Completely Randomized Design (CRD):

In the design of experiments, completely randomized designs are for studying the effects of one primary factor without the need to take other nuisance variables into account. This article describes completely randomized designs that have one primary factor. The experiment compares the values of a response variable based on the different levels of that primary factor. For completely randomized designs, the levels of the primary factor are randomly assigned to the experimental units.

Randomization and Layout of CRD

Example designing layout of experiments for $\{P1, P2, P3, P4\}$ as treatments with five replications by Completely Randomized Design (CRD)?

Solve

$$n = (t) * (r) = (4) * (5) = 20$$

$1 \rightarrow P_2$	$2 \rightarrow P_1$	$3 \rightarrow P_4$	$4 \rightarrow P_3$
$5 \rightarrow P_4$	$6 \rightarrow P_3$	$7 \rightarrow P_1$	$8 \rightarrow P_2$
$9 \rightarrow P_3$	$10 \rightarrow P_4$	$11 \rightarrow P_4$	$12 \rightarrow P_3$
$13 \rightarrow P_2$	$14 \rightarrow P_3$	$15 \rightarrow P_1$	$16 \rightarrow P_3$
$17 \rightarrow P_1$	$18 \rightarrow P_2$	$19 \rightarrow P_1$	$20 \rightarrow P_4$

Analysis of CRD Variance

Treatments(t_i)	Observations y_{ij}					$\sum y_i$	$\bar{y}_i$
t_1	y_{11}	y_{12}	y_{13}	⋯	y_{1r}	$\dot{y}_1$	$\overline{y_1}$
t_2	y_{21}	y_{22}	y_{23}	⋯	y_{2r}	$\dot{y}_2$	$\overline{y_2}$
t_3	y_{31}	y_{32}	y_{33}	⋯	y_{3r}	$\dot{y}_3$	$\overline{y_3}$
⋮	⋮	⋮	⋮	⋯	⋮	⋮	⋮
t_t	y_{t1}	y_{t2}	y_{t3}	⋯	y_{tr}	$\dot{y}_t$	$\bar{y}_t$
						$\ddot{y}_t$	

$$\bar{y}_i = \frac{\sum y_i}{r} \; in \; addition \; to \; \ddot{y}_t = \sum y_{ij}$$

$$y_{ij} = \bar{\mu} + t_i + e_{ij} \; \{^{i=1,2,3,\ldots t}_{j=1,2,3,\ldots r}$$

$$\bar{\mu} = \frac{\ddot{y_{ij}}}{(t) * (r)}$$

$$\bar{t_i} = \bar{y_i} - \bar{y}$$

$$e_{ij} = y_{ij} - y_i$$

ANOVA table:

S.O.V.	d.f.	S.S.	M.S.	Cal.F	Tab.F	
					0.05	0.01
Treatments						
Experiment Error						
Total						

Example Use a Single Factor Experiment by Completely Randomized Design (CRD) with Equal Replications ANOVA Table and L.S.D. to analysis the data which recorded underneath table , $F_{0.05} = 4.07$, $F_{0.01} = 7.59$ and $t_{0.05} = 2.306$, $t_{0.01} = 3.355$:

$K_{22} = 25$	$K_{42} = 55$	$K_{11} = 13$	$K_{31} = 37$
$K_{32} = 39$	$K_{43} = 57$	$K_{41} = 53$	$K_{23} = 27$
$K_{12} = 15$	$K_{21} = 23$	$K_{33} = 41$	$K_{13} = 19$

Solve

K_i	y_{ij}			$\sum K_i$	$\bar{K}_i$
	r_1	r_2	r_3		
K_1	13	15	19	47	15.67
K_2	23	25	27	75	25.00
K_3	37	39	41	117	39.00
K_4	53	55	57	165	55.00
				$404\ddot{y} =$	

S.O.V.	d.f.	S.S.	M.S.	Cal.F	Tab.F	
					0.05	0.01
Treatments	3	2648	882.67	165.50**	4.07	7.59
Experiment Error	8	42.67	5.33			
Total	11	2690.67				

Total d.f. = (r * t) – 1 = (3*4) – 1 = 11

Treatments d.f. = t – 1 = 4 – 1 = 3

Error d.f. = t *(r-1) = 4*(3-1) = 8

$$C.F. = \frac{(\ddot{y})^2}{(t)*(r)} = \frac{(404)^2}{(4)*(3)} = \frac{163216}{12} = 113601.33$$

$$TSS = \sum y_{ij}^2 - C.F. = y_{11}^2 + y_{12}^2 + \cdots + y_{43}^2 - C.F. = 16292 - 113601.33$$

$$= 2690.67$$

$$SS_t = \frac{\sum y_i^2}{r} - C.F. = \frac{y_1^2 + y_2^2 + y_3^2 + y_4^2}{r} - C.F. = \frac{48748}{3} - C.F.$$

$$= 16249.33 - 113601.33 = 2648$$

$$SS_e = TSS - SS_t = 2690.67 - 2648 = 42.67$$

$$MS_t = \frac{SS_t}{t-1} = \frac{2648}{3} = 882.67$$

$$MS_e = \frac{SS_e}{t*(r-1)} = \frac{42.67}{4*(3-1)} = 5.33$$

$$Cal.F = \frac{MS_t}{MS_e} = \frac{882.67}{5.33} = 165.50^{**}$$

$$LSD_{0.05} = t_{\alpha\ 0.05} * S_{\bar{y}_i - \bar{y}_i} = t_{\alpha\ 0.05} * \sqrt{\frac{2MS_e}{r}} = 3.355 * \sqrt{\frac{2*(5.33)}{3}} = 6.33$$

Treatments	t_4	t_3	t_2	t_1
$\bar{y}_i$	55	39	25	15.33
	a	b	c	d
		a	b	c
			a	b