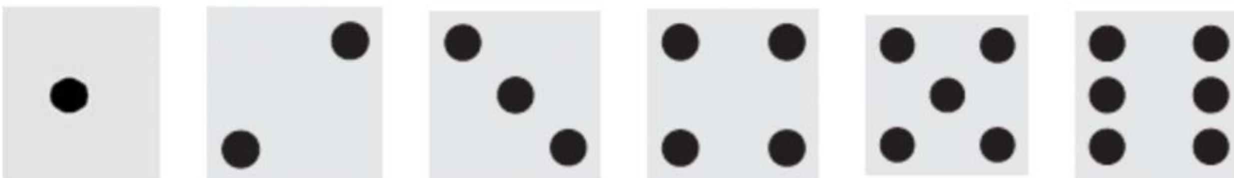
Suppose that an event E can happen in h ways out of a total of n possible equally likely ways. Then the probability of occurrence of the event (called its success) is denoted by: $P = Pr\{E\} = \frac{h}{n}$

The probability of nonoccurrence of the event (called its failure) is denoted by:

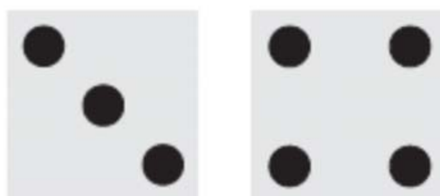
$$q = Pr\{not\ E\} = \frac{n-h}{n} = 1 - \frac{h}{n} = 1 - P = 1 - Pr\{E\}$$

Thus: $P + q = 1\ or\ Pr\{E\} + Pr\{not\ E\} = 1$

Example: When a die is tossed, there are 6 equally possible ways in which the die can fall:



The event E that a 3 or 4 turns up is:



And the probability of E is $Pr\{E\} = 2/6\ or\ 1/3$. The probability of not getting a 3 or 4 (i.e., getting a 1, 2, 5, or 6) is $Pr\{\bar{E}\} = 1 - Pr\{E\} = 2/3$

Note that the probability of an event is a number between 0 and 1. If the event cannot occur, its probability is 0. If it must occur (i.e., its occurrence is certain), its probability is 1.

If p is the probability that an event will occur, the odds in favor of its happening are p : q (read ‘‘p to q’’); the odds against its happening are q : p. Thus the odds against a 3 or 4 in a single toss of a fair die are $q\colon p = \frac{2}{3}\colon\frac{1}{3} = 2\colon 1 (i.e, 2 to 1)$

Example: If 1000 tosses of a coin result in 529 heads, the relative frequency of heads is 529/1000 = 0.529. If another 1000 tosses results in 493 heads, the relative frequency in the total of 2000 tosses is 529/2000 = 0.511.

According to the statistical definition, by continuing in this manner we should ultimately get closer and closer to a number that represents the probability of a head in

a single toss of the coin. From the results so far presented, this should be 0.5 to one significant figure. To obtain more significant figures, further observations must be made.

Terms:

If $E_1$and E_2 are two events, the probability that E_2 occurs given that E_1 has occurred is denoted by $Pr\{E_2/E_1\}$, or $Pr\{E_2\ given\ E_1\}$, and is called the conditional probability of E_2 given that E_1 has occurred.

If the occurrence or nonoccurrence of E_1 does not affect the probability of occurrence of E_2, then $Pr\{E_2/E_1\} = Pr\{E_2\}$ and we say that E_1 and E_2 are independent events; otherwise, they are dependent events.

If we denote by E_1E_2 the event that ''both E_1 and E_2 occur,'' sometimes called a compound event, then: $Pr\{E_1E_2\} = Pr\{E_1\}Pr\{E_2/E_1\}$. In particular, $Pr\{E_1E_2\} = Pr\{E_1\}Pr\{E_2\}$ for independent events.

For three events E_1, $E_2 and\ E_3$, we have :

$$Pr\{E_1E_2E_3\} = Pr\{E_1\}Pr\{E_2/E_1\}Pr\{E_1\backslash E_2\backslash E_3\}$$

Example: Let E_1 and E_2 be the events ''heads on fifth toss'' and ''heads on sixth toss'' of a coin, respectively. Then E_1 and E_2 are independent events, and thus the probability of heads on both the fifth and sixth tosses is (assuming the coin to be fair)

Solve

$$Pr\{E_1E_2\} = Pr\{E_1\}Pr\{E_2\} = \left(\frac{1}{2}\right) * \left(\frac{1}{2}\right) = \frac{1}{4}$$

Example: Suppose that a box contains 3 white balls and 2 black balls. Let E_1 be the event ''first ball drawn is black'' and E_2 the event ''second ball drawn is black,'' where the balls are not replaced after being drawn. Here E_1 and E_2 are dependent events.

The probability that the first ball drawn is black is $Pr\{E_1\} = 2/(2+3) = \frac{2}{5}$ The probability that the second ball drawn is black, given that the first ball drawn was black, is: $Pr\{E_2/E_1\} = 1/(3+1) = \frac{1}{4}$ Thus the probability that both balls drawn are black is: $Pr\{E_1E_2\} = Pr\{E_1\}Pr\{E_2/E_1\} = \frac{2}{5} * \frac{1}{4} = \frac{1}{10}$

Mutually exclusive events

Two or more events are called mutually exclusive if the occurrence of any one of them excludes the occurrence of the others. Thus if E_1 and E_2 are mutually exclusive events, then $Pr\{E_1E_2\} = 0$. if $E_1 + E_2$ denotes the event that ''either E_1 or E_2 or both occur,'' then $Pr\{E_1 + E_2\} = Pr\{E_1\} + Pr\{E_2\} - Pr\{E_1E_2\}$ In particular,
$Pr\{E_1 + E_2\} = Pr\{E_1\} + Pr\{E_2\}$ For mutually exclusive events.
As an extension of this, if $\{E_1, E_2, \cdots E_n\}$ are n mutually exclusive events having respective probabilities of occurrence $\{P_1, P_2, \cdots P_n\}$ then the probability of occurrence of either $E_1 or E_2, \cdots or\ E_n$ is $P_1 + P_2 + \cdots + P_n$ Result (5) can also be generalized to three or more mutually exclusive events.

Example: If E1 is the event ''drawing an ace from a deck of cards'' and E_2 is the event ''drawing a king,'' then $Pr\{E_1\} = \frac{4}{52} = \frac{1}{13}\ and\ Pr\{E_2\} = \frac{4}{52} = \frac{1}{13}$. The probability of drawing either an ace or a king in a single draw is:

$$Pr\{E_1 + E_2\} = Pr\{E_1\} + Pr\{E_2\} = \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$$

Since both an ace and a king cannot be drawn in a single draw and are thus mutually exclusive events:

A♣	2♣	3♣	4♣	5♣	6♣	7♣	8♣	9♣	10♣	J♣	Q♣	K♣
A♦	2♦	3♦	4♦	5♦	6♦	7♦	8♦	9♦	10♦	J♦	Q♦	K♦
A♠	2♠	3♠	4♠	5♠	6♠	7♠	8♠	9♠	10♠	J♠	Q♠	K♠
A♥	2♥	3♥	4♥	5♥	6♥	7♥	8♥	9♥	10♥	J♥	Q♥	K♥

E_1 is the event ''drawing an ace'' and E_2 is the event ''drawing a king.'' Note that E_1 and E_2 have no outcomes in common. They are mutually exclusive.

Possible Cases:

In mathematics, a proof is an inferential argument for a mathematical statement. In the argument, other previously established statements, such as theorems, can be used. In principle, a proof can be traced back to self-evident or assumed statements, known as axioms, along with accepted rules of inference. Axioms may be treated as conditions that must be met before the statement applies. Proofs are examples of

exhaustive deductive reasoning or inductive reasoning and are distinguished from empirical arguments or non-exhaustive inductive reasoning (or "reasonable expectation").

$$A = HHH, HHL, HLL, LHH, LLH, LLL$$

HHH •	HHL •	HLL •
LHH •	LLH •	LLL •

Terms:

1. Factorial n $\{n! = n * (n-1) * (n-2) * \cdots * 1\}$

2. Permutation $\left\{npr = \frac{n!}{(n-r)!}\right\}$

Example: choose 2 samples from the $\{A, B, C, D\}$ by permutation method?

Solve

$$npr = \frac{n!}{(n-r)!} = 4p2 = \frac{4!}{(4-2)!} = \frac{4*3*2*1}{(2)!} = \frac{24}{2*1} = 12$$

$$\boldsymbol{i.e., A} = \{(\boldsymbol{AB}), (\boldsymbol{AC}), (\boldsymbol{AD}), (\boldsymbol{BC}), (\boldsymbol{BD}), (\boldsymbol{CD}), (\boldsymbol{BA}), (\boldsymbol{CA}), (\boldsymbol{DA}), (\boldsymbol{CB}), (\boldsymbol{DB}), (\boldsymbol{DC})\}$$

3. Combinations $\left\{ncr\ or\ \binom{n}{r} = \frac{n!}{r!*(n-r)!}\right\}$

Example: choose 5 samples from 9 samples by Combinations method?

Solve

$$\binom{n}{r} = \frac{n!}{r!*(n-r)!} = \binom{9}{5} = \frac{9!}{5!*(9-5)!} = \frac{9!}{5!*4!} = \frac{9*8*7*6*5*4*3*2*1}{(5*4*3*2*1)*(4*3*2*1)}$$

$$= \frac{362880}{(120)*(24)} = \frac{362880}{2880} = 126$$