Orbital elements

Orbital elements are the parameters required to uniquely identify a specific orbit. In celestial mechanics these elements are generally considered in classical two-body systems, where a Kepler orbit is used. There are many different ways to mathematically describe the same orbit, but certain schemes, each consisting of a set of six parameters, are commonly used in astronomy and orbital mechanics.

A real orbit (and its elements) changes over time due to gravitational perturbations by other objects and the effects of relativity. A Keplerian orbit is merely an idealized, mathematical approximation at a particular time.

Keplerian elements

The traditional orbital elements are the six Keplerian elements, after Johannes Kepler and his laws of planetary motion.

When viewed from an inertial frame, two orbiting bodies trace out distinct trajectories. Each of these trajectories has its focus at the common center of mass. When viewed from a non-inertial frame centered on one of the bodies, only the trajectory of the opposite body is apparent; Keplerian elements describe these non-inertial trajectories. An orbit has two sets of Keplerian elements depending on which body is used as the point of reference. The reference body is called the primary, the other body is called the secondary. The primary does not necessarily possess more mass than the secondary, and even when the bodies are of equal mass, the orbital elements depend on the choice of the primary.

The main two elements that define the shape and size of the ellipse:

Eccentricity (e)—shape of the ellipse, describing how much it is elongated compared to a circle (not marked in diagram).

Semimajor axis (a)—the sum of the periapsis and apoapsis distances divided by two. For circular orbits, the semimajor axis is the distance between the centers of the bodies, not the distance of the bodies from the center of mass.

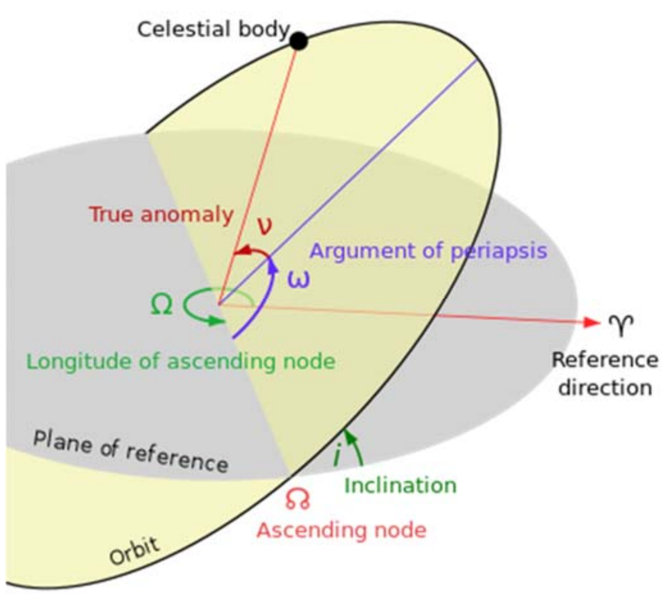
Two elements define the orientation of the orbital plane in which the ellipse is embedded:

Inclination (i)—vertical tilt of the ellipse with respect to the reference plane, measured at the ascending node (where the orbit passes upward through the reference plane, the green angle i in the diagram). Tilt angle is measured perpendicular to line of intersection between orbital plane and reference plane. Any three points on an ellipse will define the ellipse orbital plane. The plane and the ellipse are both two-dimensional objects defined in three-dimensional space.

Longitude of the ascending node (Ω)—horizontally orients the ascending node of the ellipse (where the orbit passes upward through the reference plane, symbolized by Ω) with respect to the reference frame's vernal point (symbolized by ♈). This is measured in the reference plane, and is shown as the green angle Ω in the diagram.

And finally:

Argument of periapsis (ω) defines the orientation of the ellipse in the orbital plane, as an angle measured from the ascending node to the periapsis (the closest point the satellite object comes to the primary object around which it orbits, the blue angel ω in the diagram).



True anomaly (ν, θ, or f) at epoch (M_0) defines the position of the orbiting body along the ellipse at a specific time (the "epoch").

The mean anomaly is a mathematically convenient "angle" which varies linearly with time, but which does not correspond to a real geometric angle. It can be converted into the true anomaly ν, which does represent the real geometric angle in the plane of the ellipse, between periapsis (closest approach to the central body) and the position of the orbiting object at any given time. Thus, the true anomaly is shown as the red angle ν in the diagram, and the mean anomaly is not shown.

The angles of inclination, longitude of the ascending node, and argument of periapsis can also be described as the Euler angles defining the orientation of the orbit relative to the reference coordinate system.

Note that non-elliptic trajectories also exist, but are not closed, and are thus not orbits. If the eccentricity is greater than one, the trajectory is a hyperbola. If the eccentricity is equal to one and the angular momentum is zero, the trajectory is radial. If the eccentricity is one and there is angular momentum, the trajectory is a parabola.

Required parameters

Given an inertial frame of reference and an arbitrary epoch (a specified point in time), exactly six parameters are necessary to unambiguously define an arbitrary and unperturbed orbit.

This is because the problem contains six degrees of freedom. These correspond to the three spatial dimensions which define position (x, y, z in a Cartesian coordinate system), plus the velocity in each of these dimensions. These can be described as orbital state vectors, but this is often an inconvenient way to represent an orbit, which is why Keplerian elements are commonly used instead.

Sometimes the epoch is considered a "seventh" orbital parameter, rather than part of the reference frame. If the epoch is defined to be at the moment when one of the elements is zero, the number of unspecified elements is reduced to five. (The sixth parameter is still necessary to define the orbit; it is merely numerically set to zero by

convention or "moved" into the definition of the epoch with respect to real-world clock time.)

Alternative parametrizations

Keplerian elements can be obtained from orbital state vectors (a three-dimensional vector for the position and another for the velocity) by manual transformations or with computer software.

Other orbital parameters can be computed from the Keplerian elements such as the period, apoapsis, and periapsis. (When orbiting the Earth, the last two terms are known as the apogee and perigee.) It is common to specify the period instead of the semi-major axis in Keplerian element sets, as each can be computed from the other provided the standard gravitational parameter, GM, is given for the central body.

Instead of the mean anomaly at epoch, the mean anomaly M, mean longitude, true anomaly ν_0, or (rarely) the eccentric anomaly might be used.

Using, for example, the "mean anomaly" instead of "mean anomaly at epoch" means that time t must be specified as a seventh orbital element. Sometimes it is assumed that mean anomaly is zero at the epoch (by choosing the appropriate definition of the epoch), leaving only the five other orbital elements to be specified.

Different sets of elements are used for various astronomical bodies. The eccentricity, e, and either the semi-major axis, a, or the distance of periapsis, q, are used to specify the shape and size of an orbit. The angle of the ascending node, Ω, the inclination, i, and the argument of periapsis, ω, or the longitude of periapsis, ϖ, specify the orientation of the orbit in its plane. Either the longitude at epoch, L_0, the mean anomaly at epoch, M_0, or the time of perihelion passage, T_0, are used to specify a known point in the orbit. The choices made depend whether the vernal equinox or the node are used as the primary reference. The semi-major axis is known if the mean motion and the gravitational mass are known.

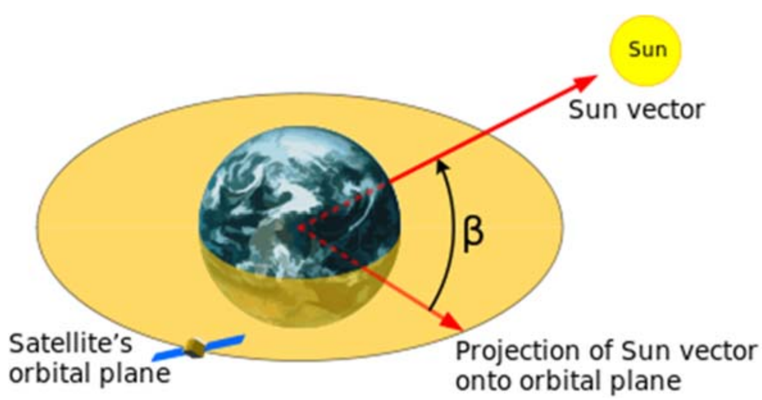
It is also quite common to see either the mean anomaly (M) or the mean longitude (L) expressed directly, without either M_0 or L_0 as intermediary steps, as

a polynomial function with respect to time. This method of expression will consolidate the mean motion (n) into the polynomial as one of the coefficients. The appearance will be that L or M are expressed in a more complicated manner, but we will appear to need one fewer orbital element.

Mean motion can also be obscured behind citations of the orbital period *P*.

Beta angle

The beta angle (β) is a measurement that is used most notably in orbital spaceflight. The beta angle determines the percentage of time that a satellite in low Earth orbit (LEO) spends in direct sunlight, absorbing solar energy. The term is defined as the angle between the orbital plane of the satellite and the vector to the Sun (i.e., the direction from which the Sun is shining). The beta angle is the smaller of the two angles between the Sun vector and the plane of the object's orbit. The beta angle does not define a unique orbital plane; all satellites in orbit with a given beta angle at a given altitude have the same exposure to the Sun, even though they may be orbiting in completely different planes around Earth.



The beta angle varies between +90° and −90°, and the direction in which the satellite orbits its primary body determines whether the beta angle sign is positive or negative. An imaginary observer standing on the Sun defines a beta angle as positive if the satellite in question orbits in a counterclockwise direction and negative if it revolves clockwise. The maximum amount of time that a satellite in a normal LEO mission can spend in Earth's shadow occurs at a beta angle of 0°. A satellite in such an orbit spends no more than 59% of its period in sunlight.