Nonlinear regression:

In statistics, nonlinear regression is a form of regression analysis in which observational data are modeled by a function which is a nonlinear combination of the model parameters and depends on one or more independent variables. The data are fitted by a method of successive approximations.

In statistics, polynomial regression is a form of regression analysis in which the relationship between the independent variable x and the dependent variable y is modelled as an nth degree polynomial in x. Polynomial regression fits a nonlinear relationship between the value of x and the corresponding conditional mean of y, denoted E(y |x), and has been used to describe nonlinear phenomena such as the growth rate of tissues, the distribution of carbon isotopes in lake sediments, and the progression of disease epidemics. Although polynomial regression fits a nonlinear model to the data, as a statistical estimation problem it is linear, in the sense that the regression function E(y | x) is linear in the unknown parameters that are estimated from the data. For this reason, polynomial regression is considered to be a special case of multiple linear regression.

The explanatory (independent) variables resulting from the polynomial expansion of the "baseline" variables are known as higher-degree terms. Such variables are also used in classification settings.

$$\hat{Y} = \beta_0 + \beta_1 X_1 \qquad m = 1$$

$$\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 {X_1}^2 \qquad m = 2$$

$$\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 {X_1}^2 + \beta_2 {X_1}^3 \qquad m = 3$$

Example: Use Polynomial nonlinear regression with $\boldsymbol{m = 2}$ to analysis data in the underneath, where F tabulation at 0.05 level = 4.26, 0.01 level = 8.02:

X_1	8	10	6	11	8	7	10	9	10	6	12	9
Y	64	71	53	67	55	58	77	57	56	51	76	68

Solve

	X_1	X_2	Y_i	X_1^2	X_2^2	${Y_i}^2$	$X_1 * X_2$	$X_1 * Y_i$	$X_2 * Y_i$
	8	64	64	64	4096	4096	512	512	4096
	10	100	71	100	10000	5041	1000	710	7100
	6	36	53	36	1296	2809	216	318	1908
	11	121	67	121	14641	4489	1331	737	8107
	8	64	55	64	4096	3025	512	440	3520
	7	49	58	49	2401	3364	343	406	2842
	10	100	77	100	10000	5929	1000	770	7700
	9	81	57	81	6561	3249	729	513	4617
	10	100	56	100	10000	3136	1000	560	5600
	6	36	51	36	1296	2601	216	306	1836
	12	144	76	144	20736	5776	1728	912	10944
	9	81	68	81	6561	4624	729	612	5508
Σ	106	976	753	976	91684	48139	9316	6796	63778

$$S_{X_1}^2 = \sum(X_1-\bar{X}_1)^2 = \sum X_1^2 - \frac{(\sum X_1)^2}{n} = 976 - \frac{(106)^2}{12} = \frac{11712-11236}{12} = 39.667$$

$$S_{X_2}^2 = \sum(X_2-\bar{X}_2)^2$$

$$= \sum X_2^2 - \frac{(\sum X_2)^2}{n} = 91684 - \frac{(976)^2}{12} = \frac{1100208-952576}{12} = 12302.667$$

$$S_Y^2 = \sum(Y-\bar{Y})^2 = \sum Y^2 - \frac{(\sum Y)^2}{n} = 48139 - \frac{(753)^2}{12} = \frac{577668-567009}{12} = 888.25$$

$$S_{X_1}Y = \sum X_1Y_i - \frac{(\sum X_1)(\sum Y_i)}{n} = 6796 - \frac{(106)*(753)}{12} = \frac{81552-79818}{12} = 144.5$$

$$S_{X_2}Y = \sum X_2Y_i - \frac{(\sum X_2)(\sum Y_i)}{n} = 63778 - \frac{(976)*(753)}{12} = \frac{765336-734928}{12} = 2534$$

$$S_{X_1X_2} = \sum X_1X_2 - \frac{(\sum X_1)(\sum X_2)}{n} = 9316 - \frac{(106)*(976)}{12} = \frac{111792-103456}{12} = 694.667$$

$$b_1 = \left\{\frac{\left[(S_{X_2}^2)*(S_{X_1}Y) - (S_{X_1X_2})*(S_{X_2}Y)\right]}{\left[(S_{X_1}^2)*(S_{X_2}^2) - (S_{X_1X_2})^2\right]}\right\}$$

$$= \left\{\frac{[(12302.667)*(144.5) - (694.667)*(2534)]}{[(39.667)*(12302.667) - (694.667)^2]}\right\}$$

$$= \frac{1777735.3815 - 1760286.178}{488009.891889 - 482562.240889} = \frac{17449.20349}{5447.65098} = 3.203$$

$$b_2 = \left\{\frac{[(S^2_{X_1}) * (S_{X_2}Y) - (S_{X_1X_2}) * (S_{X_1}Y)]}{[(S^2_{X_1}) * (S^2_{X_2}) - (S_{X_1X_2})^2]}\right\} = \left\{\frac{[(39.667) * (2534) - (694.667) * (144.5)]}{[(39.667) * (12302.667) - (694.667)^2]}\right\}$$

$$= \frac{100516.178 - 100379.3815}{488009.891889 - 482562.240889} = \frac{136.79649}{5447.65098} = 0.025$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{753}{12} = 62.75 \quad , \bar{x}_1 = \frac{\sum x_1}{n} = \frac{106}{12} = 8.833 \quad , \bar{x}_2 = \frac{\sum x_2}{n} = \frac{976}{12} = 81.333$$

$$a = \bar{y} - b_1\bar{x}_1 - b_2\bar{x}_2 = 62.75 - [(3.203) * (8.833) - (0.025) * (81.333)] = 32.425$$

$$a \to \beta_0$$

$$\therefore \hat{Y} = 32.425 + 3.203X + 0.025X^2$$

$$SSR = b_1 * \sum X_1Y_i + b_2 * \sum X_2Y_i = (3.203 * 144.5) + (0.025 * 2534) = 526.1835$$

$$SS_e = S^2_Y - SSR = 888.25 - 526.1835 = 362.0665$$

$$R^2 = \frac{SSR}{S^2_Y} = \frac{526.1835}{888.25} = 0.5924^*$$

$$Cal.F = \frac{MS_R}{MS_e}$$

S.O.V.	d.f.	SS	MS	Cal.F	F_tab	
					0.05	0.01
Regression	2	526.1835	263.09175	6.54^*	4.26	8.02
Residual(Error)	9	362.0665	40.23			
Total	11	888.25				