Generalized randomized block design(RCBD):

In randomized statistical experiments, generalized randomized block designs (GRBDs) are used to study the interaction between blocks and treatments. For a GRBD, each treatment is replicated at least two times in each block; this replication allows the estimation and testing of an interaction term in the linear model (without making parametric assumptions about a normal distribution for the error).

Functional models for block-treatment interactions:

Non-replicated experiments are used by knowledgeable experimentalists when replications have prohibitive costs. When the block-design lacks replicates, interactions have been modeled. For example, Tukey's F-test for interaction (non-additivity) has been motivated by the multiplicative model of Mandel (1961); this model assumes that all treatment-block interactions are proportion to the product of the mean treatment-effect and the mean block-effect, where the proportionality constant is identical for all treatment-block combinations. Tukey's test is valid when Mandel's multiplicative model holds and when the errors independently follow a normal distribution.

Tukey's F-statistic for testing interaction has a distribution based on the randomized assignment of treatments to experimental units. When Mandel's multiplicative model holds, the F-statistics randomization distribution is closely approximated by the distribution of the F-statistic assuming a normal distribution for the error, according to the 1975 paper of Robinson. The rejection of multiplicative interaction need not imply the rejection of non-multiplicative interaction, because there are many forms of interaction.

Generalizing earlier models for Tukey's test are the “bundle-of-straight lines” model of Mandel (1959) and the functional model of Milliken and Graybill (1970), which assumes that the interaction is a known function of the block and treatment main-effects. Other methods and heuristics for block-treatment interaction in unreplicated studies are surveyed in the monograph Milliken & Johnson (1989).

Randomization and Layout of (RCBD)

Example designing layout of experiments for $\{t_1 , t_2 , t_3 , t_4, t_5, t_6\}$as treatments within three block by Generalized randomized block design(RCBD)?

Solve

designing block as fallowing:

B_I B_{II} B_{III}

Analysis of (RCBD) Variance

Treatments	Blocks					Σy_i	$\bar{y}_i$
	1	2	3	⋯	r		
t_1	y_{11}	y_{12}	y_{13}	⋯	y_{1r}	Σy_{t1}	$\bar{y}_{t1}$
t_2	y_{21}	y_{22}	y_{23}	⋯	y_{2r}	Σy_{t2}	$\bar{y}_{t2}$
t_3	y_{31}	y_{32}	y_{33}	⋯	y_{3r}	Σy_{t3}	$\bar{y}_{t3}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
t_t	y_{t1}	y_{t2}	y_{t3}	⋯	y_{tr}	Σy_{tr}	$\bar{y}_t$
Σy_j	Σy_{r1}	Σy_{r2}	Σy_{r3}	⋯	Σy_{tr}	$\ddot{y}$	$\bar{y}_i$
$\bar{y}_j$	$\bar{y}_{r1}$	$\bar{y}_{r2}$	$\bar{y}_{r3}$	⋯	$\bar{y}_r$	$\bar{y}_j$	$\bar{y}$

$$y_{ij} = \mu + t_i + R_j + e_{ij} \begin{Bmatrix} i = 1\,,2\,,3\,,\cdots,t \\ j = 1,2\,,3\,,\cdots,r \end{Bmatrix}$$

$$\mu = \bar{y} = \frac{\ddot{y}}{t * r}$$

$$e_{ij} = y_{ij} - \mu - t_i - R_i$$

$$e_{ij} = y_{ij} - \bar{y} - (\bar{y}_i - \bar{y}) - (\bar{y}_j - \bar{y}) = y_{ij} - \bar{y} - \bar{y}_i + \bar{y} - \bar{y}_j + \bar{y}$$

$$\therefore e_{ij} = y_{ij} - \bar{y}_i - \bar{y}_j + \bar{y}$$

Example: Use a Single Factor Experiment by (RCBD) ANOVA Table and L.S.D. to analysis the data which recorded underneath table ? , where $F_{0.05} = 3.26$, $F_{0.01} = 5.41$ and $t_{0.05} = 1.782$, $t_{0.01} = 2.681$:

R1	R2	R3	R4
$\emptyset_4 = 31$	$\emptyset_2 = 9$	$\emptyset_3 = 25$	$\emptyset_5 = 50$
$\emptyset_2 = 11$	$\emptyset_5 = 42$	$\emptyset_5 = 45$	$\emptyset_2 = 18$
$\emptyset_3 = 23$	$\emptyset_3 = 22$	$\emptyset_1 = 5$	$\emptyset_3 = 26$
$\emptyset_5 = 43$	$\emptyset_1 = 2$	$\emptyset_4 = 34$	$\emptyset_1 = 6$
$\emptyset_1 = 3$	$\emptyset_4 = 29$	$\emptyset_2 = 14$	$\emptyset_4 = 38$

Solve

$\emptyset_i$	Blocks				$\sum \emptyset_i$	$\bar{\emptyset}_i$
	R1	R2	R3	R4		
$\emptyset_1$	3	2	5	6	16	4
$\emptyset_2$	11	9	14	18	52	13
$\emptyset_3$	23	22	25	26	96	24
$\emptyset_4$	31	29	34	38	132	33
$\emptyset_5$	43	42	45	50	180	45
$\sum r_j$	111	104	123	138	$\ddot{y} = 476$	

S.O.V.	d.f.	S.S.	M.S.	Cal.F	Tab.F	
					0.05	0.01
Treatments($\emptyset$)	4	4171.2	1042.8	744.86^{**}	3.26	5.41
Blocks	3	133.2	44.4			
Error	12	16.8	1.4			
Total	19	4321.2				

$Total\ d.f = [(\emptyset) * (r)] - 1 = [(5) * (4)] - 1 = 20 - 1 = 19$

$Treatments\ d.f. = \emptyset - 1 = 5 - 1 = 4$

$Blocks\ d.f. = r - 1 = 4 - 1 = 3$

$Error\ d.f. = (r - 1) * (\emptyset - 1) = (3) * (4) = 12$

Or $Error\ d.f. = Total\ d.f - Blocks\ d.f. - Treatments\ d.f. = 19 - 3 - 4 = 12$

$$c.f. = \frac{(\ddot{y})^2}{(\emptyset) * (r)} = \frac{(476)^2}{(5) * (4)} = \frac{226576}{20} = 11328.8$$

$$Total\ SS(TSS) = \sum y_{ij}^2 - c.f. = [y_{11}^2 + y_{12}^2 + y_{13}^2 + \cdots + y_{54}^2] - c.f.$$
$$= 15650 - 11328.8 = 4321.2$$

$$SS_r = \frac{\sum r_j^2}{\emptyset} - c.f. = \frac{\sum r_1^2 + \sum r_2^2 + \sum r_3^2 + \sum r_4^2}{\emptyset} - c.f.$$
$$= \frac{(111)^2 + (104)^2 + (123)^2 + (138)^2}{4} - c.f.$$
$$= \frac{12321 + 10816 + 15129 + 19044}{5} - c.f. = \frac{57310}{5} - c.f.$$
$$= 11462 - 11328.8 = 133.2$$

$$Treatments\ SS = SS_{\emptyset} = \frac{\sum \emptyset_i^2}{r} - c.f. = \frac{\sum \emptyset_1^2 + \sum \emptyset_2^2 + \sum \emptyset_3^2 + \sum \emptyset_4^2 + \sum \emptyset_5^2}{r} - c.f.$$

$$= \frac{(16)^2 + (52)^2 + (96)^2 + (132)^2 + (180)^2}{4} - c.f.$$

$$= \frac{256 + 2704 + 9216 + 17424 + 32400}{4} - c.f. = \frac{62000}{4} - c.f.$$

$$= 15500 - 11328.8 = 4171.2$$

$$ErrorSS = SS_e = TSS - SS_r - SS_{\emptyset} = 4321.2 - 133.2 - 4171.2 = 16.8$$

$$MS_r = \frac{SS_r}{r-1} = \frac{133.2}{3} = 44.4$$

$$MS_{\emptyset} = \frac{SS_{\emptyset}}{\emptyset - 1} = \frac{4171.2}{4} = 1042.8$$

$$MS_e = \frac{SS_e}{(r-1)*(\emptyset-1)} = \frac{16.8}{(3)*(4)} = 1.4$$

$$Cal.F = \frac{MS_{\emptyset}}{MS_e} = \frac{1042.8}{1.4} = 744.86^{**}$$

$$LSD_{0.01} = t_{\alpha\ 0.01} * \sqrt{\frac{2MS_e}{r}} = 2.681 * \sqrt{\frac{2*(1.4)}{4}} = 2.681 * 0.84 = 2.24$$

$\bar{C}$	$\overline{\emptyset_5}$	$\overline{\emptyset_4}$	$\overline{\emptyset_3}$	$\overline{\emptyset_2}$	$\overline{\emptyset_1}$
	45	33	24	13	4
	a	b	c	d	e
		a	b	c	d
			a	b	c
				a	b