AL-Karkh University of Science
College of Geophysics and Remote Sensing
Department of Remote Sensing

Electromagnetism

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Electromagnetic Spectrum (1)

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Light and the Electromagnetic Spectrum

- **The terms *light*, *radiation*, and *electromagnetic wave* can all be used to explain the same concept**

- **Light comes in many forms and it took physicists some time to realize that x-rays, visible light, radio waves, etc. are all the same phenomena**

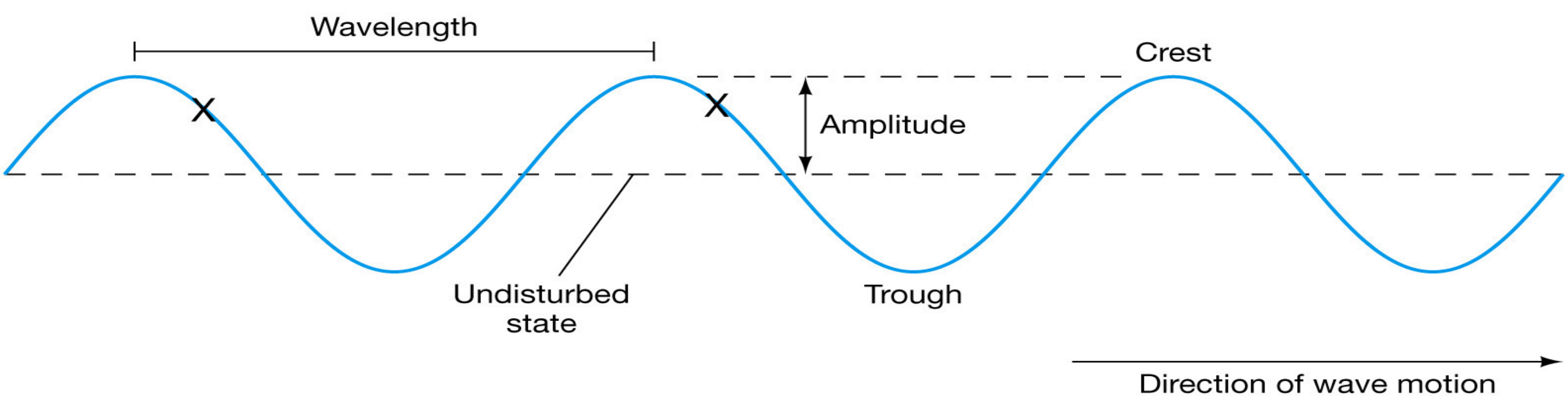
- **By using these different tools, astronomers are able to gain a lot of information on various objects**

Light as a Wave

- **One way to think about light is as a traveling wave**
- **A wave is just a disturbance in some medium (water, air, space)**
- **A wave travels through a medium but does not transport material**
- **A wave can carry both energy and information**

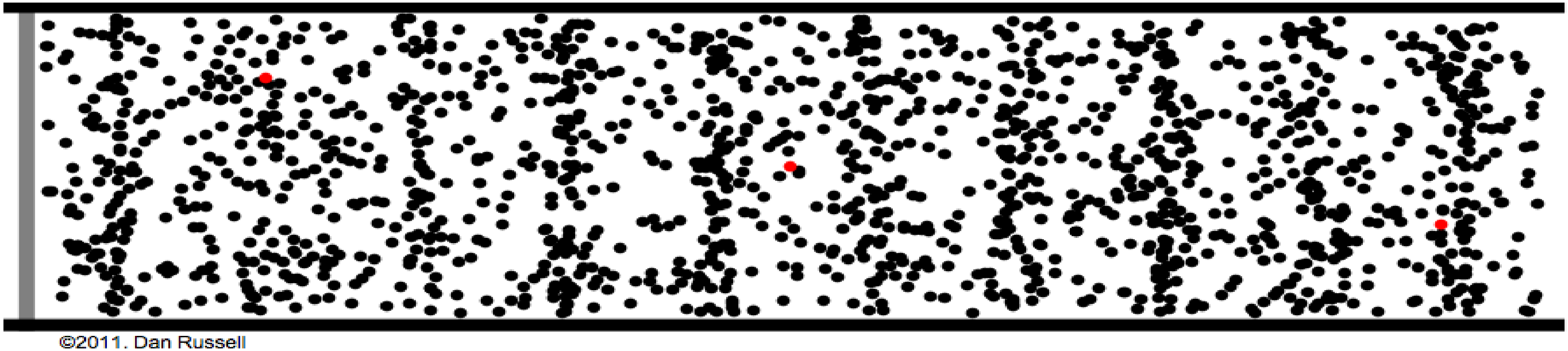
Wave Terminology

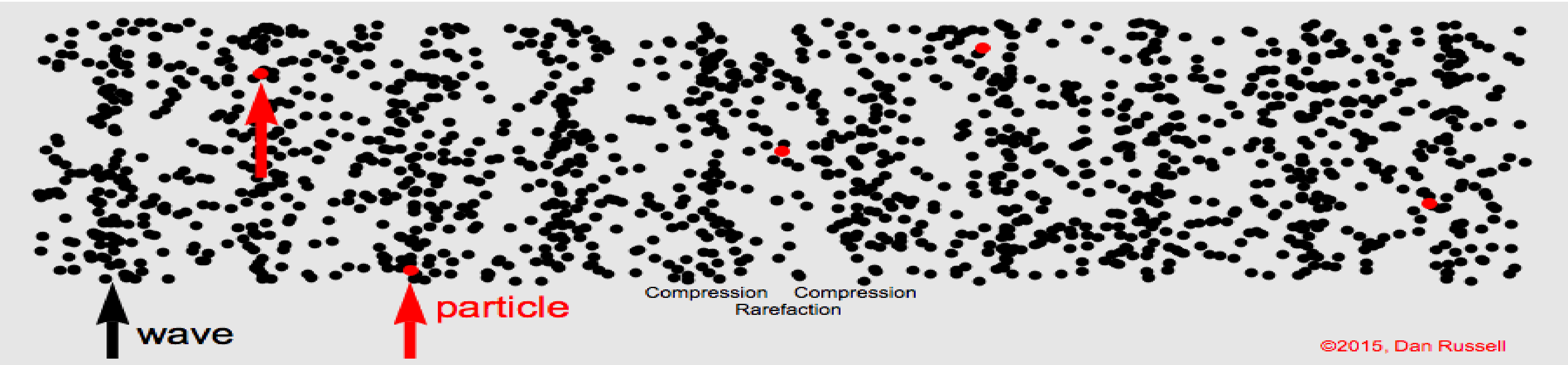
- *Wavelength* - **distance between two like points on the wave**
- *Amplitude* - **the height of the wave compared to undisturbed state**
- *Period* - **the amount of time required for one wavelength to pass**
- *Frequency* - **the number of waves passing in a given amount of time**



Longitudinal Waves

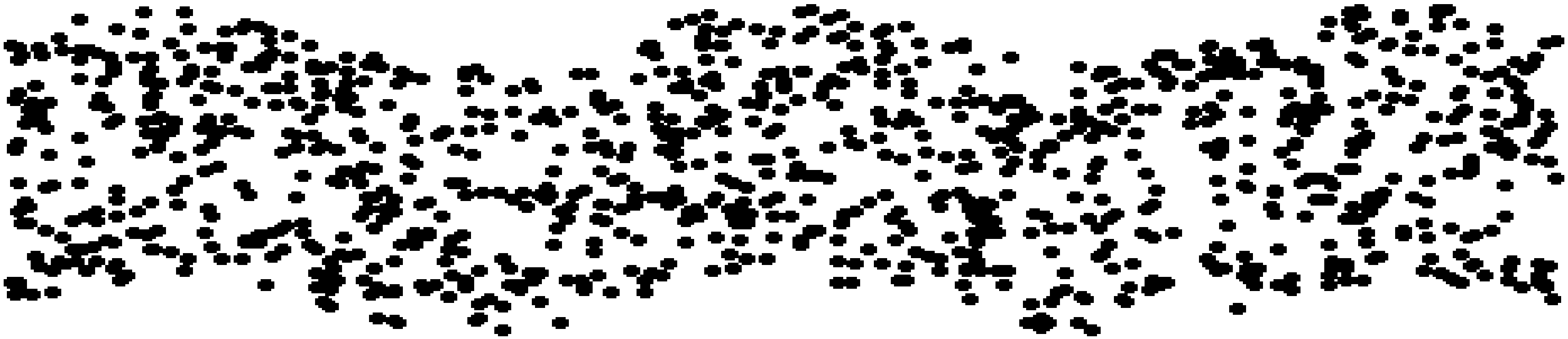
In a longitudinal wave the particle displacement is parallel to the direction of wave propagation. The animation at right shows a one-dimensional longitudinal plane wave propagating down a tube. The particles do not move down the tube with the wave; they simply oscillate back and forth about their individual equilibrium positions.



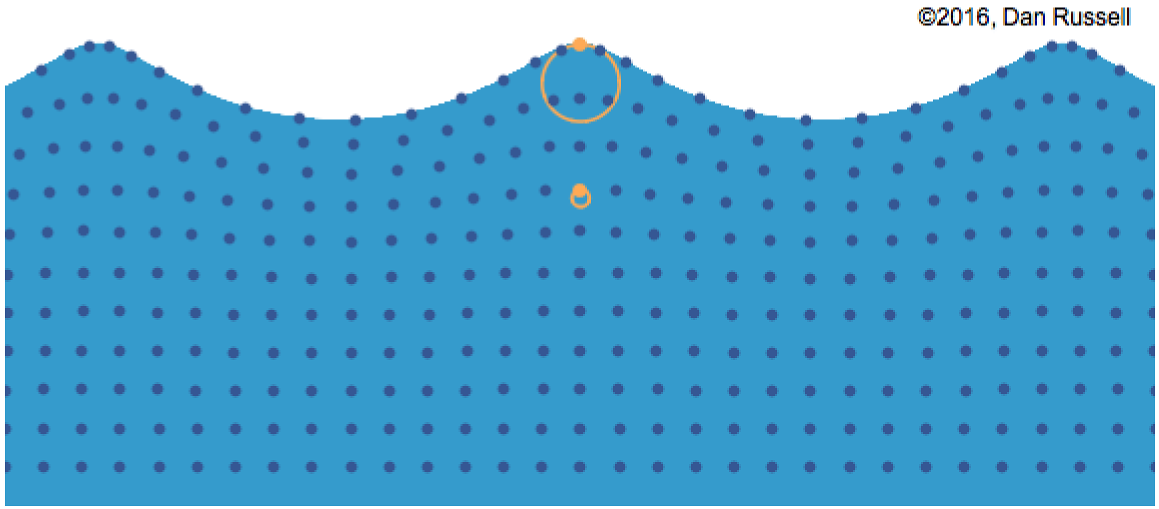


Transverse Waves

In a transverse wave the particle displacement is perpendicular to the direction of wave propagation. The animation below shows a one-dimensional transverse plane wave propagating from left to right.



Water waves are an example of waves that involve a combination of both longitudinal and transverse motions.



Longitudinal waves are waves in which the displacement of the medium is in the same direction as, or the opposite direction to, the direction of propagation of the wave.
Transverse wave is a moving wave that consists of oscillations occurring perpendicular (right angled) to the direction of energy transfer (or the propagation of the wave).

- **Most waves are either longitudinal or transverse.**
- **Sound waves are longitudinal.**
- **But all electromagnetic waves are transverse…**

Wave Relationships

- **Notice from the definitions we can relate the properties of a wave to one another**

$$frequency = {}^{1}\!/_{period}$$

$$velocity = \frac{wavelength}{period} = wavelength \times frequency$$

Wave Relationships

- **Frequency is usually expressed in the unit of Hertz**
  - **This unit is named after a German scientist who studied radio waves**

$$1Hz = \frac{1}{s}$$

  - **For example, if a wave has a period of 10 seconds, the frequency of the wave would be 1/10 Hz, or 0.1 Hz**

- **Note that light is always traveling at the same speed ($c \sim 3 \times 10^8$ m/s)**

  - **Remember:** *velocity* = *wavelength* x *frequency*

    - **If frequency increases, wavelength decreases**
    - **If frequency decreases, wavelength increases**

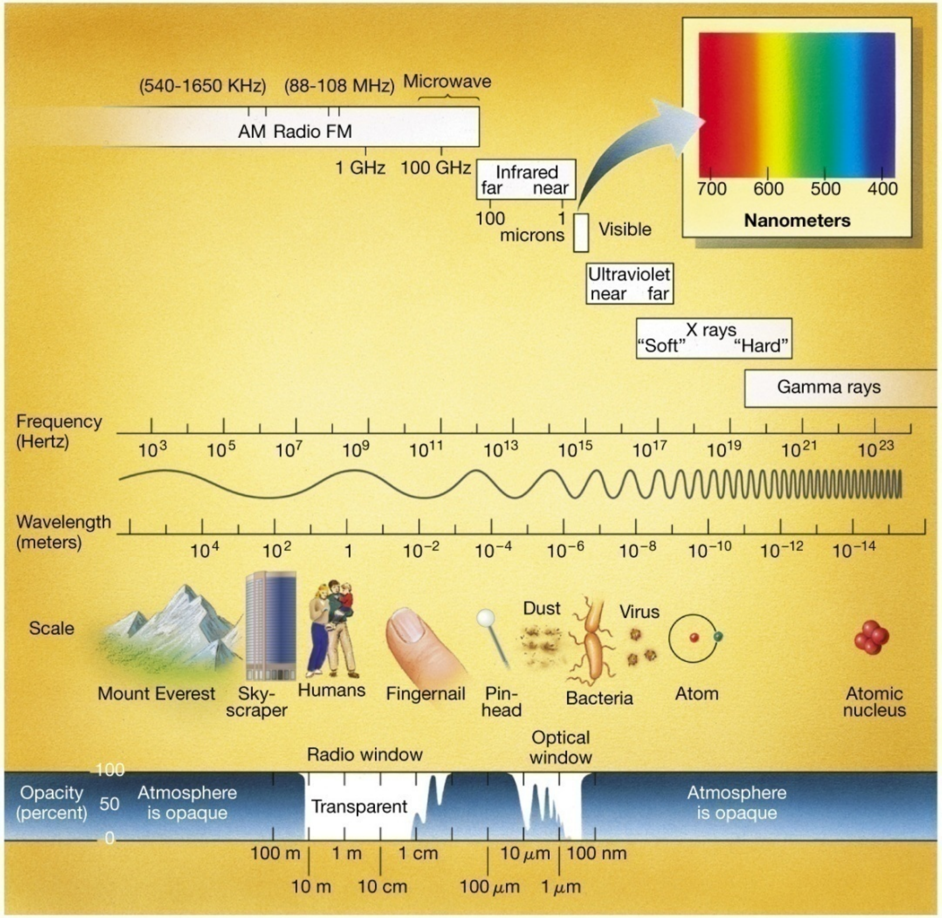
The Electromagnetic Spectrum

- Human eyes are only able to process information from the visible part of the spectrum

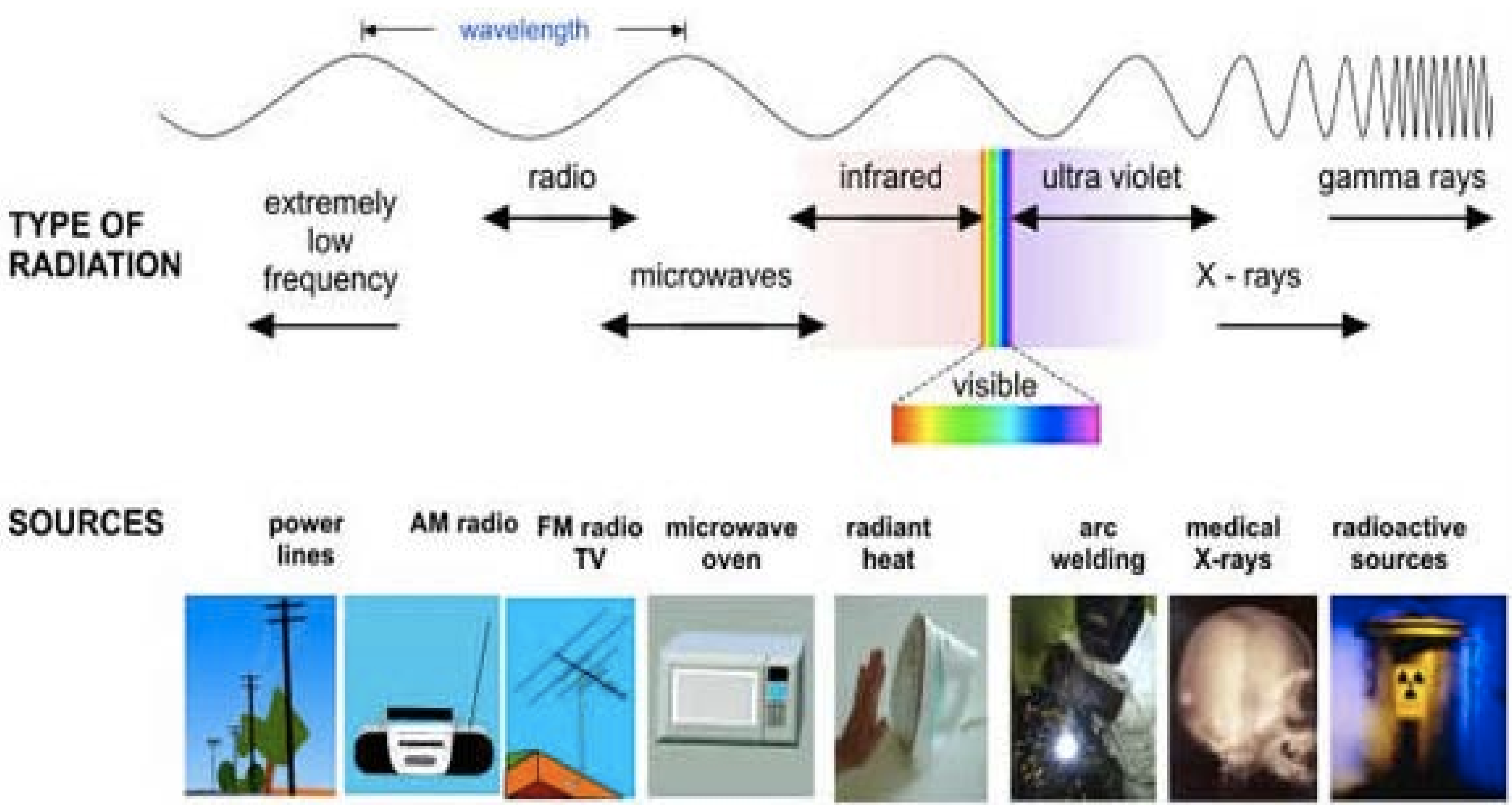
- Toward longer wavelengths, the spectrum includes infrared light, microwaves, and radio

- Toward shorter wavelengths, the spectrum includes ultraviolet light, X-rays, and gamma rays

- All of these are forms of electromagnetic radiation



What does this picture show about the relationship between frequency and wavelength?



Electromagnetic waves

- **Produced by the movement of electrically charged particles.**
- **Can travel in a "vacuum" (they do NOT need a medium).**
- **Travel at the speed of light.**
- **Also known as EM waves.**

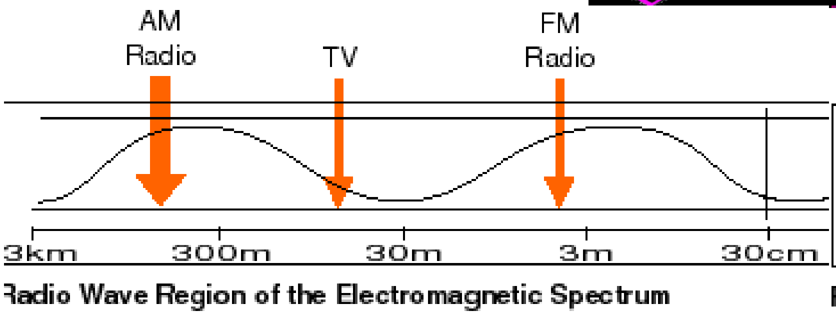
Types of EM Radiation

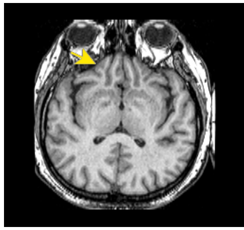
❖ **Radio waves**

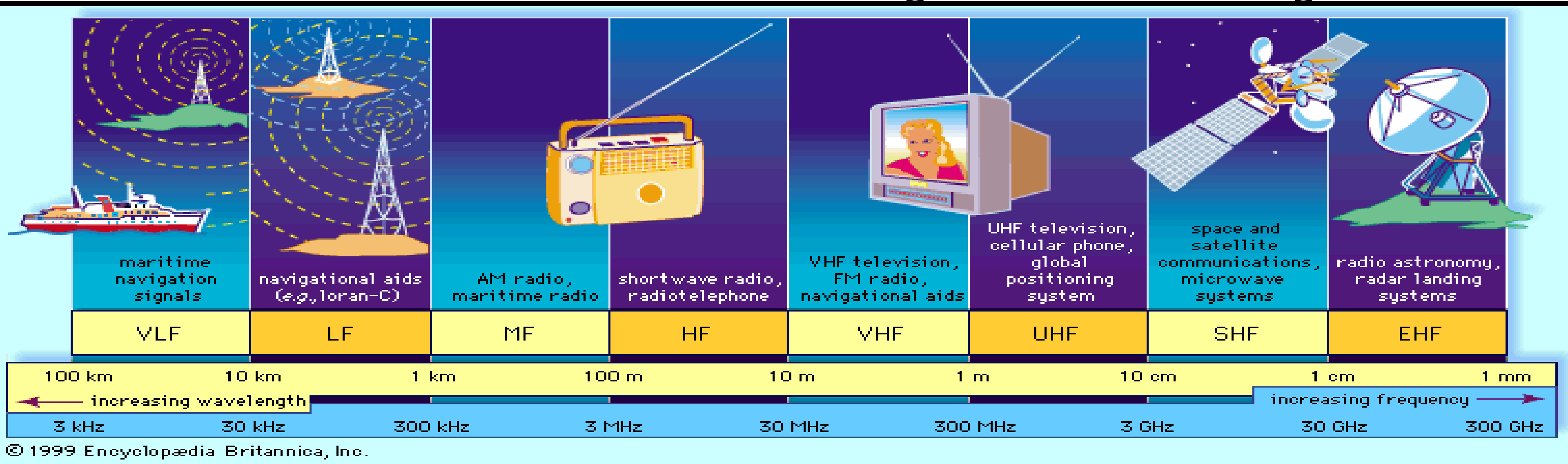
Longest wavelength EM waves

Uses:

- **TV broadcasting**
- **AM and FM broadcast radio**
- **Heart rate monitors**
- **Cell phone communication**
- **MRI (MAGNETIC RESONACE IMAGING)**
  - **Uses Short wave radio waves with a magnet to create an image**

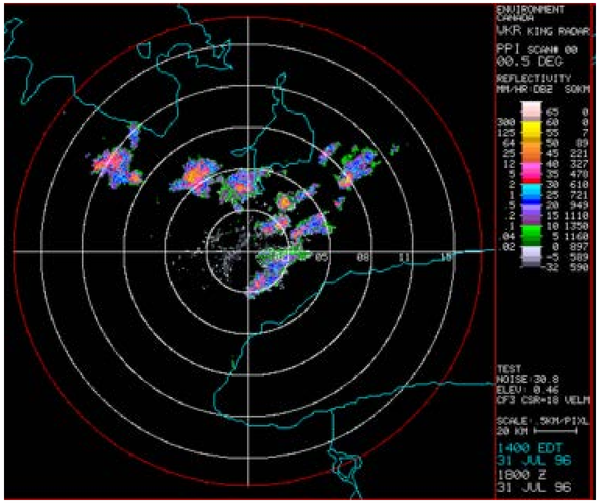


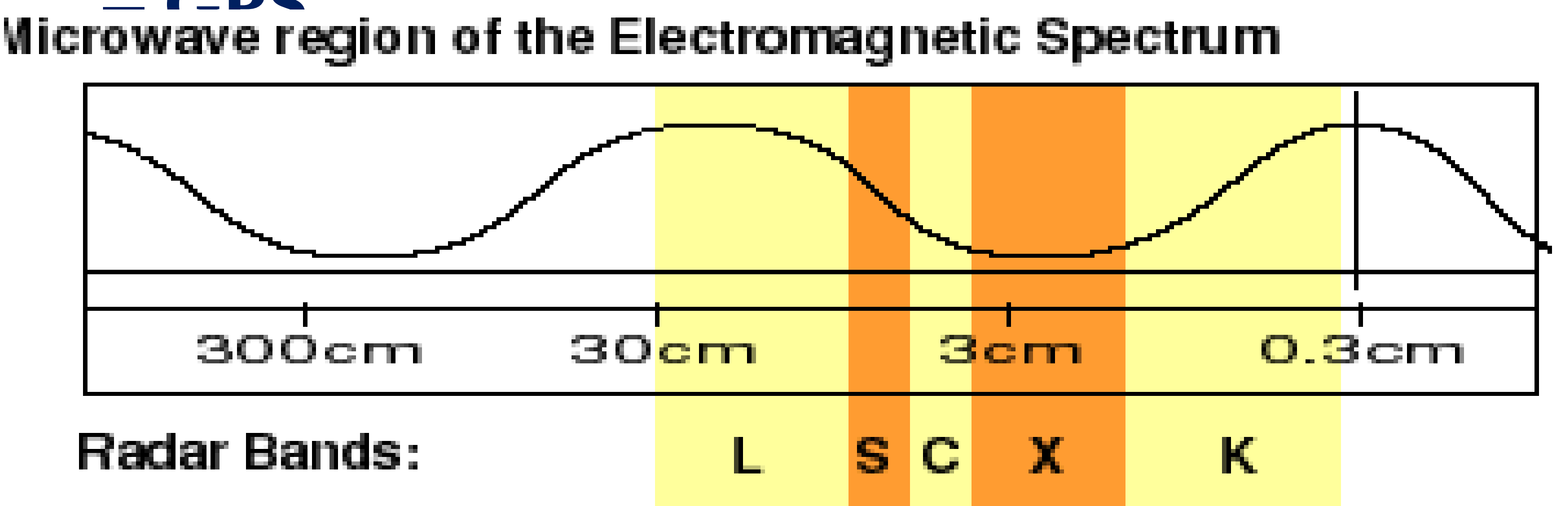




Microwaves

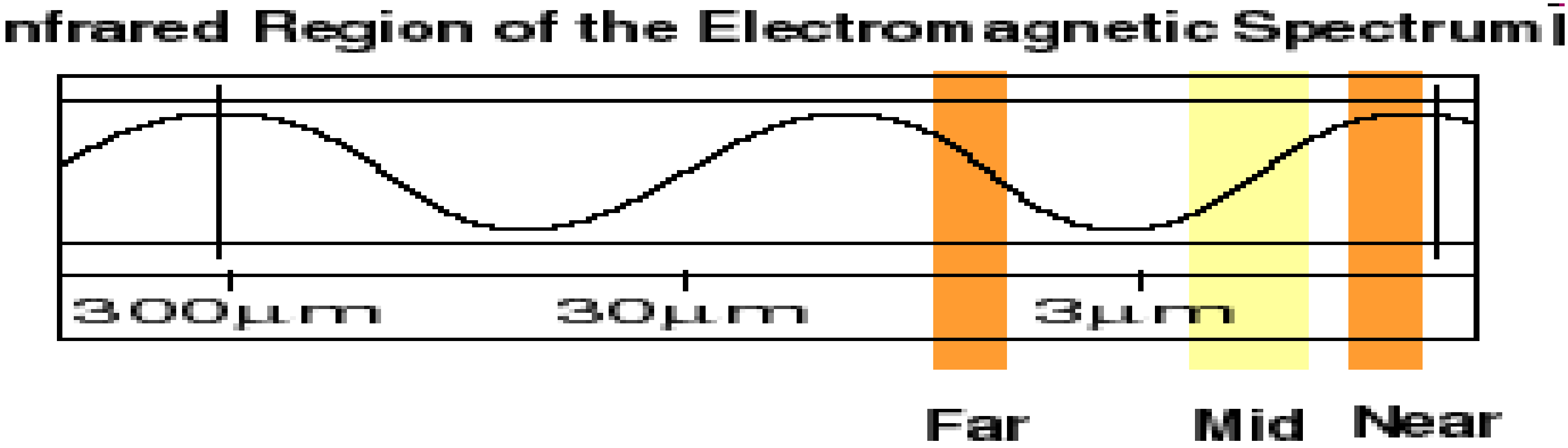
- **Wavelengths from 1 mm- 1 m**
- **Uses:**
  - – **Microwave ovens**
  - – **Bluetooth headsets**
  - – **Broadband Wireless Internet**
  - – **Radar**
  - – **GPS**





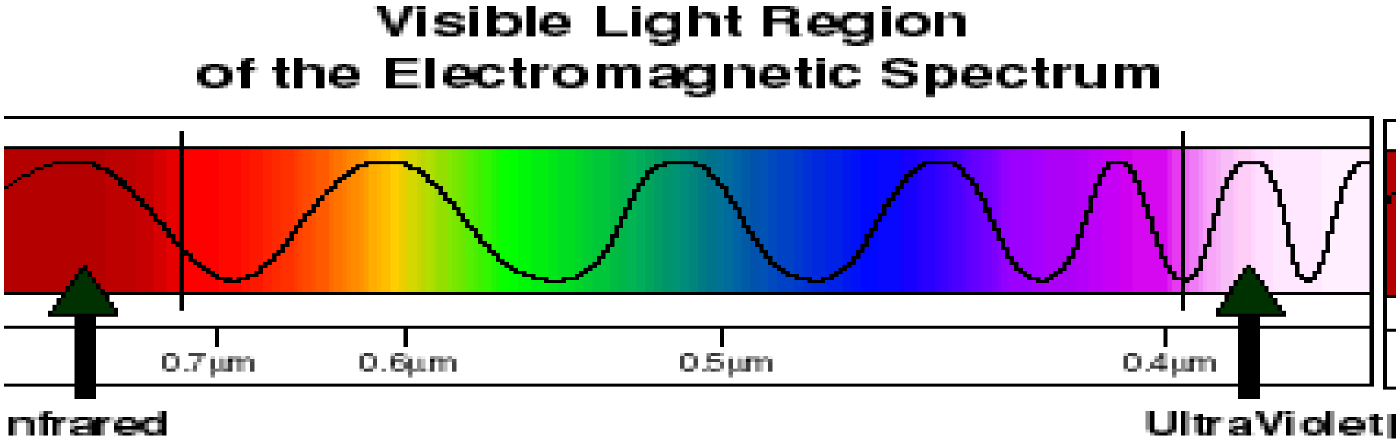
Infrared Radiation

- **Wavelengths in between microwaves and visible light**
- **Uses:**
  - **Night vision goggles**
  - **Remote controls**



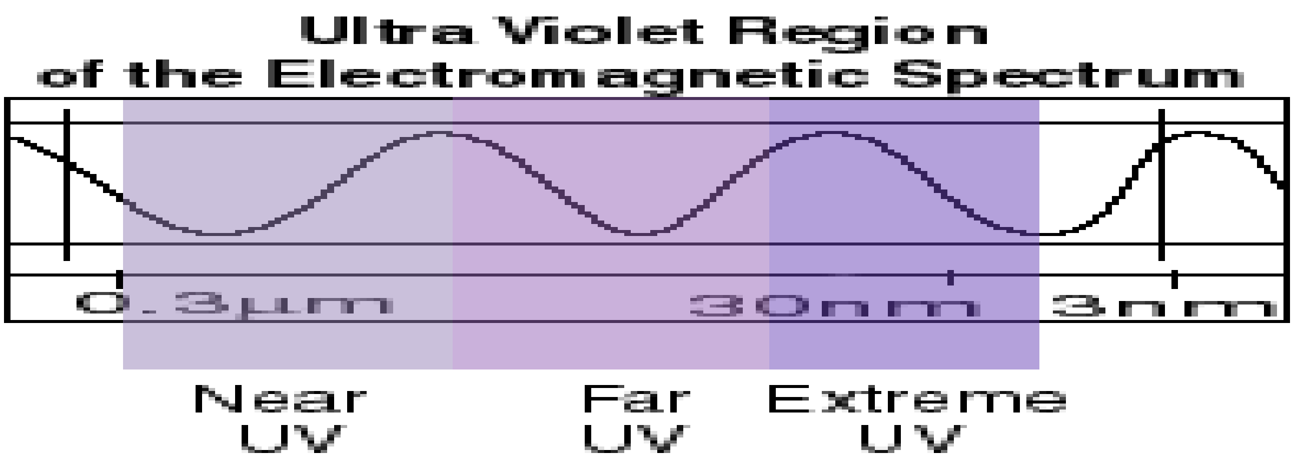
Visible light

- Only type of EM wave able to be detected by the human eye
- Violet is the highest frequency light
- Red light is the lowest frequency light



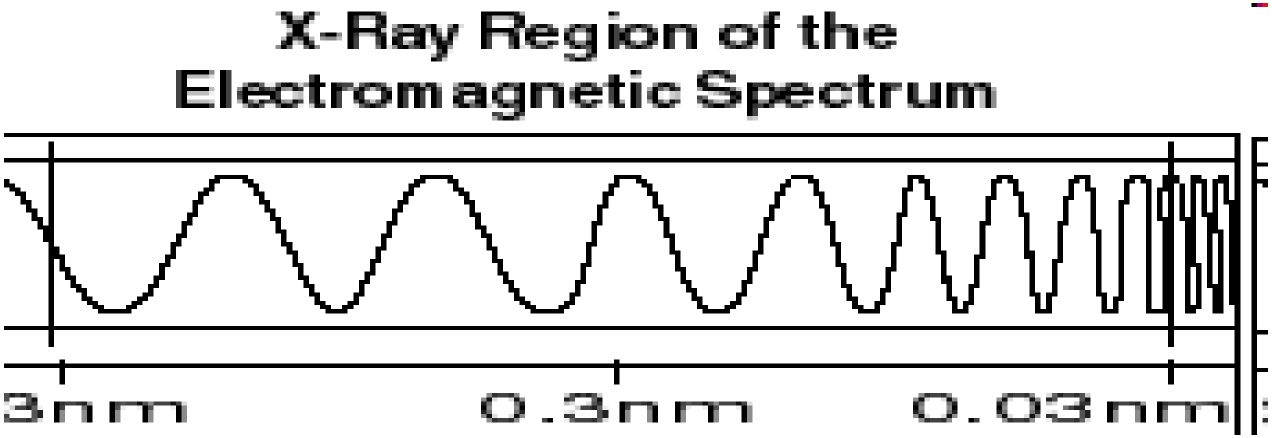
Ultraviolet

- Shorter wavelengths than visible light
- Uses:
  - Black lights
  - Security images on money
  - Harmful to living things
    - Used to sterilize medical equipment
    - Too much causes sun burn



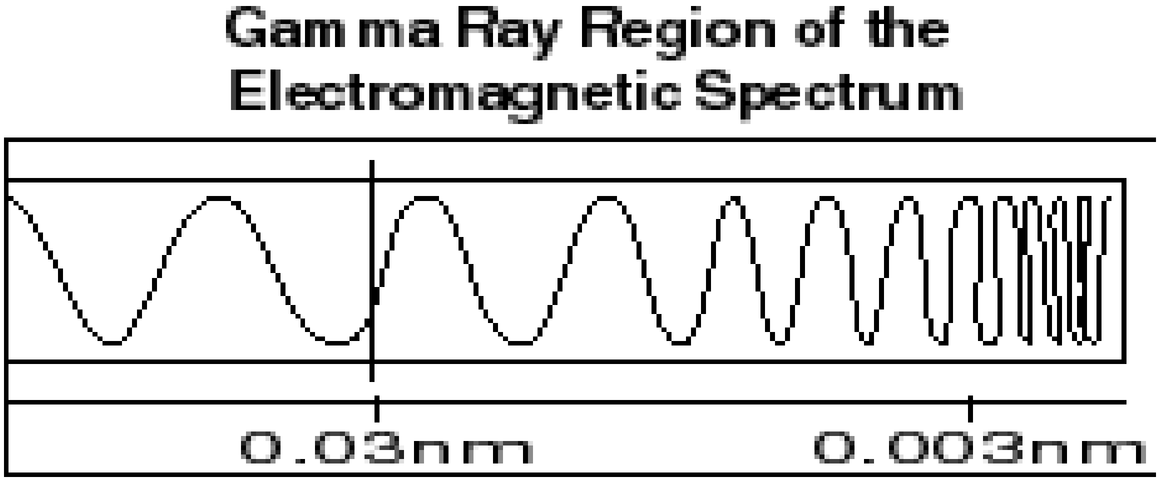
X-rays

- **Tiny wavelength, high energy waves**
- **Uses:**
  - **– Medical imaging**
  - **– Airport security**
    - **Moderate dose can damaging to cells**



Gamma Rays

- **Smallest wavelengths, highest energy EM waves**
- **Uses**
  - **Sterilizes medical equipment**
  - **Cancer treatment to kill cancer**
    - **Kills nearly all living cells.**



The Electric Field

- **Since EM waves consist of electric and magnetic fields, the physical nature of these fields is reviewed here.**
- **By the term ‘field’, is simply meant any quantity which exists at every point in space, e.g., a ‘temperature field’. More specifically, electric and magnetic fields, or taken together, the electromagnetic field, is associated with the electromagnetic interaction or force, between charged particles.**

- **(The letter q (or Q) is used as a general symbol for electric charge.)**
- **Equivalently, the electric force per unit charge, i.e., FE/|q| at each point in space and time is constant, and independent of the size of |q|. This ratio is used as a measure of the strength, E, of the electric field at each point, i.e., the Electric Field Strength is given by:**

$$\mathbf{E = FE/|q|}$$

This is usually written:

$$\mathbf{FE = E \cdot |q|}$$

The S.I. units of E are obviously N/C.

- **The electric field has magnitude and direction, i.e., it is a vector quantity.**
- **where there is an electric field, all positive charges will experience an electric force in the same direction and all negative charges will experience a force in the opposite direction.**
- **Of these two opposite directions, the direction of the force on positive charges is chosen arbitrarily, i.e., the direction of the electric field vector E is defined as the direction of the electric force on a positive charge.**

Example 1-1:

At a certain point, a charge $q = +\ 1.0\ \mu C$ experiences an electric force $\boldsymbol{F}_E$ of 2.0 N toward the south-west.

a) What is the electric field at this point?

The magnitude of $\boldsymbol{E}$ is: $E = F_E/|q| = 2.0/1.0\times10^{-6}\ C = 2.0\times10^{6}\ N/C$; i.e.,
$\boldsymbol{E} = 2.0\times10^{6}\ N/C$ toward the south-west.

b) If an electron ($|q| = e = 1.60\times10^{-19}$ C) is placed at the above spot, what electric force $\boldsymbol{F}_E$ acts on it?

$$
\begin{aligned}
|\boldsymbol{F}_E| \text{ on the electron} &= |q|E = (1.60\times10^{-19}\ C)(2.0\times10^{6}\ N/C) \\
&= 3.2\times10^{-13}\ N
\end{aligned}
$$

Since the electron is negative, the force on it is opposite to the direction of $\boldsymbol{E}$, i.e.,

$\boldsymbol{F}_E = 3.2\times10^{-13}$ N toward the north-east.

- Recall that a ‘volt’, the unit of electrical potential (the potential energy per unit charge), is equivalent to a joule/coulomb (J/C) and a joule is a ‘newton-meter’.
- Verify that the units for electric field, i.e., N/C, are equivalent to volts/meter (V/m) (See Problem 1-2).
- Electric field strengths are frequently expressed in these alternate units.
- In Example 1 we could say that E = 2.0×106 V/m = 2.0×103 kV/m = 2.0 MV/m.
- Since an electric field can exert a force on a charged particle, it can cause it to move, i.e., it can do work on the particle and give it energy.
- Thus, electric fields possess energy which must of course come from the source of the fields, which have not yet been discussed.
- Experiments and theory show that the electric field energy density uE, i.e., the field energy per unit volume (J/m3) at any point is proportional to the square of the field strength (E2) at that point. Thus, for fields in vacuum:

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$$u_E = \tfrac{1}{2}\varepsilon_0 E^2$$

where $\tfrac{1}{2}\varepsilon_0$ is a proportionality constant. (It is simply an historical convention that this constant is written with the ½ factored out.) The constant ε_0 is called the 'permittivity of a vacuum' and (as you should be able to show; see Problem 1-3) has units of $C^2/N\cdot m^2$; it is an important electrical constant of nature; its value is:

$$\varepsilon_0 = 8.85\times10^{-12}\ C^2/N\cdot m^2$$

Example 1-2:

In Example 1-1, the field $E = 2.0\times10^6$ N/C. What is the energy density at that point?

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}(8.85\times10^{-12}\ C^2/Nm^2)(2.0\times10^6\ N/C)^2 = 18\ J/m^3$$

An electric field is produced in two ways:

(a) by charged particles.

(b) by magnetic fields that are changing with time (electromagnetic induction)

2.1 *COULOMB'S LAW*

It is found experimentally that the force between two stationary electric *point* charges Q_a and Q_b (a) acts along the line joining the two charges, (b) is proportional to the product Q_aQ_b, and (c) is inversely proportional to the square of the distance r separating the charges.

If the charges are extended, the situation is more complicated in that the "distance between the charges" has no definite meaning. Moreover, the presence of Q_b can modify the charge distribution within Q_a, and vice versa, leading to a complicated variation of force with distance.

We thus have *Coulomb's law* for stationary point charges:

$$\mathbf{F}_{ab} = \frac{1}{4\pi\epsilon_0}\frac{Q_aQ_b}{r^2}\mathbf{r}_1, \tag{2-1}$$

where $\mathbf{F}_{ab}$ is the force exerted *by* Q_a *on* Q_b, and $\mathbf{r}_1$ is a unit vector pointing in the direction *from* Q_a *to* Q_b, as in Fig. 2-1. The force is repulsive if Q_a and Q_b are of the same sign; it is attractive if they are of different signs. As usual, F is measured in newtons, Q in *coulombs*, and r in meters. The constant ϵ_0

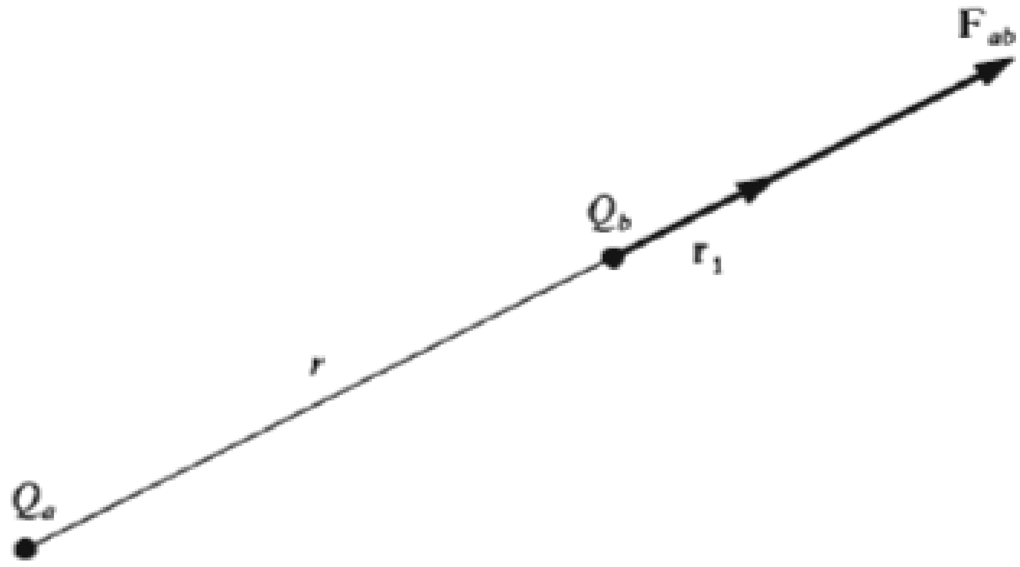


Figure 2-1 Charges Q_a and Q_b separated by a distance r. The force exerted on Q_b by Q_a is $\mathbf{F}_{ab}$ and is in the direction of $\mathbf{r}_1$ along the line joining the two charges.

is the *permittivity of free space*:

$$\epsilon_0 = 8.854\,187\,82 \times 10^{-12} \text{ farad/meter.}^\dagger \tag{2-2}$$

$$\varepsilon_0 = 8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}$$

2.2 THE ELECTRIC FIELD INTENSITY E

We think of the force between the point charges Q_a and Q_b in Coulomb's law as the product of Q_a and the *field* of Q_b, or vice versa. We define the *electric field intensity* **E** to be the force per unit charge exerted on a test charge in the field. Thus the electric field intensity due to the point charge Q_a is

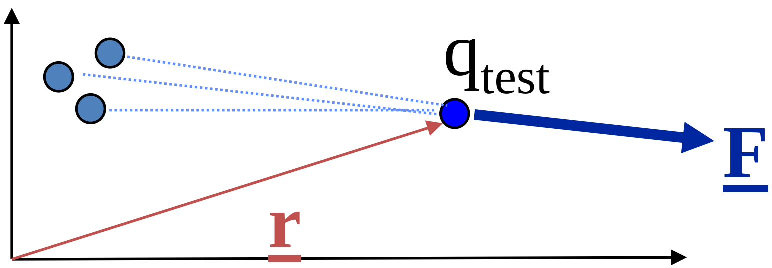
$$\mathbf{E}_a = \frac{\mathbf{F}_{ab}}{Q_b} = \frac{Q_a}{4\pi\epsilon_0 r^2}\mathbf{r}_1. \tag{2-5}$$

The electric field intensity is measured in volts per meter.

The electric field intensity due to the point charge Q_a is the same, whether the test charge Q_b is in the field or not, even if Q_b is large compared to Q_a.

The Electric Field

- **Group of fixed charges exert a force $\underline{F}$, given by Coulomb's law, on a test charge q_{test} at position $\underline{r}$.**

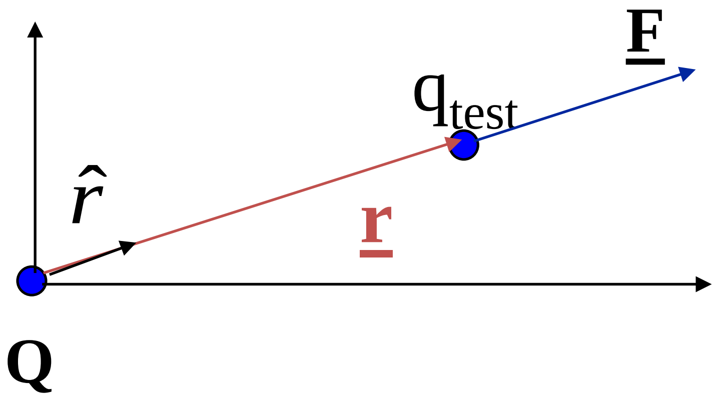


- **The electric field $\underline{E}$ (at a given point in space) is the force per unit charge that would be experienced by a test charge at that point.**

$$\underline{E} = \underline{F} / q_{test}$$

This is a vector function of position.

Electric Field of a Point Charge

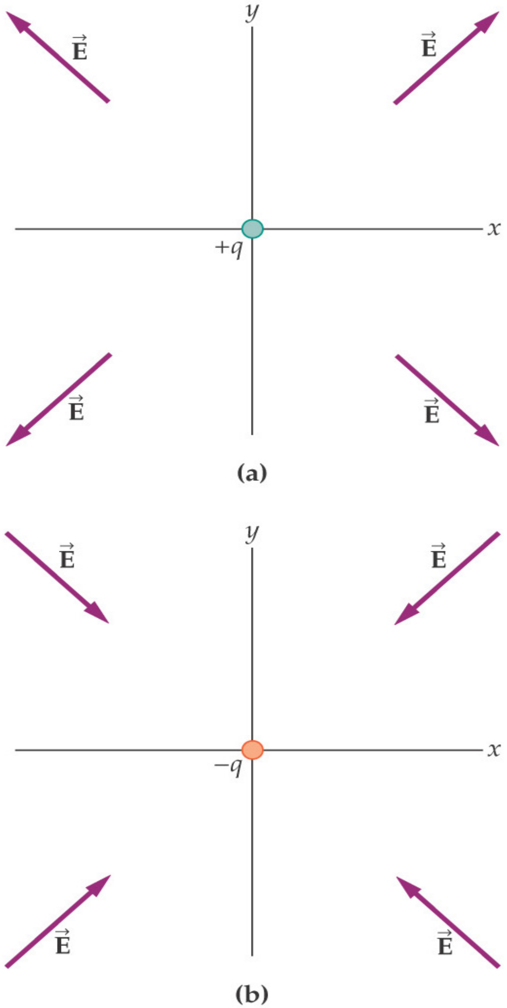


$$\vec{\mathbf{F}} = \frac{1}{4\pi\varepsilon_0}\frac{Qq_{test}}{r^2}\,\hat{\mathbf{r}}$$

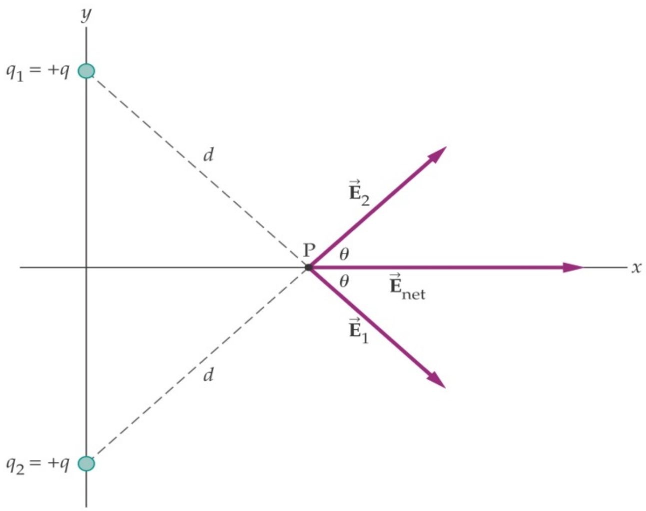
- Dividing out q_{test} gives the electric field at **r**:

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\,\hat{\mathbf{r}}$$

The Electric Field



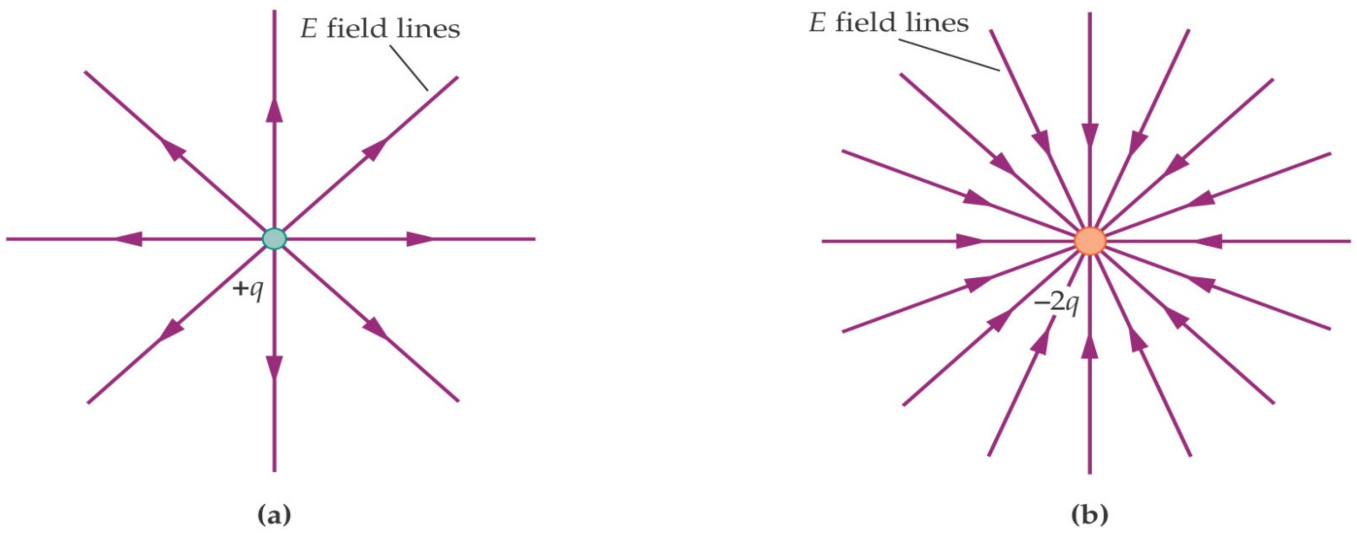
Direction of the electric field



Superposition of electric field

Electric Field Lines

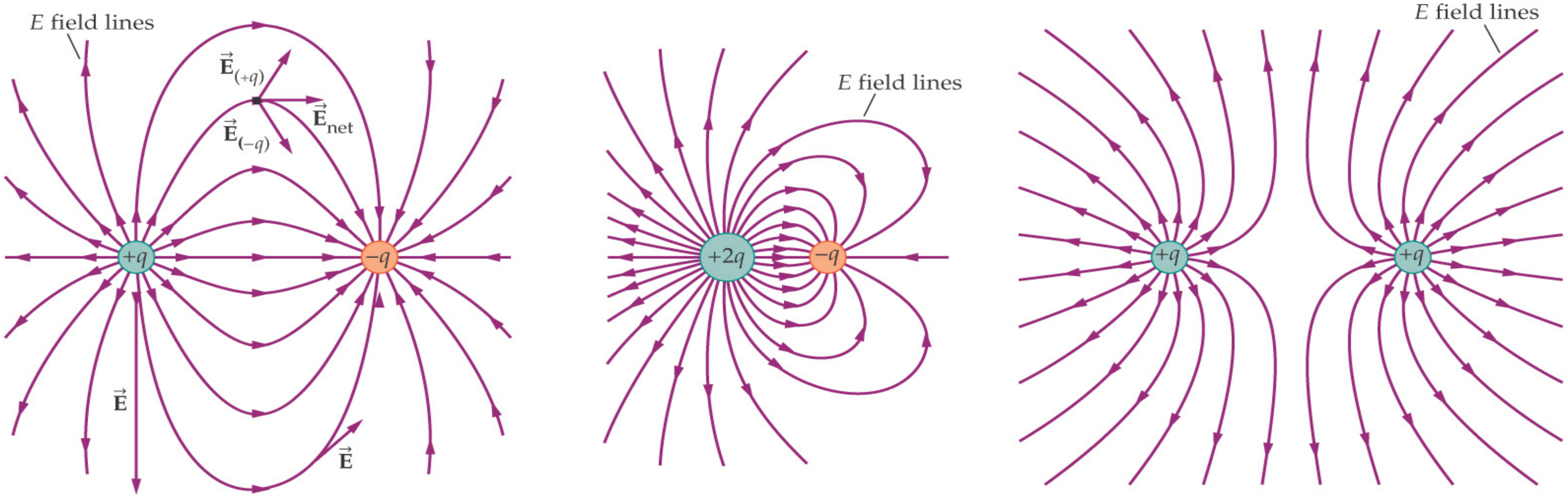
Electric field lines (lines of force) are continuous lines whose direction is everywhere that of the electric field



Electric field lines:

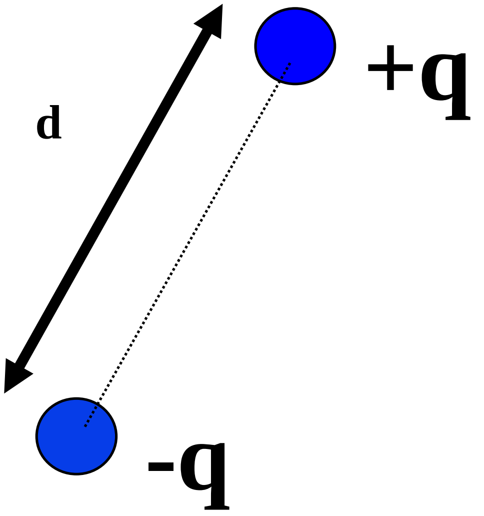
1) **Point in the direction of the electric field E**
2) **Start at positive charges or at infinity**
3) **End at negative charges or at infinity**
4) **Are more dense where the field has greater magnitude**

Electric Field Lines



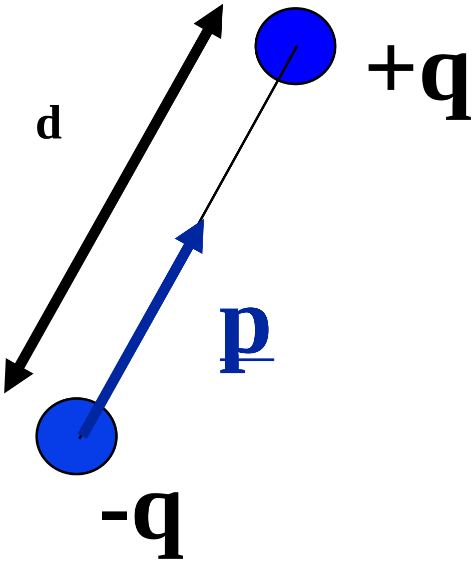
(a) (b) (c)

The Electric Dipole



An electric dipole consists of two equal and opposite charges (**q** and **-q**) separated a distance **d.**

The Electric Dipole

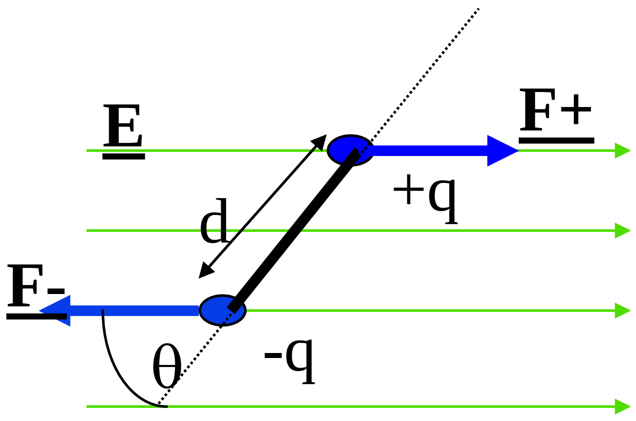


We define the **Dipole Moment** **p**

p → magnitude = **qd**,

p → direction = from -q to +q

The Electric Dipole

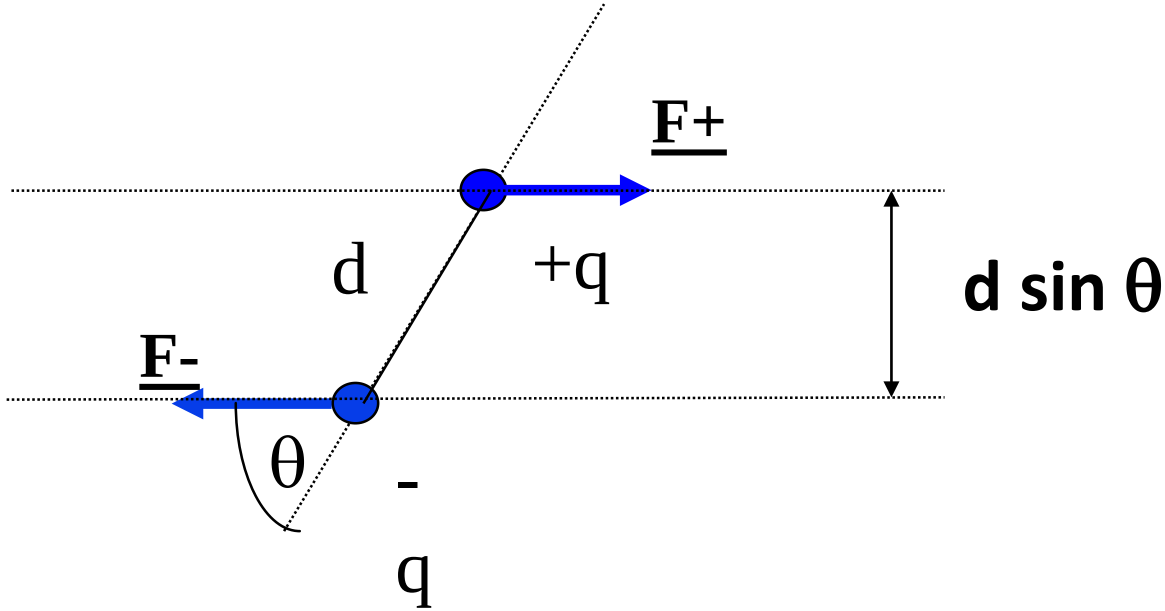


What is the total force acting on the dipole?

Zero, because the force on the two charges cancel: both have magnitude qE. The center of mass does not accelerate.

But the charges start to move (rotate). Why?

There's a torque because the forces aren't colinear.



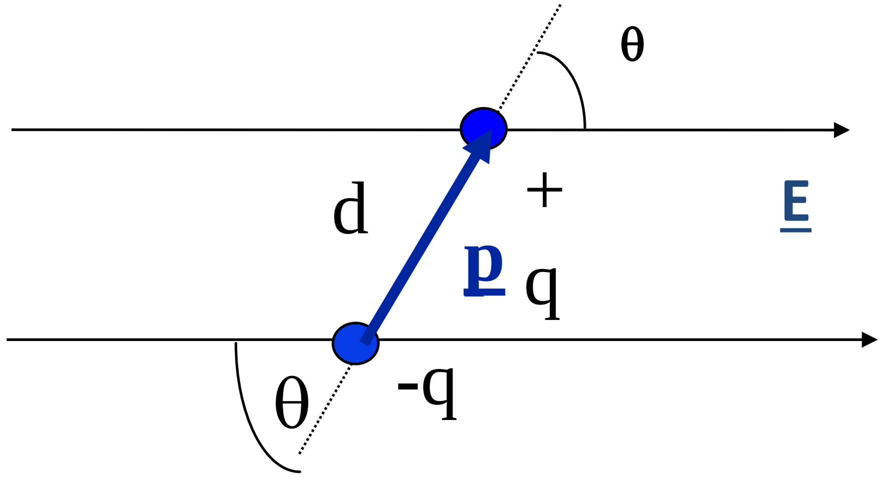
The torque is:

τ = (magnitude of force) (moment arm)

$$\tau = (qE)(d \sin \theta)$$

and the direction of $\underline{\tau}$ is (in this case)

into the page



but we have defined : **p = q d**

and the direction of **p** is from **-**q to +q

Then, the **torque** can be written as:

$$\underline{\tau} = \underline{p} \times \underline{E} \qquad \tau = p\,E\sin\theta$$

with an associated **potential energy**

$$U = -\,\underline{p}\cdot\underline{E}$$

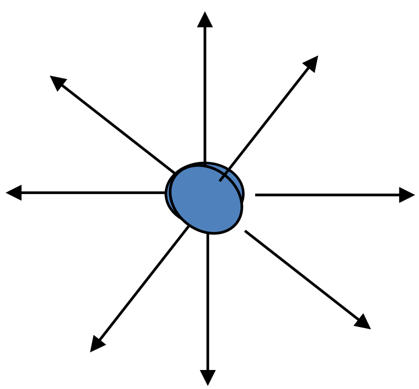
Electric fields due to various charge distributions

- The electric field is a vector which obeys the superposition principle.
- The electric field of a charge distribution is the sum of the fields produced by individual charges, or by differential elements of charge

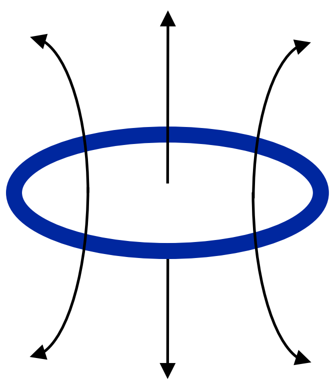
Electric Fields From Continuous Distributions of Charge

Up to now we have only considered the electric field of point charges.

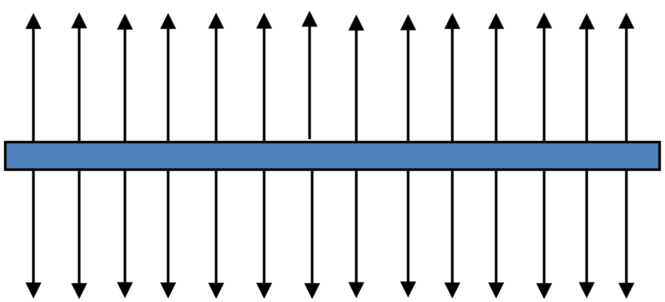
Now let's look at continuous distributions of charge
lines - surfaces - volumes of charge
and determine the resulting electric fields.



Sphere



Ring



Sheet

2.5 THE ELECTRIC POTENTIAL V

Consider a test point charge Q' that can be moved about in an electric field. The work W required to move it at a constant speed from a point P_1 to a point P_2 along a given path is

$$W = -\int_{P_1}^{P_2} \mathbf{E}Q' \cdot d\mathbf{l}. \tag{2-8}$$

The negative sign is required to obtain the work done *against* the field. Here again, we assume that Q' is so small that the charge distributions are not appreciably disturbed by its presence.

If the path is closed, the total work done is

$$W = -\oint \mathbf{E}Q' \cdot d\mathbf{l}. \tag{2-9}$$

Let us evaluate this integral. To simplify matters, we first consider the electric field produced by a single point charge Q. Then

$$\oint \mathbf{E}Q' \cdot d\mathbf{l} = \frac{QQ'}{4\pi\epsilon_0}\oint \frac{(\mathbf{r}_1 \cdot d\mathbf{l})}{r^2}. \tag{2-10}$$

Figure 2-3 shows that the term under the integral on the right is simply dr/r^2 or $-d(1/r)$. The sum of the increments of $(1/r)$ over a closed path is zero, since r has the same value at the beginning and at the end of the path. Then the line integral is zero, and the net work done in moving a point charge Q' around any closed path in the field of a point charge Q, which is fixed, is zero.

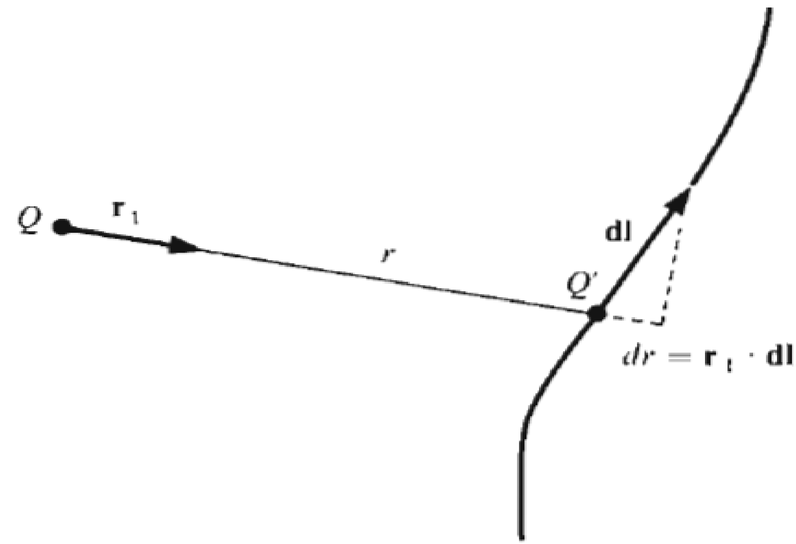


Figure 2-3 The product $\mathbf{r}_1 \cdot \mathbf{dl}$ is simply dr.

If the electric field is produced, not by a single point charge Q, but by some fixed charge distribution, then the line integrals corresponding to each individual charge of the distribution are all zero. Thus, for any distribution of fixed charges,

$$\oint \mathbf{E} \cdot \mathbf{dl} = 0. \tag{2-11}$$

Then, from Stokes's theorem at all points in space,

$$\nabla \times \mathbf{E} = 0, \tag{2-12}$$

and we can write that

$$\mathbf{E} = -\nabla V, \tag{2-13}$$

where V is a scalar point function, since $\nabla \times \nabla V \equiv 0$.

We can thus describe an electrostatic field completely by means of the function $V(x,y,z)$, which is called the *electric potential*. The negative sign is required in order that the electric field intensity $\mathbf{E}$ point toward a *decrease* in potential, according to the usual convention.

It is important to note that V is not uniquely defined; we can add to it any quantity that is independent of the coordinates without affecting $\mathbf{E}$ in any way.

As shown in Prob. 1-23, the work done in moving a test charge a constant speed from a point P_1 to a point P_2 is independent of the pa

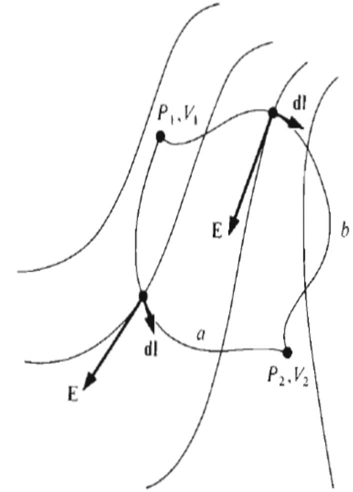


Figure 2-4 Two paths in a conservative electric field **E**, with endpoints P_1, at potential V_1, and P_2, at potential V_2. The integral of $\mathbf{E} \cdot d\mathbf{l}$ along path a is the same as along path b and is $V_1 - V_2$.

According to Eq. 2-13,

$$\mathbf{E} \cdot d\mathbf{l} = -\nabla V \cdot d\mathbf{l} = -dV. \tag{2-14}$$

Then, for any two points P_1 and P_2 as in Fig. 2–4,

$$V_2 - V_1 = -\int_1^2 \mathbf{E} \cdot d\mathbf{l} = \int_2^1 \mathbf{E} \cdot d\mathbf{l}. \tag{2-15}$$

Note that the electric field intensity $\mathbf{E}(x, y, z)$ determines only the *difference* between the potentials at two different points. When we wish to speak of the electric potential at a given point, we must therefore arbitrarily define the potential in a given region of space to be zero. It is usually convenient to choose the potential at infinity to be zero. Then the potential V at the point 2 is

$$V = \int_2^\infty \mathbf{E} \cdot d\mathbf{l}. \tag{2-16}$$

When the field is produced by a single point charge Q, the potential at a distance r is

$$V = \int_r^{\infty} \frac{Q}{4\pi\epsilon_0} \frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0 r}. \tag{2-17}$$

It will be observed that the sign of the potential V is the same as that of Q.

The principle of superposition applies to the electric potential V as well as to the electric field intensity **E**. The potential V at a point P due to a charge distribution of density ρ is therefore

$$V = \frac{1}{4\pi\epsilon_0} \int_{\tau'} \frac{\rho\, d\tau'}{r}, \tag{2-18}$$

where r is the distance between the point P and the element of charge $\rho\, d\tau'$.

The points in space that are at a given potential define an *equipotential surface*. For example, an equipotential surface about a point charge is a concentric sphere as in Fig. 2-5. We can see from Eq. 2-13 that **E** is everywhere normal to the equipotential surfaces (Sec. 1.5).

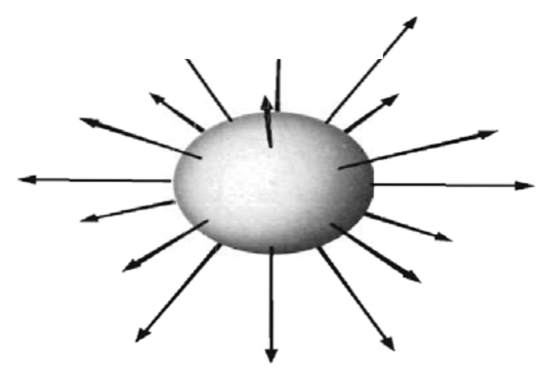


Figure 2-5 Equipotential surface and lines of force near a point charge.

EXAMPLE: THE ELECTRIC DIPOLE

The *electric dipole* shown in Fig. 2-6 is one type of charge distribution that is encountered frequently. We shall return to it in Chapter 6.

The electric dipole consists of two charges, one positive and the other negative, of the same magnitude, separated by a distance s. We shall calculate V and $\mathbf{E}$ at a distance r that is large compared to s.

At P,

$$V = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right), \tag{2-19}$$

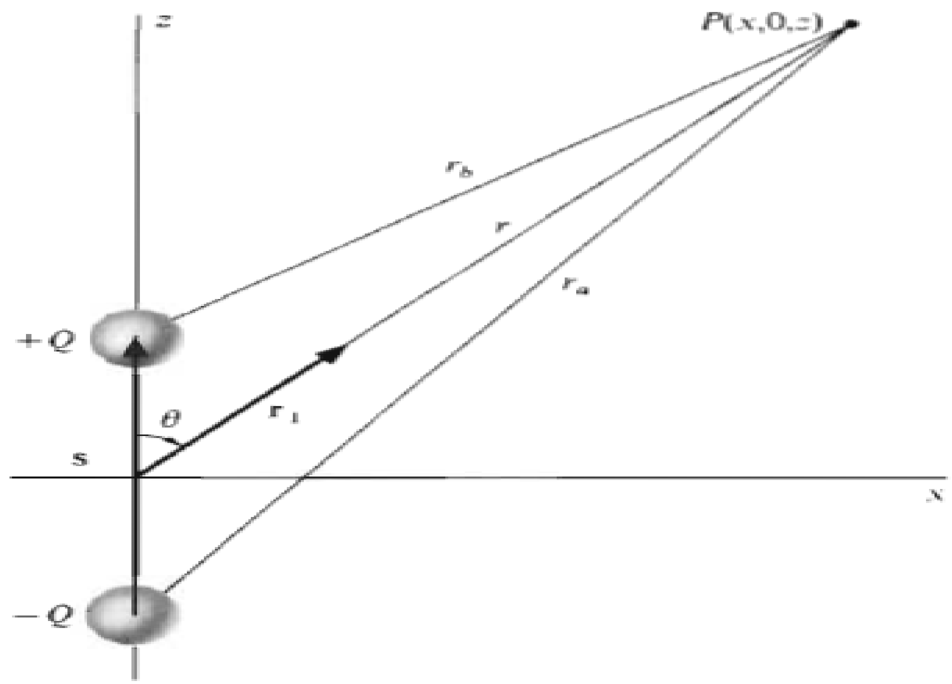


Figure 2-6 The two charges $+Q$ and $-Q$ form a dipole. The electric potential at P is the sum of the potentials due to the individual charges. The vector $\mathbf{s}$ points from $-Q$ to $+Q$, and $\mathbf{r}_1$ is a unit vector pointing from the origin to the point of observation P.

where

$$r_a^2 = r^2 + \left(\frac{s}{2}\right)^2 + rs\cos\theta, \tag{2-20}$$

$$r_a = r\left[1 + \left(\frac{s}{2r}\right)^2 + \frac{s}{r}\cos\theta\right]^{1/2}. \tag{2-21}$$

$$\approx r\left(1 + \frac{s}{r}\cos\theta\right)^{1/2}. \tag{2-22}$$

$$\frac{r}{r_a} \approx \frac{1}{\left(1 + \frac{s}{r}\cos\theta\right)^{1/2}}, \tag{2-23}$$

$$\approx 1 - \frac{s}{2r}\cos\theta.^{\dagger} \tag{2-24}$$

Similarly,

$$\frac{r}{r_b} \approx 1 + \frac{s}{2r}\cos\theta, \tag{2-25}$$

and

$$V = \frac{Qs}{4\pi\epsilon_0 r^2}\cos\theta. \tag{2-26}$$

This expression is valid for $r^3 \gg s^3$.

It is interesting to note that the potential due to a dipole falls off as $1/r^2$, whereas the potential from a single point charge varies only as $1/r$. This comes from the

* Remember that

$$(1 + a)^n = 1 + na + \frac{n(n-1)}{2!}a^2 + \frac{n(n-1)(n-2)}{3!}a^3 + \cdots$$

If n is a positive integer, the series stops when the coefficient is zero. Try, for example, $n = 1$ and $n = 2$. If n is not a positive integer, the series converges for $a^2 < 1$.

This series is very often used when $a \ll 1$. Then

$$(1 + a)^n \approx 1 + na.$$

For example,

$$(1 + a)^2 \approx 1 + 2a, \qquad (1 + a)^{-1/2} \approx 1 - \frac{a}{2}, \text{ etc.}$$

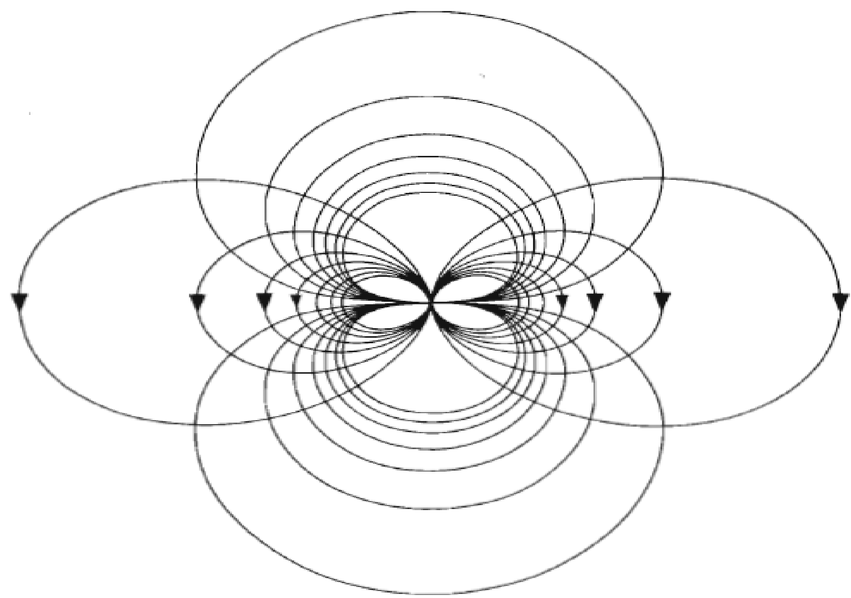


Figure 2-7 Lines of force (arrows) and equipotential lines for the dipole of Fig. 2-6 The dipole is vertical, at the center of the figure, with the positive charge close to and above the negative charge. In the central region the lines come too close together to be shown.

Let us now calculate **E** at the point $P(x,0,z)$ in the plane $y = 0$ as in Fig. 2-6. We have that

$$V = \frac{p}{4\pi\epsilon_0}\frac{\cos\theta}{r^2} = \frac{p}{4\pi\epsilon_0}\frac{z}{r^3} \tag{2-28}$$

and, at $P(x,0,z)$,

$$E_x = -\frac{\partial V}{\partial x} = \frac{3p}{4\pi\epsilon_0}\frac{xz}{r^5} = \frac{3p}{4\pi\epsilon_0}\frac{\sin\theta\cos\theta}{r^3}, \tag{2-29}$$

$$E_y = -\frac{\partial V}{\partial y} = \frac{3p}{4\pi\epsilon_0}\frac{yz}{r^5} = 0, \tag{2-30}$$

since $y = 0$, and

$$E_z = -\frac{\partial V}{\partial z} = -\frac{p}{4\pi\epsilon_0}\left(\frac{1}{r^3} - \frac{3z^2}{r^5}\right), \tag{2-31}$$

$$= \frac{p}{4\pi\epsilon_0}\frac{3\cos^2\theta - 1}{r^3}. \tag{2-32}$$

Figure 2-7 shows lines of force for the electric dipole.

2.6 SUMMARY

It is found empirically that the force exerted *by* a point charge Q_a *on* a point charge Q_b is

$$\mathbf{F}_{ab} = \frac{1}{4\pi\epsilon_0}\frac{Q_a Q_b}{r^2}\mathbf{r}_1, \tag{2-1}$$

where r is the distance between the charges and $\mathbf{r}_1$ is a unit vector pointing from Q_a to Q_b. This is *Coulomb's law*. We consider the force $\mathbf{F}_{ab}$ as being the product of Q_b by the *electric field intensity* due to Q_a,

$$\mathbf{E}_a = \frac{Q_a}{4\pi\epsilon_0 r^2}\mathbf{r}_1, \tag{2-5}$$

or vice versa.

According to the *principle of superposition*, two or more electric field intensities acting at a given point add vectorially. For an extended charge distribution,

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int_{\tau'} \frac{\rho \mathbf{r}_1}{r^2}\, d\tau'. \tag{2-6}$$

The electrostatic field is conservative,

$$\nabla \times \mathbf{E} = 0, \tag{2-12}$$

hence

$$\mathbf{E} = -\nabla V, \tag{2-13}$$

where

$$V = \frac{1}{4\pi\epsilon_0} \int_{\tau'} \frac{\rho\, d\tau'}{r} \tag{2-18}$$

is the *electric potential*; $\rho\, d\tau'$ is the element of charge contained within the element of volume $d\tau'$, and r is the distance between this element and the point where V is calculated.

Homework

Q1

a) Calculate the electric field intensity that would be just sufficient to balance the gravitational force of the earth on an electron.

b) If this electric field were produced by a second electron located below the first one, what would be the distance between the two electrons?

The charge on an electron is -1.6×10^{-19} coulomb and its mass is 9.1×10^{-31} kilogram.

Q2

A charge $+Q$ is situated at $x = a$, $y = 0$, and a charge $-Q$ at $x = -a$, $y = 0$. Calculate the electric field intensity at the point $x = 0$, $y = a$.

Q3

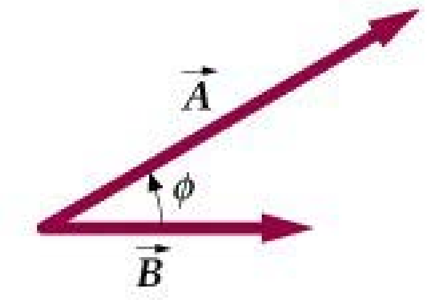
A circular disk of charge has a radius R and carries a surface charge density σ. Find the electric field intensity E at a point P on the axis at a distance a. What is the value of E when $a \ll R$?

You can solve this problem by calculating the field at P due to a ring of charge of radius r and width dr, and then integrating from $r = 0$ to $r = R$.

Vector Notation

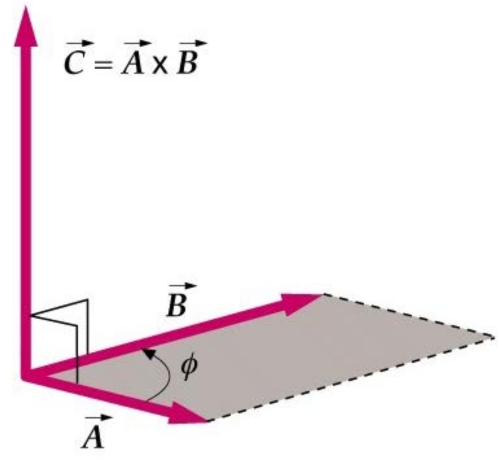
- **The DOT product**

$$\vec{\mathbf{C}} = \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \equiv C = AB\cos\phi$$



- **The CROSS product**

$$\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}} \equiv C = AB\sin\phi$$



The Force Exerted by a Magnetic Field

- **Key Concept – Magnetic fields apply a force to moving charges**

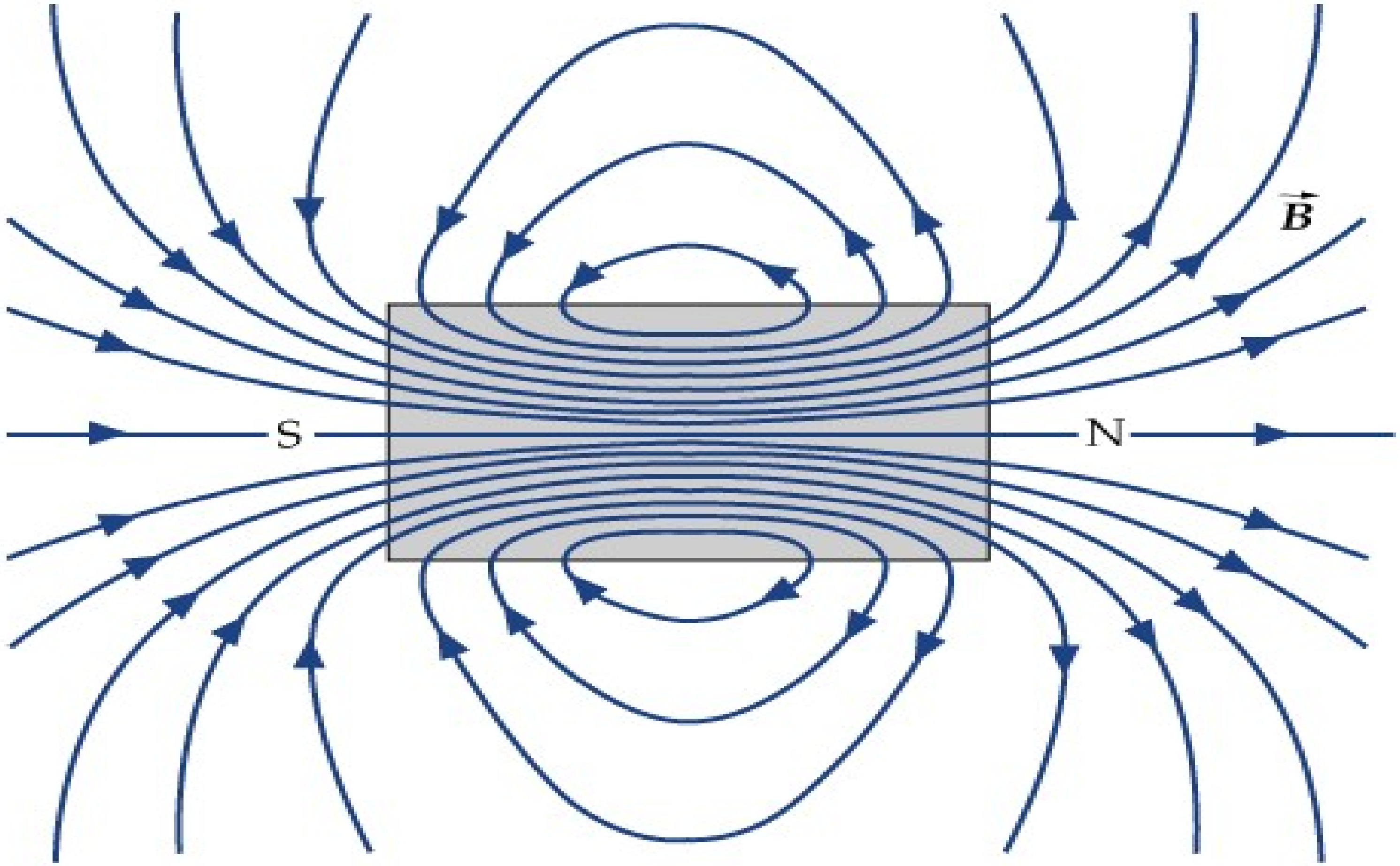
$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

Current element

$$\overrightarrow{d\mathbf{F}} = I\,\overrightarrow{d\mathbf{l}} \times \vec{\mathbf{B}}$$

Magnetic Field

-
-

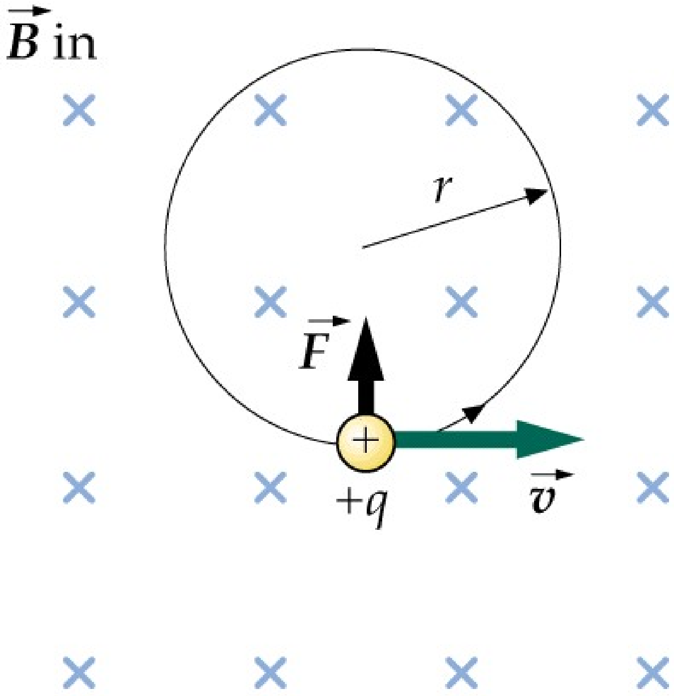


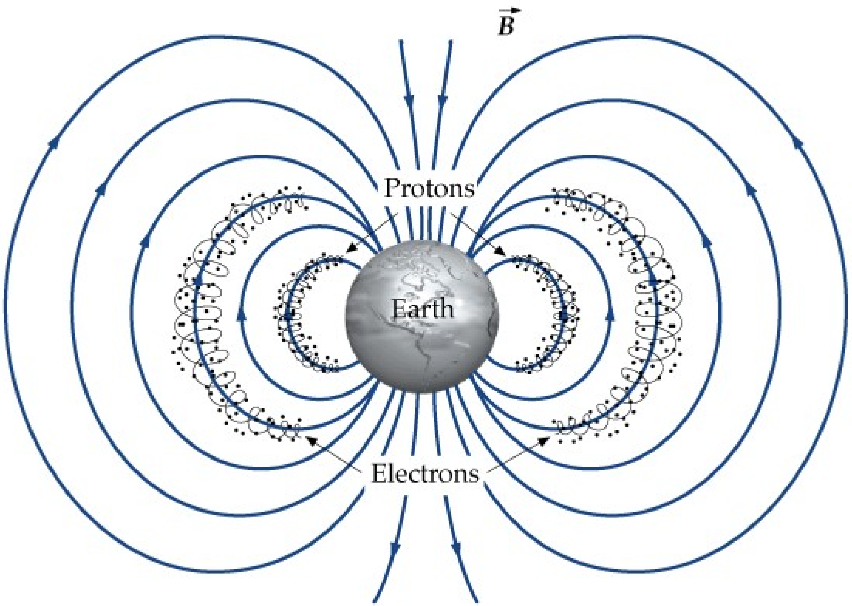
(a)

Motion of a Point Charge in a Magnetic Field

- **Force is perpendicular to field direction and velocity**
- **Therefore, magnetic fields do no work on particles**
- **There is no change in magnitude of velocity, just direction**

Motion of a Point Charge in a Magnetic Field





Motion of a Point Charge in a Magnetic Field

- **Radius of circular orbit**

$$r = \frac{mv}{qB}$$

- **Cyclotron period**

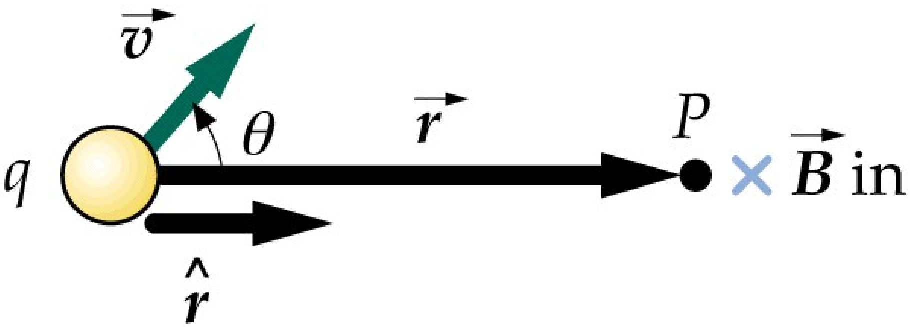
$$T = \frac{2\pi m}{qB}$$

- **Cyclotron frequency**

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

The Magnetic Field of Moving Point Charges

- **Point charge q moving with velocity v produces a field B at point P**



$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{q\vec{\mathbf{v}}\times\hat{\mathbf{r}}}{r^2}$$

μ_o= **permeability of free space**

μ_o= **4π x 10^{-7} T·m·A^{-1}**

The Magnetic Field

8.1 THE MAGNETIC INDUCTION B

Figure 8-1 shows a circuit carrying a current I. We define the *magnetic induction* at the point P as follows:

$$\mathbf{B} = \frac{\mu_0}{4\pi} I \oint \frac{\mathbf{dl} \times \mathbf{r}_1}{r^2}, \qquad (8\text{-}1)$$

where the integration is carried out over the closed circuit. As usual, the unit vector $\mathbf{r}_1$ points *from* the source *to* the point of observation: it points *from* the element $\mathbf{dl}$ *to* the point P. Magnetic inductions are expressed in *teslas*.

The constant μ_0 is *defined* as follows:

$$\mu_0 \equiv 4\pi \times 10^{-7} \text{ tesla meter/ampere}, \qquad (8\text{-}2)$$

and is called the *permeability of free space*.

If the current I is distributed in space with a current density $\mathbf{J}$ amperes per square meter, then I becomes $J\,da$ and must be put under the integral

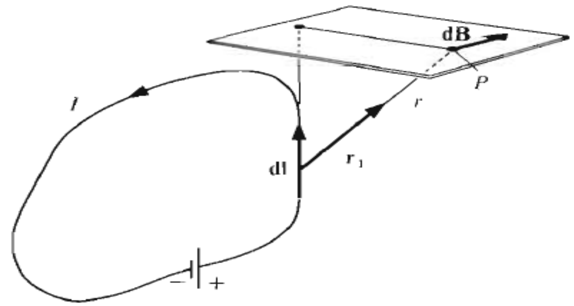


Figure 8-1 The magnetic induction $\mathbf{dB} = (\mu_0/4\pi)I\,\mathbf{dl} \times \mathbf{r}_1/r^2$ produced by an element $I\,\mathbf{dl}$ of the current I in a circuit.

sign. Then $J\,da\,\mathbf{dl}$ can be written as $\mathbf{J}\,d\tau'$, where $d\tau'$ is an element of volume, and

$$\mathbf{B} = \frac{\mu_0}{4\pi} \int_{\tau'} \frac{\mathbf{J} \times \mathbf{r}_1}{r^2}\, d\tau', \tag{8-3}$$

as in Fig. 8-2. The integration is carried out over the volume τ' occupied by the currents.

We assume that $\mathbf{J}$ is not a function of time and that there are no magnetic materials in the field.

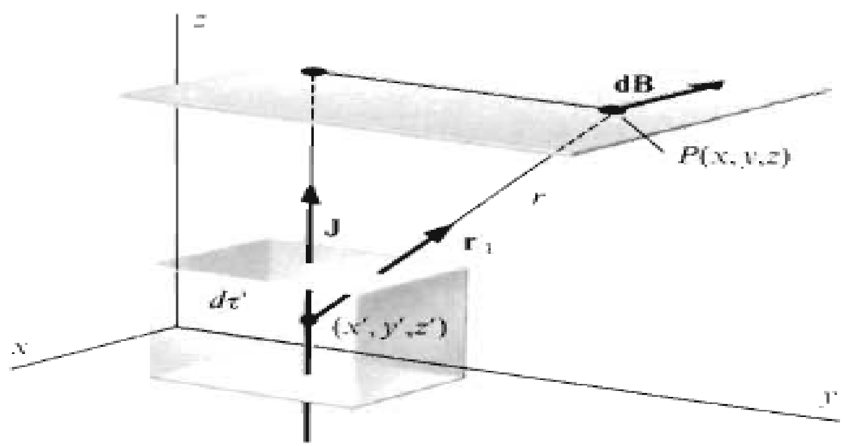


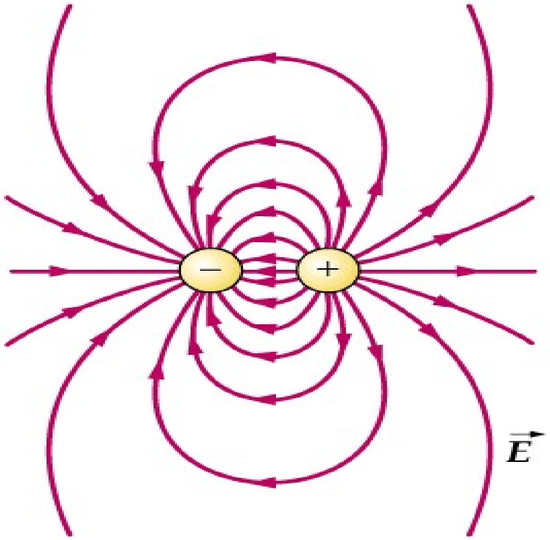
Figure 8-2 The magnetic induction $\mathbf{dB}$ due to an element $\mathbf{J}\,d\tau'$ of a volume distribution of current.

As in electrostatics, we can describe a magnetic field by drawing *lines of* **B** that are everywhere tangent to **B**.

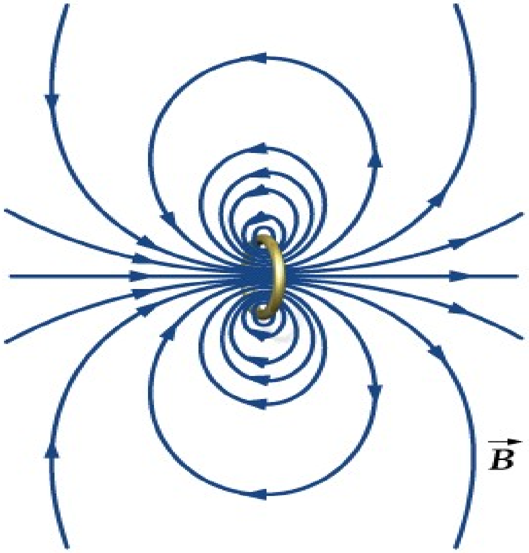
Similarly, it is convenient to use the concept of flux, the *flux of the magnetic induction* **B** through a surface S, defined as the normal component of **B** integrated over S:

$$\Phi = \int_S \mathbf{B} \cdot \mathbf{da}. \tag{8-4}$$

The flux Φ is expressed in *webers*. Thus the tesla is one weber per square meter.



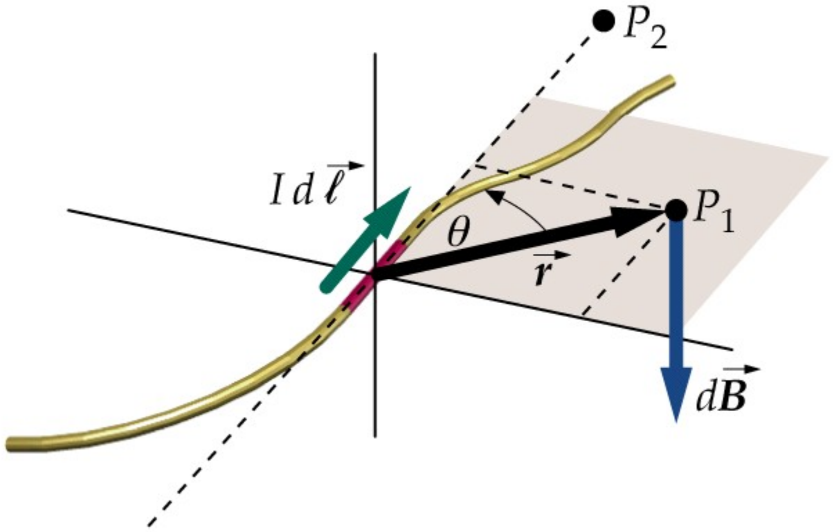
Electric dipole



Magnetic dipole (or current loop)

The Magnetic Field of Currents:

- **current as a series of moving charges – replace $q\underline{\mathbf{v}}$ by $I\underline{\mathbf{dl}}$**



$$d\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{Id\vec{\mathbf{l}}\times\hat{\mathbf{r}}}{r^2}$$

Add each element to get total B field

Gauss' Law for Magnetism

- **The net flux of magnetic field lines through a closed surface is zero (i.e. no magnetic monopoles)**

$$\phi_{m,\,net} = \oint_S B_n dA = 0$$

Magnetic flux

Ampère's Law

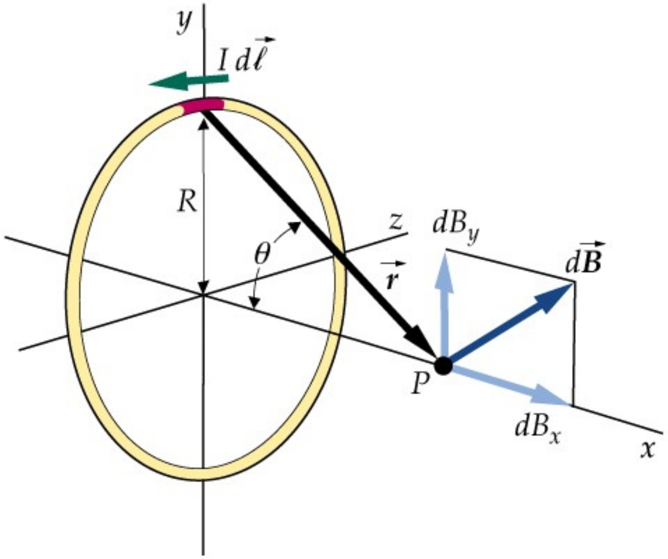
- **like Gauss' law for electric field, uses symmetry to calculate B field around a closed curve C**

$$\oint_C \vec{\mathbf{B}} . d\vec{\mathbf{l}} = \mu_o I_C$$

Field due to a current loop

$$\left|d\vec{\mathbf{B}}\right| = \frac{\mu_o}{4\pi}\frac{I\left|d\vec{\mathbf{l}}\times\hat{\mathbf{r}}\right|}{r^2}$$

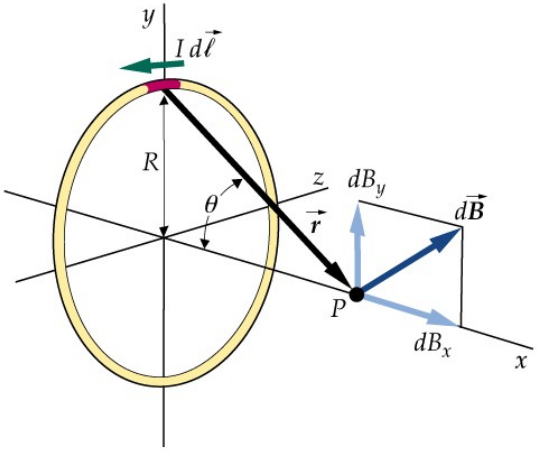
$$\left|d\vec{\mathbf{B}}\right| = \frac{\mu_o}{4\pi}\frac{Idl}{x^2+R^2}$$



Field due to a current loop

$$dB_x = \frac{\mu_o}{4\pi}\frac{Idl}{x^2+R^2}\frac{R}{\sqrt{x^2+R^2}}$$

$$B_x = \oint dB_x = \frac{\mu_o}{4\pi}\frac{IR}{\left(x^2+R^2\right)^{3/2}}\oint dl$$



$2\pi R$

$$B_x = \frac{\mu_o}{4\pi}\frac{2\pi R^2 I}{\left(x^2+R^2\right)^{3/2}}$$

An element **dl** of a long straight wire carrying a current I, as in Fig. 8-3, produces a magnetic induction

$$\mathbf{dB} = \frac{\mu_0 I}{4\pi} \frac{dl \sin \theta}{r^2} \boldsymbol{\varphi}_1. \tag{8-5}$$

where $\boldsymbol{\varphi}_1$ is the unit vector pointing in the azimuthal direction. The positive directions for $\boldsymbol{\varphi}_1$ and I are related by the right-hand screw rule.

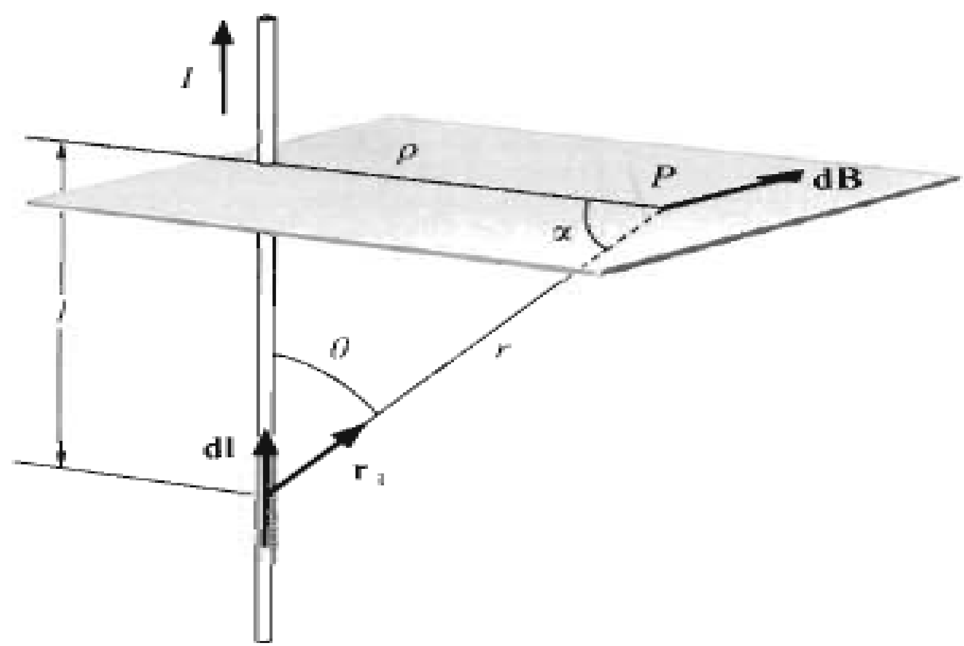


Figure 8-3 The magnetic induction **dB** produced by an element I **dl** of the current I in a long straight wire.

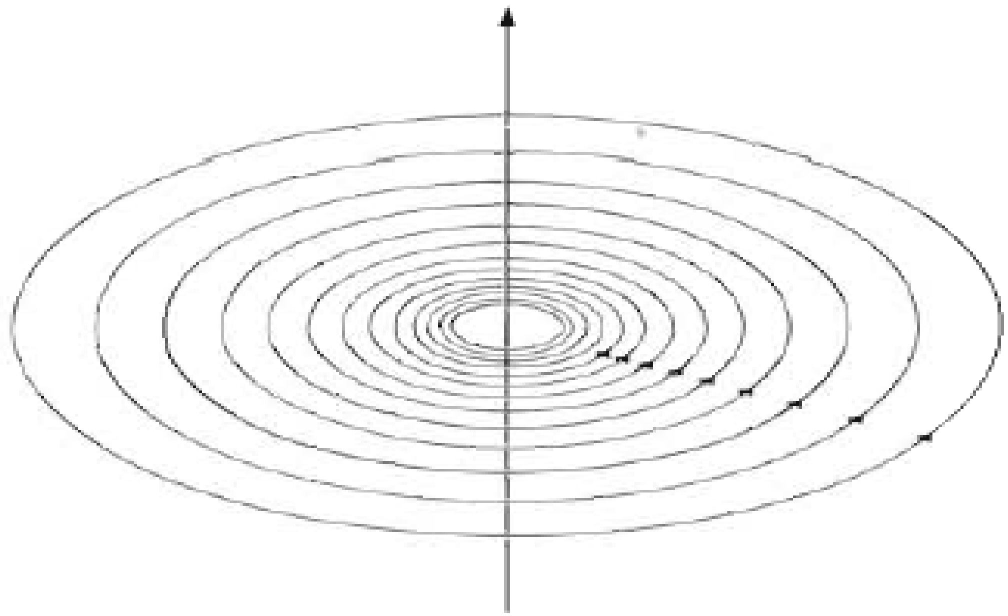


Figure 8-4 Lines of **B** in a plane perpendicular to a long straight wire carrying a current I. The density of the lines is inversely proportional to the distance to the wire. Lines close to the wire are not shown.

Expressing dl, $\sin\theta$, and r^2 in terms of α and ρ,

$$\mathbf{B} = \frac{\mu_0 I}{4\pi\rho}\int_{-\pi/2}^{+\pi/2}\cos\alpha\, d\alpha\, \boldsymbol{\varphi}_1 = \frac{\mu_0 I}{2\pi\rho}\boldsymbol{\varphi}_1. \qquad (8\text{-}6)$$

The magnitude of **B** thus falls off inversely as the first power of the distance from an infinitely long wire. The lines of **B** are concentric circles lying in a plane perpendicular to the wire, as in Fig. 8-4.

EXAMPLE: CIRCULAR LOOP. MAGNETIC DIPOLE MOMENT m

A circular loop of radius a carries a current I, as in Fig. 8-5.

An element $I\,\mathbf{dl}$ of current produces a $\mathbf{dB}$ having a component dB_z on the axis, as indicated in the figure. By symmetry, the total $\mathbf{B}$ is along the axis, and

$$dB_z = \frac{\mu_0 I}{4\pi}\frac{dl}{r^2}\cos\theta, \tag{8-7}$$

$$B_z = \frac{\mu_0 I}{4\pi}\frac{2\pi a}{r^2}\cos\theta = \frac{\mu_0 I a^2}{2(a^2 + z^2)^{3/2}}. \tag{8-8}$$

Therefore, on the axis, the magnetic induction is maximum at the center of the ring and drops off as z^3 for $z^2 \gg a^2$.

Figure 8-6 shows lines of $\mathbf{B}$ in a plane containing the axis of the loop.

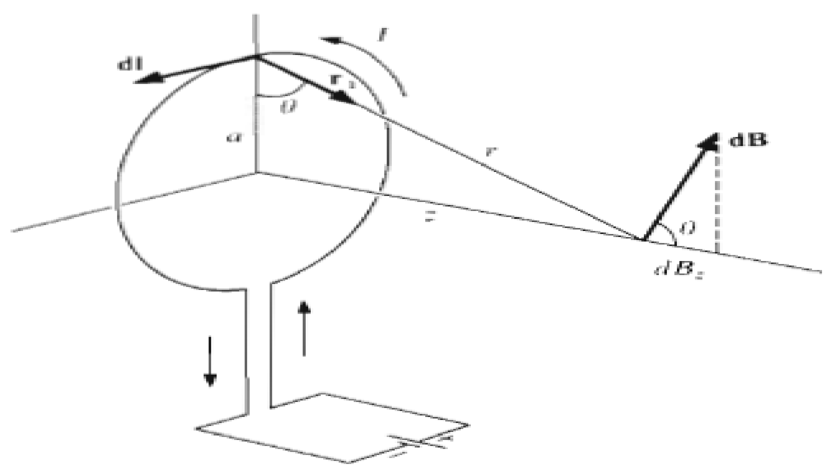


Figure 8-5 The magnetic induction **dB** produced by an element I **dl** at a point on the axis of circular current loop of radius a. The projection of **dB** on the axis is dB_z

SUMMARY

The *magnetic induction* **B** due to a circuit C carrying a current I is defined as follows:

$$\mathbf{B} = \frac{\mu_0}{4\pi} I \oint_C \frac{\mathbf{dl} \times \mathbf{r}_1}{r^2}. \qquad (8\text{-}1)$$

Magnetic inductions are measured in *teslas*, and μ_0 is defined to be *exactly* $4\pi \times 10^{-7}$ tesla meter per ampere.

The *magnetic flux* through a surface S is

$$\Phi = \int_S \mathbf{B} \cdot \mathbf{da}. \qquad (8\text{-}4)$$

Magnetic flux is measured in *webers*, and a tesla is one weber per square meter.

The divergence of **B** is zero:

$$\boxed{\nabla \cdot \mathbf{B} = 0,} \qquad (8\text{-}12)$$

since the magnetic flux through a closed surface is always identically equal to zero, if magnetic monopoles do not exist. This is one of Maxwell's equations.

It follows from this that

$$\mathbf{B} = \nabla \times \mathbf{A}, \qquad (8\text{-}13)$$

where **A** is the *vector potential*

$$\mathbf{A} = \frac{\mu_0 I}{4\pi} \oint \frac{\mathbf{dl}}{r}. \qquad (8\text{-}18)$$

The line integral of $\mathbf{A} \cdot \mathbf{dl}$ over any closed curve C is equal to the magnetic flux through any surface S bounded by C:

$$\oint_C \mathbf{A} \cdot \mathbf{dl} = \Phi. \qquad (8\text{-}30)$$

Homework

8-1E *MAGNETIC FIELD ON THE AXIS OF A CIRCULAR LOOP*

Plot a curve of B as a function of z on the axis of a circular loop of 100 turns having a mean radius of 100 millimeters and carrying a current of one ampere.

8-2 *SQUARE CURRENT LOOP*

Compute the magnetic induction B at the center of a square current loop of side a carrying a current I.

SUMMARY

In Cartesian coordinates a *vector* quantity is written in the form

$$\mathbf{A} = A_x\,\mathbf{i} + A_y\,\mathbf{j} + A_z\,\mathbf{k}, \tag{1-1}$$

where **i**, **j**, **k** are *unit vectors* directed along the x, y, z axes respectively.

The *magnitude* of the vector **A** is the scalar

$$A = (A_x^2 + A_y^2 + A_z^2)^{1/2}. \tag{1-2}$$

Vectors can be added and subtracted:

$$\mathbf{A} + \mathbf{B} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} + (A_z + B_z)\mathbf{k}, \tag{1-3}$$

$$\mathbf{A} - \mathbf{B} = (A_x - B_x)\mathbf{i} + (A_y - B_y)\mathbf{j} + (A_z - B_z)\mathbf{k}. \tag{1-4}$$

The scalar product (Fig. 1-3) is

$$\mathbf{A} \cdot \mathbf{B} = AB\cos(\varphi - \theta), \tag{1-5}$$

$$= A_xB_x + A_yB_y + A_zB_z. \tag{1-11}$$

It is commutative,

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}, \tag{1-6}$$

and distributive,

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}. \tag{1-7}$$

The *vector product* (Fig. 1-3) is

$$\mathbf{A} \times \mathbf{B} = \mathbf{C} \tag{1-14}$$

with

$$C = |AB \sin (\varphi - \theta)|. \tag{1-15}$$

Another equivalent definition of the vector product is

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}. \tag{1-22}$$

The vector product follows the distributive rule

$$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{C}), \tag{1-17}$$

but *not* the commutative rule:

$$\mathbf{A} \times \mathbf{B} = -(\mathbf{B} \times \mathbf{A}). \tag{1-16}$$

The *time derivative* of $\mathbf{A}$ is

$$\frac{d\mathbf{A}}{dt} = \frac{dA_x}{dt}\mathbf{i} + \frac{dA_y}{dt}\mathbf{j} + \frac{dA_z}{dt}\mathbf{k}. \tag{1-25}$$

The *del operator* is defined as follows:

$$\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{j}\frac{\partial}{\partial z}, \tag{1-29}$$

and the *gradient* of a scalar function f is

$$\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}. \tag{1-30}$$

The gradient gives the maximum rate of change of f with distance at the point considered, and it points toward larger values of f.

The *surface integral* is a common type of *double integral*. It is used for integrating over two coordinates, say x and y. It is composed of an integral within an integral. The inner integral is evaluated first.

The *volume integral* is one form of *triple integral*. It is used for integrating functions of three space coordinates, say x, y, z. In this case, we have an integral that is within an integral that is within an integral. The innermost integral is evaluated first, leaving a double integral, and so on.

The *flux* Φ of a vector $\mathbf{A}$ through a surface S is

$$\Phi = \int_S \mathbf{A} \cdot \mathbf{da}. \tag{1-53}$$

For a closed surface the vector $\mathbf{da}$ points outward.

The *divergence* of $\mathbf{A}$,

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}, \tag{1-59}$$

is the outward flux of $\mathbf{A}$ per unit volume at the point considered.

The *divergence theorem* states that

$$\int_\tau \nabla \cdot \mathbf{A}\, d\tau = \int_S \mathbf{A} \cdot \mathbf{da}, \tag{1-60}$$

where τ is the volume bounded by the surface S.

The *line integral*

$$\int_a^b \mathbf{A} \cdot \mathbf{dl}$$

over a specified curve is the sum of the terms $\mathbf{A} \cdot \mathbf{dl}$ for each element $\mathbf{dl}$ of the curve between the points a and b.

For a closed curve C that bounds a surface S, we have *Stokes's theorem*:

$$\oint_C \mathbf{A} \cdot \mathbf{dl} = \int_S (\nabla \times \mathbf{A}) \cdot \mathbf{da}, \tag{1-85}$$

where

$$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} \tag{1-75}$$

is the *curl* of the vector function $\mathbf{A}$.

The *Laplacian* is the divergence of the gradient:

$$\nabla \cdot \nabla f = \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}. \tag{1-89}$$

Maxwell's equations (2)

Dr.Hind I. Abdulgafour

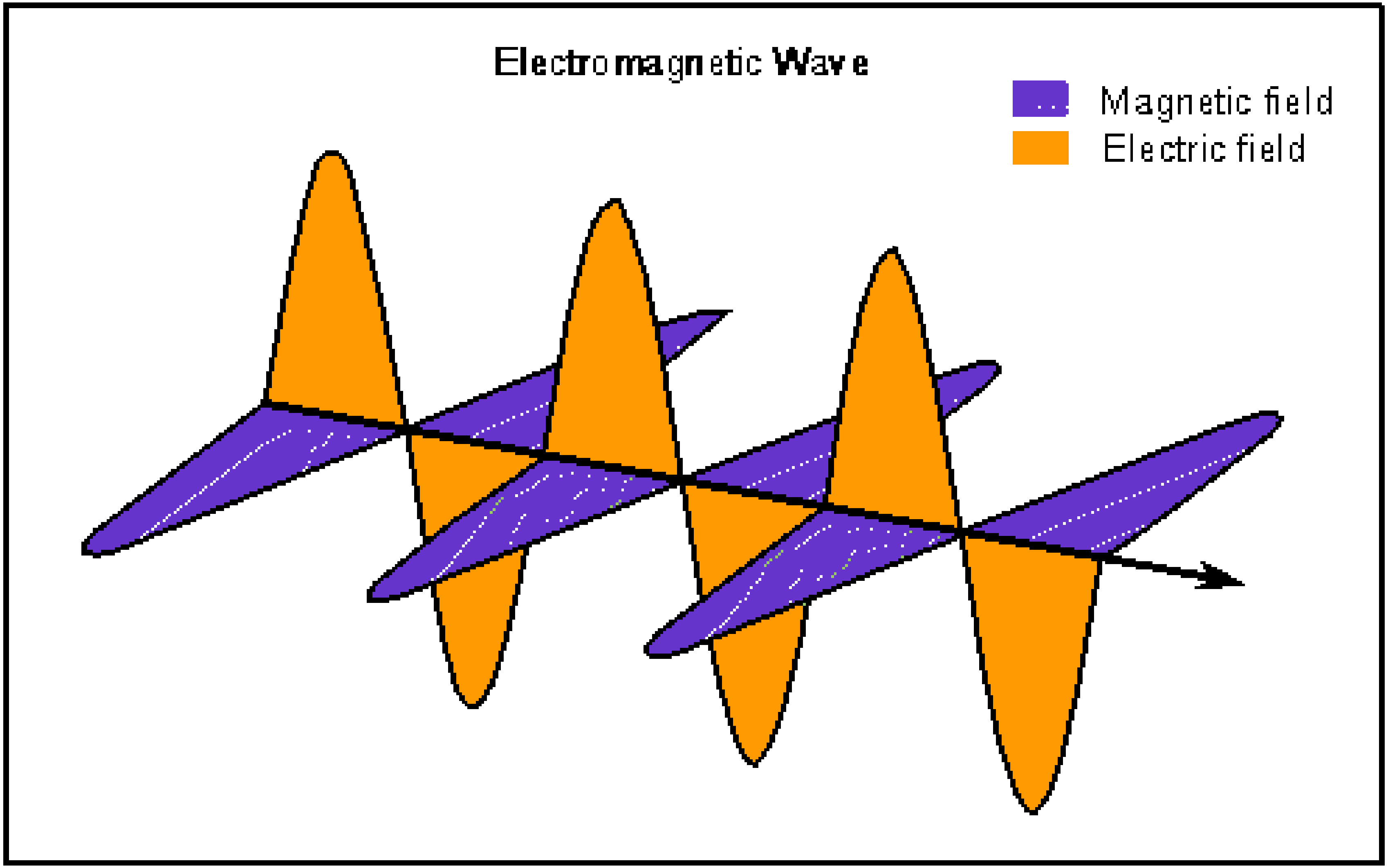
Induction of Electric and Magnetic Fields

- **According to Faraday:**
  - **Electric fields are created in any region of space where a magnetic field is changing with time.**
- **According to Maxwell:**
  - **A magnetic field is created in any region of space where an electric field is changing with time.**
- **These laws are inverses of each other and lead to the concept of electromagnetic waves**

Electromagnetic Waves

- **Composed of vibrating electric and magnetic fields the regenerate each other**
  - **Waves move outward from a vibrating charge**
  - **E.F. is always perpendicular to the M.F. and both are perpendicular to the direction of the moving wave**

Electromagnetic Waves



Electromagnetic Waves

- All electromagnetic waves move at the speed of light
  - Discovered by Maxwell
- Changing electric fields constantly induce changing magnetic fields and vice versa
  - If the waves traveled at less than the speed of light, they would rapidly die out

Electromagnetic waves

Electromagnetic Waves

A changing electric field gives rise to a changing magnetic field, which gives rise to a changing electric field, which gives rise to a changing magnetic field, which ...

$\Rightarrow$ You don't need any charges or currents around to produce "waving" E and B fields.

All electromagnetic waves move at the speed

$$c = 299792458 \text{ m/sec}$$

The "speed of light"

$$c = \lambda \times f$$

Changing E produces B

Changing magnetic field $\Rightarrow$ electric field

Changing electric field $\Rightarrow$ magnetic field ?

A beautiful symmetry in nature: $\vec{E} \Leftrightarrow \vec{B}$

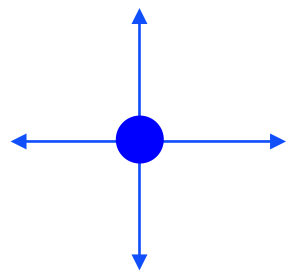
A changing magnetic flux produces an Electric field

A changing electric flux produces a Magnetic field

The Equations of Electromagnetism

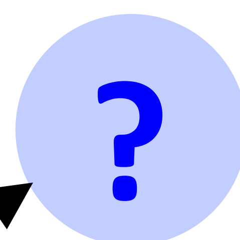
Gauss's Laws

..monopole..



1 $$\oint \underline{E} \bullet \underline{dA} = \frac{q}{\varepsilon_0}$$

2 $$\oint \underline{B} \bullet \underline{dA} = 0$$



...there's no magnetic monopole....!!

The Equations of Electromagnetism

Faraday's Law

3 $$\oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt}$$

.. if you change a magnetic field you induce an electric field.........

Ampere's Law

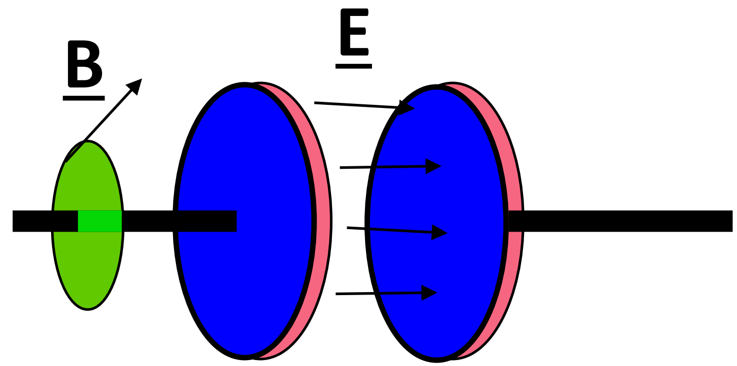
4 $$\oint \underline{B} \bullet \underline{dl} = \mu_0 I$$

.......is the reverse true..?

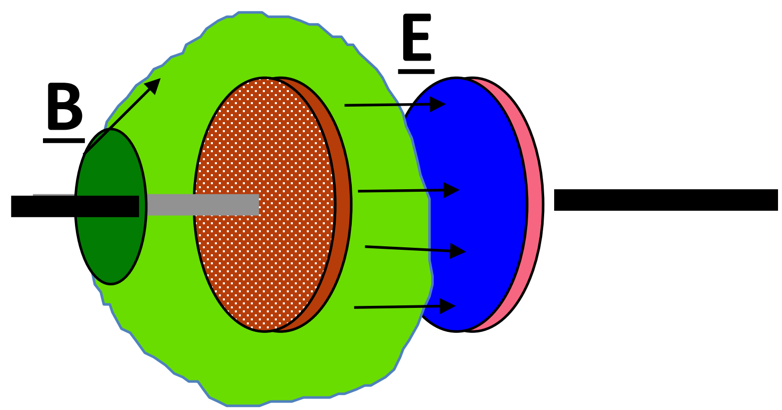
...lets take a look at charge flowing into a capacitor...

...when we derived Ampere's Law we assumed constant current...

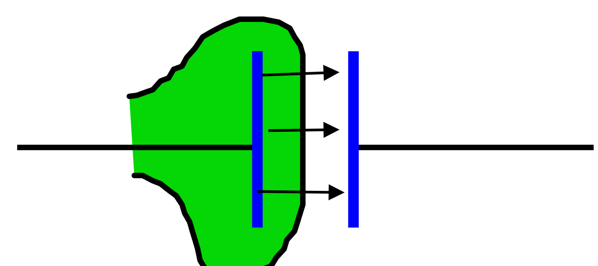
$$\boxed{\oint \underline{B} \bullet \underline{dl} = \mu_0 I}$$



.. if the loop encloses one plate of the capacitor..there is a problem … I = 0

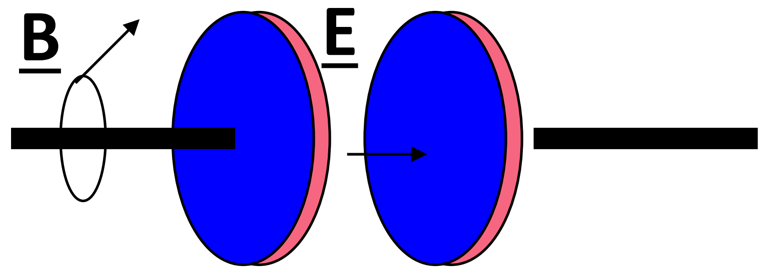


Side view: (Surface is now like a bag:)

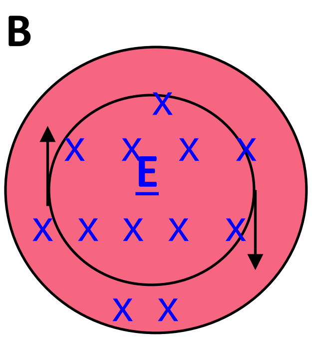


Maxwell solved this problem by realizing that….

Inside the capacitor there must be an induced magnetic field...



How?. Inside the capacitor there is a changing E $\Rightarrow$



A changing electric field induces a magnetic field

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0 I_d$$

where I_d is called the ***displacement current***

Therefore, Maxwell's revision of Ampere's Law becomes….

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Derivation of Displacement Current

For a capacitor, $q = \varepsilon_0 EA$ **and** $I = \dfrac{dq}{dt} = \varepsilon_0 \dfrac{d(EA)}{dt}$.

Now, the electric flux is given by EA, so: $I = \varepsilon_0 \dfrac{d(\Phi_E)}{dt}$**, where this current , not being associated with charges, is called the "Displacement current", I_d.**

Hence:

$$I_d = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

and:

$$\oint \underline{B} \bullet \underline{ds} = \mu_0 (I + I_d)$$

$$\Rightarrow \oint \underline{B} \bullet \underline{ds} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell's Equations of Electromagnetism in Vacuum (no charges, no masses)

Consider these equations in a vacuum.....

......no mass, no charges. no currents.....

$$\oint \underline{E} \bullet \underline{dA} = \frac{q}{\varepsilon_0} \quad \longrightarrow \quad \oint \underline{E} \bullet \underline{dA} = 0$$

$$\oint \underline{B} \bullet \underline{dA} = 0 \quad \longrightarrow \quad \oint \underline{B} \bullet \underline{dA} = 0$$

$$\oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt} \quad \longrightarrow \quad \oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt}$$

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \quad \longrightarrow \quad \oint \underline{B} \bullet \underline{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell's Equations of Electromagnetism in Vacuum (no charges, no masses)

$$\boxed{\oint \underline{E} \bullet \underline{dA} = 0}$$

$$\boxed{\oint \underline{B} \bullet \underline{dA} = 0}$$

$$\boxed{\oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt}}$$

$$\boxed{\oint \underline{B} \bullet \underline{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}}$$

Maxwell's equations (1)

Integral form of Maxwell's equations:

$$\oint \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon_0}$$

$$\oint \vec{B} \cdot d\vec{S} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

① **source of *E***

② **no magnetic charges / monopoles**

③ **changing *B* → *E***

④ **changing $E \rightarrow B$**

Maxwell's equations (2)

Differential form of Maxwell's equations:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

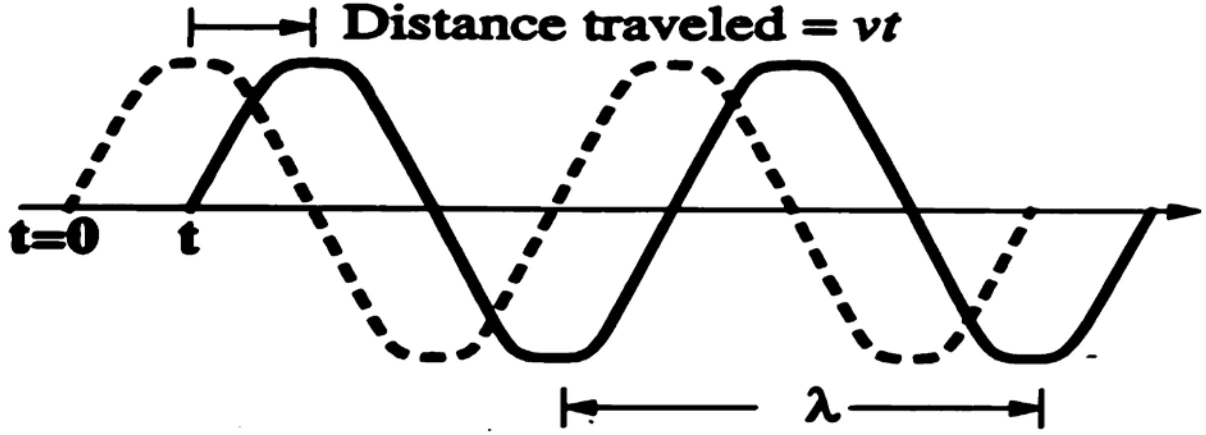
1) **Basic equations for all electromagnetism**

2) **As fundamental as Newton's laws**

3) **Important outcome: electromagnetic waves**

Properties of Waves

$$y(x, t) = y_m \sin(kx - \omega t)$$



Oscillates in *time* with *period* $T = 2\pi/\omega$

Frequency f defined as $f = 1/T$

Oscillates in *space* with *wavelength* $\lambda = 2\pi/k$

Moves a distance x in a time t with a *speed*

$$v = \frac{x}{t} = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$$

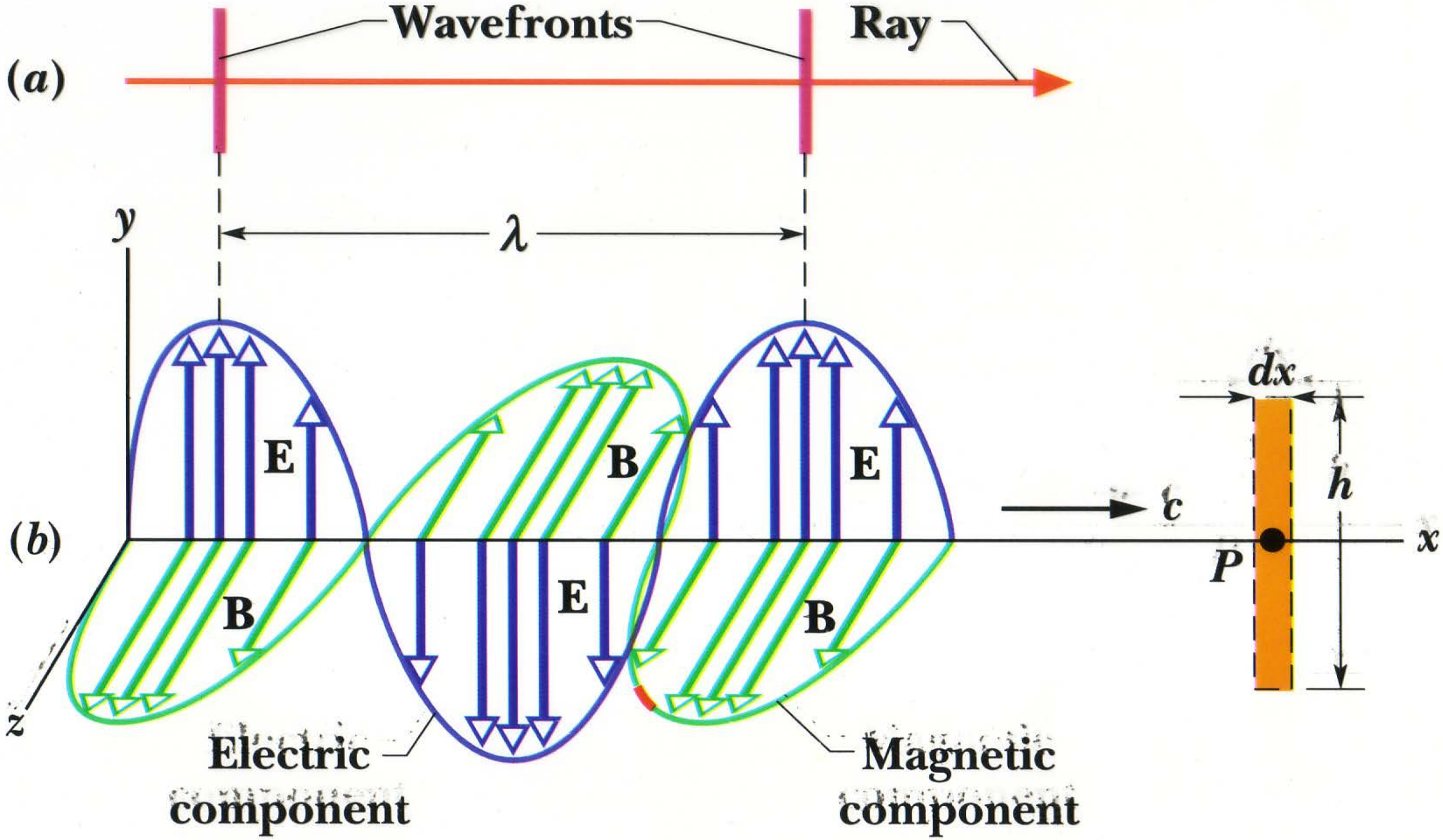
$\Rightarrow$ Speed of wave = wavelength $\times$ frequency

ELECTROMAGNETIC WAVES

- $E = E_{\mathrm{m}} \sin(kx - wt)$
- $B = B_{\mathrm{m}} \sin(kx - wt)$

$$k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f = \frac{2\pi}{T}$$

$$\text{wave speed} = c = \frac{\omega}{k} = \lambda f$$



Wavefronts
Ray
(a)
y
λ
E
B
E
B
E
B
c
dx
h
(b)
x
P
z
Electric component
Magnetic component

Electromagnetic Waves

Faraday's law: **dB/dt** $\longrightarrow$ **electric field**

Maxwell's modification of Ampere's law

dE/dt $\longrightarrow$ **magnetic field**

$$\boxed{\oint \underline{B} \bullet \underline{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}}$$

$$\boxed{\oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt}}$$

These two equations can be solved simultaneously.

The result is:

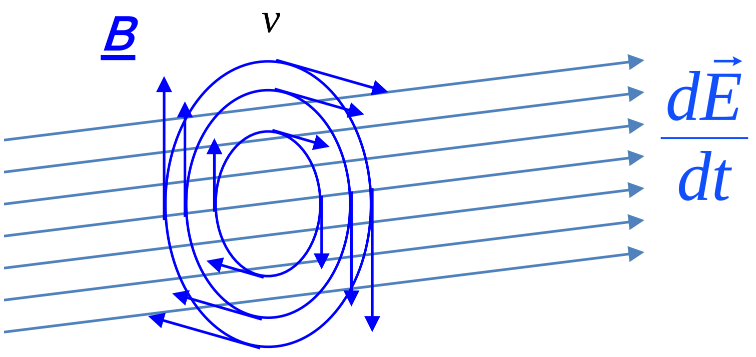
$$\underline{\mathbf{E}}(x, t) = E_P \sin(kx - \omega t)\ \hat{j}$$

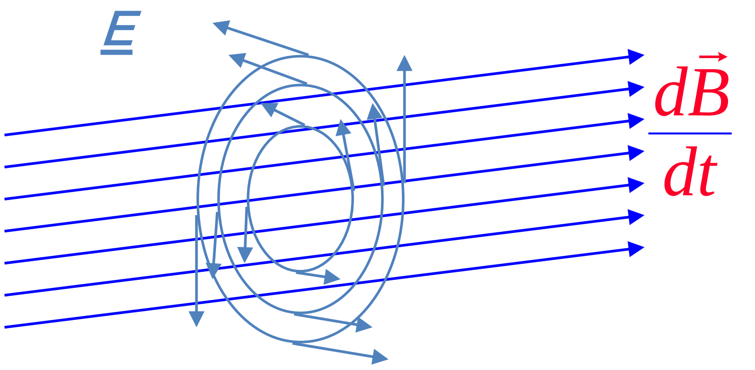
$$\underline{\mathbf{B}}(x, t) = B_P \sin(kx - \omega t)\ \hat{z}$$

Electromagnetic Waves

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt}$$





Special case..PLANE WAVES...

$$\vec{E} = E_y(x,t)\hat{j} \qquad \vec{B} = B_z(x,t)\hat{k}$$

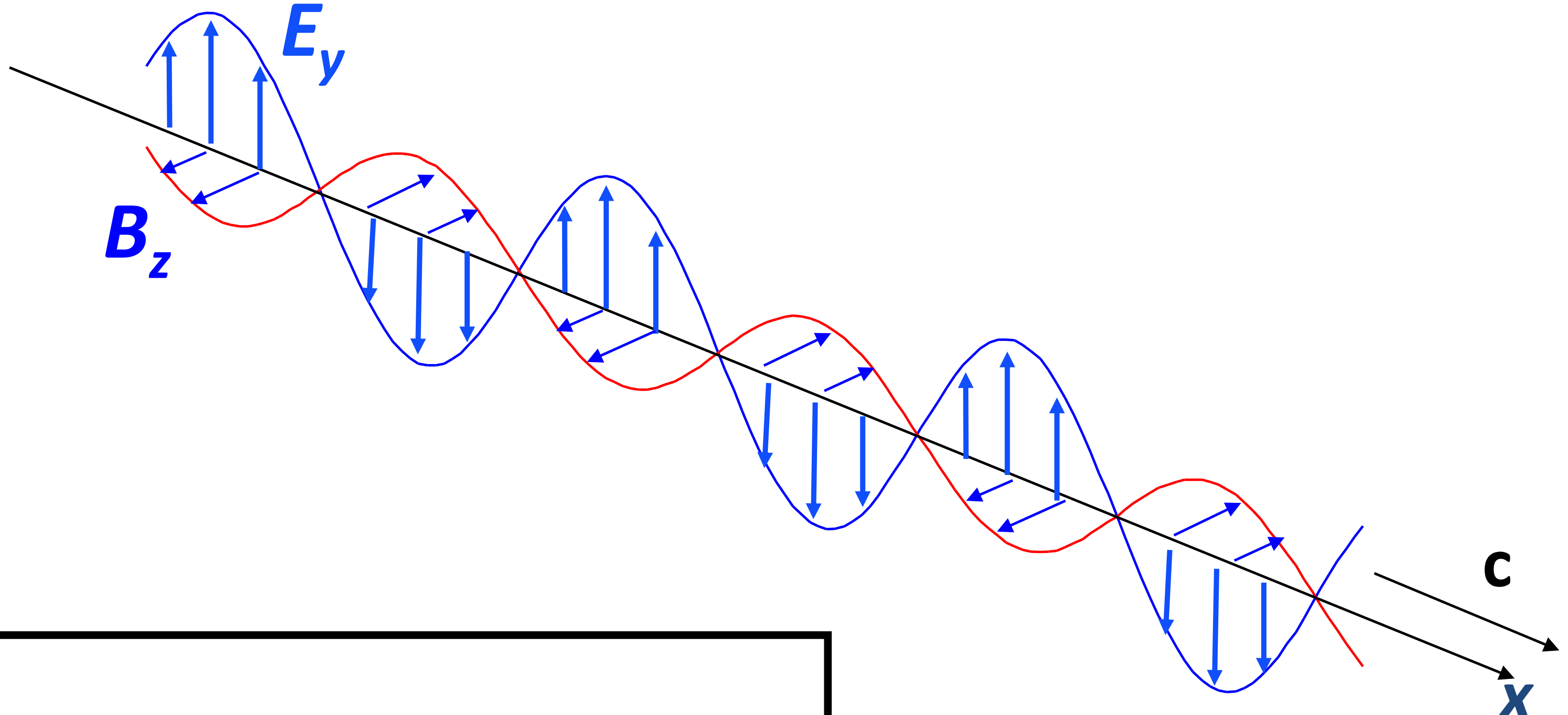
satisfy the wave equation

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

Maxwell's Solution

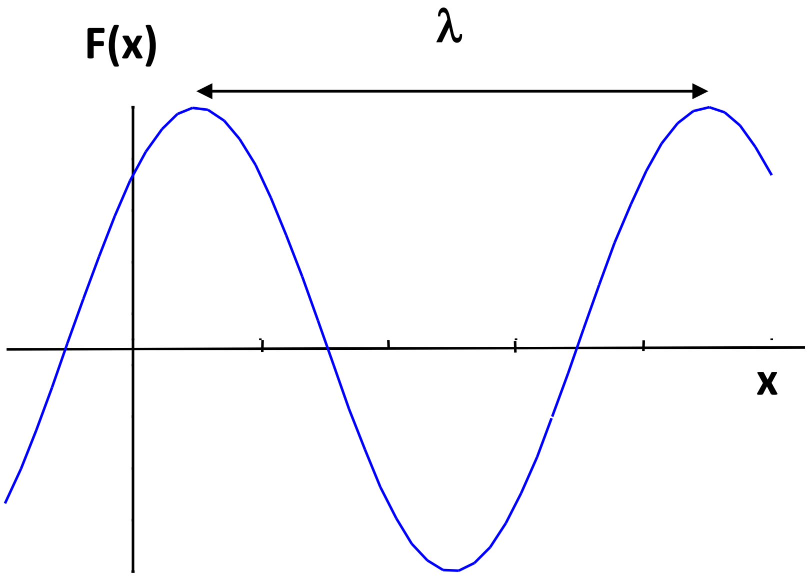
$$\boxed{\psi = A\sin(\omega t + \phi)}$$

Plane Electromagnetic Waves



$$\underline{E}(x, t) = E_P \sin(kx - \omega t)\ \hat{j}$$

$$\underline{B}(x, t) = B_P \sin(kx - \omega t)\ \hat{z}$$



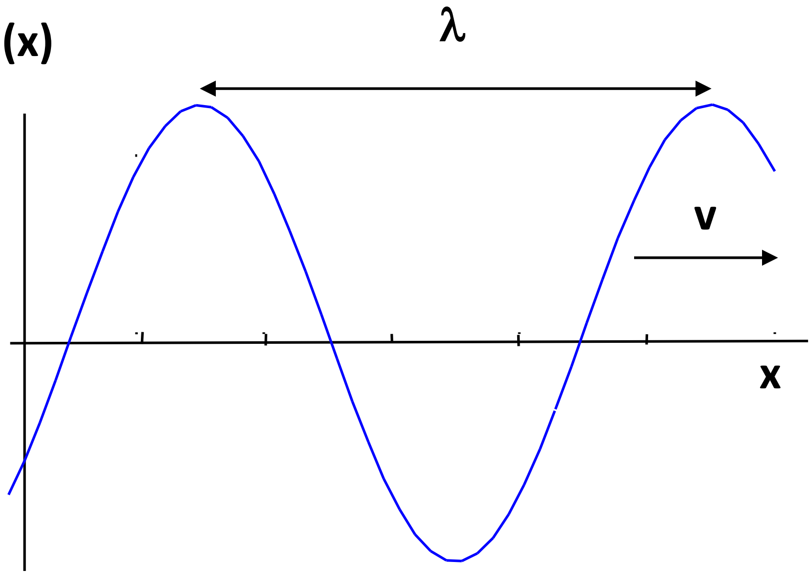
Static wave

$F(x) = F_P \sin(kx + \phi)$

$k = 2\pi / \lambda$

k = wavenumber

λ = wavelength



Moving wave

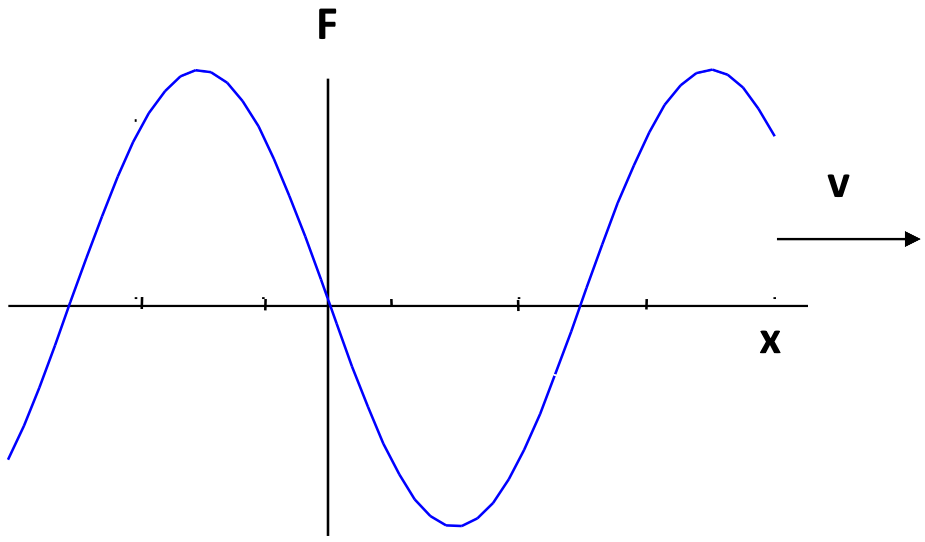
$F(x, t) = F_P \sin(kx - \omega t)$

$\omega = 2\pi / f$

ω = angular frequency

f = frequency

$v = \omega / k$



Moving wave

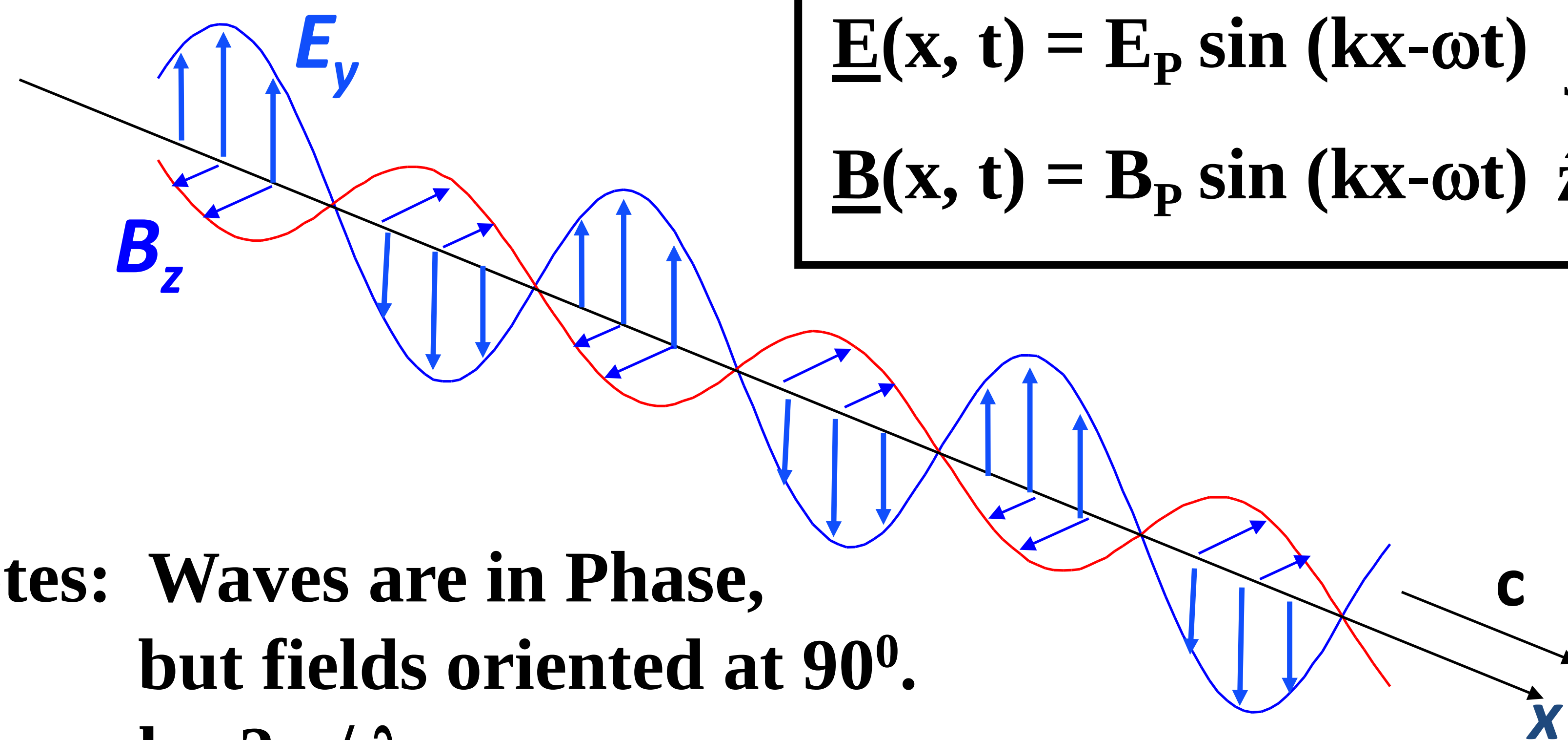
$F(x, t) = F_P \sin(kx - \omega t)$

What happens at x = 0 as a function of time?

$F(0, t) = F_P \sin(-\omega t)$

For x = 0 and t = 0 $\Rightarrow$ $F(0, 0) = F_P \sin(0)$
For x = 0 and t = t $\Rightarrow$ $F(0, t) = F_P \sin(0 - \omega t) = F_P \sin(-\omega t)$
This is equivalent to: $kx = -\omega t \Rightarrow x = -(\omega/k)\, t$
F(x=0) at time t is the same as $F[x=-(\omega/k)t]$ at time 0
The wave moves to the right with speed ω/k

Plane Electromagnetic Waves



$$\underline{E}(x, t) = E_P \sin(kx-\omega t)\ \hat{j}$$

$$\underline{B}(x, t) = B_P \sin(kx-\omega t)\ \hat{z}$$

Notes: Waves are in Phase,

but fields oriented at 90^0.

$k = 2\pi / \lambda$

Speed of wave is c=w/k (= *f* l)

$$c = 1/\sqrt{\varepsilon_0 \mu_0} = 3 \times 10^8 m/s$$

At all times E=cB.

Properties of EM wave

$$c^2\frac{\partial^2\vec{E}}{\partial x^2}=\frac{\partial^2\vec{E}}{\partial t^2},\qquad c^2\frac{\partial^2\vec{B}}{\partial x^2}=\frac{\partial^2\vec{B}}{\partial t^2}$$

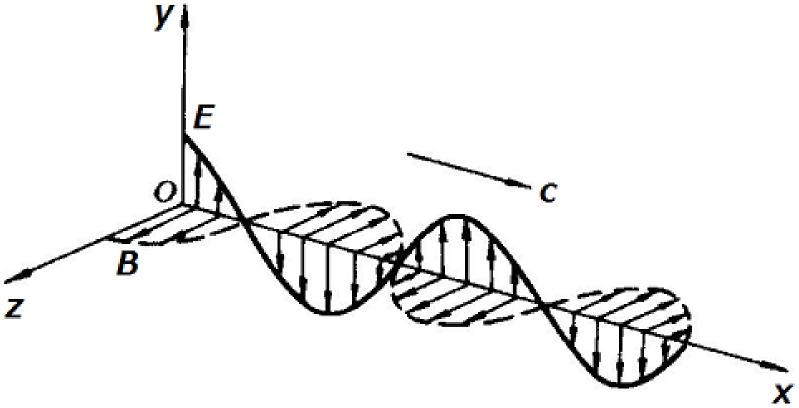
Particular solutions:

$$\begin{cases} E=E_0\cos\omega(t-\dfrac{x}{c}) \\ B=B_0\cos\omega(t-\dfrac{x}{c}) \end{cases}$$

1) Transverse wave

2) In phase: $\boxed{\dfrac{E}{B}=c}$

3) $\vec{E}\perp\vec{B}$



In general, the magnitude of the ***E***-field of the wave depends on distance r from the dipole, time t, and angle θ. This wave can be modeled by the travelling wave equation:

$$\begin{aligned} E(r,\theta,t) &= \left(\frac{E_1 \sin\theta}{r}\right)\sin\left(\frac{2\pi t}{T} - \frac{2\pi r}{\lambda}\right) \\ &= E_0(r,\theta)\sin(\omega t - kr) \end{aligned} \qquad [1\text{-}8]$$

where: E_1 = a constant with units of N·m/C which is proportional to the amplitude of the wave at r = 1 m and θ = 90°; this constant is a measure of the radiating strength of the dipole.

$E_0(r,\theta) \equiv E_1 \sin\theta / r$ is the amplitude of the wave (N/C) at a given r and θ. Further:

$T = 1/f$ is the period of oscillation (s)

$\omega = 2\pi f = 2\pi/T$ is the angular frequency (radians/s)

λ is the wavelength (m)

$k = 2\pi/\lambda$ is the 'wavevector' (radians/m)

Similarly, for the ***B***-field:

$$B(r,\theta,t) = B_0(r,\theta)\sin(\omega t - kr) \qquad [1\text{-}9]$$

where, from Eq. [1-7], the amplitude $B_0(r, \theta) = E_0(r, \theta)/c$. Thus, at any point r, angle θ and time t, $B = E/c$.

The pattern shown in Fig. 1-10 is for one instant; it is not static. The fields oscillate so that the sinusoidal pattern and associated energy propagates outward with the

speed common to all EM waves, that is, the *speed of light*, $c = 2.998\times10^{8}$ m/s (to 4 significant figures).[14]
Further, as for any periodic wave, c, λ, T and f are related:

$$c = \frac{\lambda}{T} = \lambda f = \frac{\omega}{k} \qquad [1\text{-}10]$$

Example

What is the wavelength of the EM waves broadcast by radio station CFRB (Toronto), where, as mentioned earlier (see footnote 12), $f = 1.010\times10^{6}$ Hz?

$$\lambda = \frac{c}{f} = \frac{2.998\times10^{8}\ \text{m/s}}{1.010\times10^{6}\ \text{Hz}} = 297\ \text{m}$$

Example

A measurement of the electric field 2.0 km horizontally from the CFRB transmitter shows that its amplitude is 100 mV/m.

a) What is the equation of the EM wave?

b) What is the electric field amplitude 3.0 km away horizontally and 1.0 km above the ground?

$$E(r,\theta,t) = \left(\frac{E_1 \sin\theta}{r}\right)\sin\left(\frac{2\pi t}{T} - \frac{2\pi r}{\lambda}\right) = E_0(r,\theta)\sin(\omega t - kr)$$

a) Since $f = 1.01\times10^{6}$ Hz, then $\omega = 2\pi f = 6.35\times10^{6}$ rad/s
$\lambda = 297$ m (from Example 2-6), $k = 2\pi/\lambda = 2.12\times10^{-2}$ rad/m

$E_1 \sin\theta / r = (E_1 \times \sin 90)/2000 = 100\times10^{-3}$
therefore $E_1 = 200$ Nm/C

$E(r,\theta,t) = (200\sin\theta / r)\sin(6.35\times10^{6}t - 2.12\times10^{-2}r)$

b) $\tan(90 - \theta) = 1/3$ therefore $\theta = 71.6°$
and $r = (3^2 + 1^2)^{1/2} = 10^{1/2} = 3.16$ km

$E_0(3.16\ \text{km}, 71.6°) = 200 \sin 71.6/3160 = 0.06$ V/m

Poynting vector $(\overrightarrow{P})$

- **Poynting vector represent the rate of energy flow per unit area in a plane electromagnetic wave.**

$$\therefore \overrightarrow{P} = \frac{1}{\mu_o} \overrightarrow{E} \times \overrightarrow{B}$$

The direction of $(\overrightarrow{P})$ gives the direction in which the energy is transferred. Unit: W/m^2

Energy in EM wave and The Poynting Vector

The Vector **P = E x B** has interpreted as representing the amount of field energy passing through the unit area of surface in unit time normally to the direction of flow of energy. This statement is termed as *Poynting's theorem* and the vector *P* is called *Poynting Vector.*

- Light waves (and all electromagnetic waves) carry energy

$$u = u_E + u_B = \frac{EB}{\mu_o c}$$

- A wave has an intensity

$$I = u_{average} c$$

Poynting Vector

$$I = \frac{1}{2}\frac{E_o B_o}{\mu_o} = \left|\vec{S}\right|_{av}$$

$$\vec{S} = \frac{1}{\mu_o}\vec{E} \times \vec{B}$$

33.5: Energy Transport and the Poynting Vector

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}\varepsilon_0 (cB)^2. = \tfrac{1}{2}\varepsilon_0 \frac{1}{\mu_0\varepsilon_0} B^2 = \frac{B^2}{2\mu_0} = u_B$$

Energy flux: $$S = uc$$

$$S = \left(\frac{\text{energy/time}}{\text{area}}\right)_{\text{inst}} = \left(\frac{\text{power}}{\text{area}}\right)_{\text{inst}}.$$

$$\boxed{S = \frac{1}{c\mu_0}E^2}$$

$$S = \frac{1}{\mu_0}EB,$$

$$\boxed{\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \qquad \text{(Poynting vector).}}$$

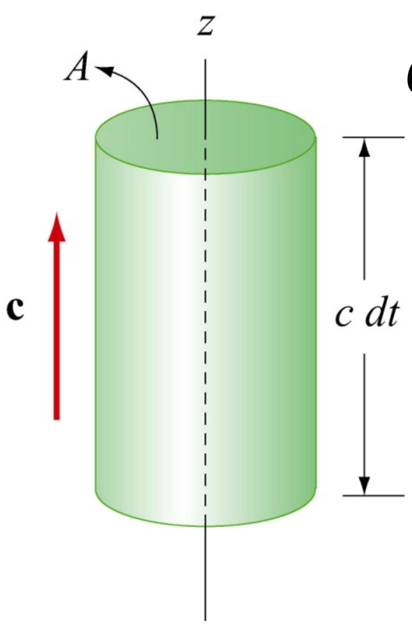
The direction of the Poynting vector $\vec{S}$ of an electromagnetic wave at any point gives the wave's direction of travel and the direction of energy transport at that point.

Example 1: Energy in EM Waves

Energy densities:

$$u_E = \frac{1}{2}\varepsilon_0 E^2, \; u_B = \frac{1}{2\mu_0} B^2$$

Consider cylinder:



$$dU = (u_E + u_B) A dz = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right) A c dt$$

What is rate of energy flow per unit area?

$$S = \frac{1}{A}\frac{dU}{dt} = \frac{c}{2}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right) = \frac{c}{2}\left(\varepsilon_0 c E B + \frac{EB}{c\mu_0}\right)$$

$$= \frac{EB}{2\mu_0}\left(\varepsilon_0 \mu_0 c^2 + 1\right) = \frac{EB}{\mu_0}$$

Example2: Radiation from the Sun reaches the Earth at a rate about 1350W/m^2. Assume it is a single EM wave, calculate E_0 and B_0.

Solution: Rate → time averaged S / intensity

$$\bar{S} = \frac{1}{2}\frac{E_0 B_0}{\mu_0} = \frac{1}{2}\varepsilon_0 c E_0^2 \quad \Rightarrow E_0 = \sqrt{\frac{2\bar{S}}{\varepsilon_0 c}} = 1.01\times 10^3 V/m$$

$$= \bar{u}\cdot c = \frac{1}{2}u_m c \qquad\qquad B_0 = \frac{E_0}{c} = 3.37\times 10^{-6} T$$

Example 3

Bright sunlight gives an irradiance of about 1000 W/m^2. Assuming it is a sinusoidal wave, what is the amplitude of the electric and magnetic fields in the wave?

This quantity, the irradiance, is often called the 'intensity' of the radiation.

From Eq. [1-14]:

$$I = \tfrac{1}{2}\varepsilon_0 c E_0^2 \qquad [1\text{-}14]$$

$$E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}} = \sqrt{\frac{2(1000)}{(8.85\times10^{-12})(3\times10^{8})}} = 868\ \mathrm{N/C}$$

(All quantities in this calculation are in S.I. units; check that the calculation does give E_0 in N/C.)

From Eq. [1-7], $B = E/c$, so the ***B***-field amplitude is:

$$B_0 = E_0/c = (868\ \mathrm{N/C})/(3.00\times10^{8}\ \mathrm{m/s}) = 2.89\times10^{-6}\ \mathrm{T}$$

Worked Example

- **An electromagnetic wave of frequency f = 3.0 MHz passes from vacuum into a non – magnetic medium with relative permittivity 4. Calculate the increment in its wavelength. Assume that for a non – magnetic medium μ_r=1.**

Solution

- Frequency of the em wave = f = 3.0 MHz = 3×10^6 Hz
- Relative permittivity of the non – magnetic medium = $\varepsilon_r = 4$
- Relative permeability of the non – magnetic medium = $\mu_r = 1$

- Velocity of em wave in vacuum = $C = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

- Wavelength of the EM wave in vacuum = λ = $$\frac{C}{f} = \frac{1}{f} \cdot \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

Velocity of em wave in non- magnetic medium = $$C' = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}}$$

- Wavelength of the em wave in non-magnetic medium =

$$\lambda' = \frac{C'}{f} = \frac{1}{f} \cdot \frac{1}{\sqrt{\mu_o \mu_r \varepsilon_o \varepsilon_r}}$$

- Therefore the change in wavelength =

$$\lambda' - \lambda = \frac{1}{f} \cdot \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \left(\frac{1}{\sqrt{\mu_r \varepsilon_r}} - 1 \right)$$

$$= \quad \frac{3 \times 10^8}{3 \times 10^6} \left[\frac{1}{\sqrt{4}} - 1 \right] = -50 \ \text{m}$$

i.e. the wavelength decreased by 50 m.

Homework

Q1

a) Calculate the electric field intensity of the radiation at the surface of the sun from the following data: power radiated by the sun, 3.8×10^{26} watts; radius of the sun, 7.0×10^{8} meters.

b) What is the electric field intensity of solar radiation at the surface of the earth? The average distance between the sun and the earth is 1.5×10^{11} meters.

c) Calculate the value of $\mathscr{S}$ at the surface of the earth, neglecting absorption in the atmosphere.

Q2

Express the following quantities in terms of kilograms, meters, seconds, and amperes:

joule, watt,
coulomb, volt, ohm, siemens, farad,
weber, tesla, henry.

Appendix

Relation of the speed of light and electric and magnetic vacuum constants

$$c = 1/\sqrt{\epsilon_0\mu_0} = 299792458 \text{ m/s}$$

$$\epsilon_0 \approx 8.854 \cdot 10^{-12} \text{ As/Vm},$$

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ Vs/Am},$$

ε_0	permittivity of free space, also called the electric constant	As/Vm or F/m (farad per meter)
μ_0	permeability of free space, also called the magnetic constant	Vs/Am or H/m (henry per meter)

Differential operators

$\nabla \cdot$ the divergence operator div

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$\nabla \times$ the curl operator curl, rot

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Other notation used

$$\partial_x = \frac{\partial}{\partial x}$$

$\frac{\partial}{\partial t}$ the partial derivative with respect to time

Reflection, Refraction, Polarization

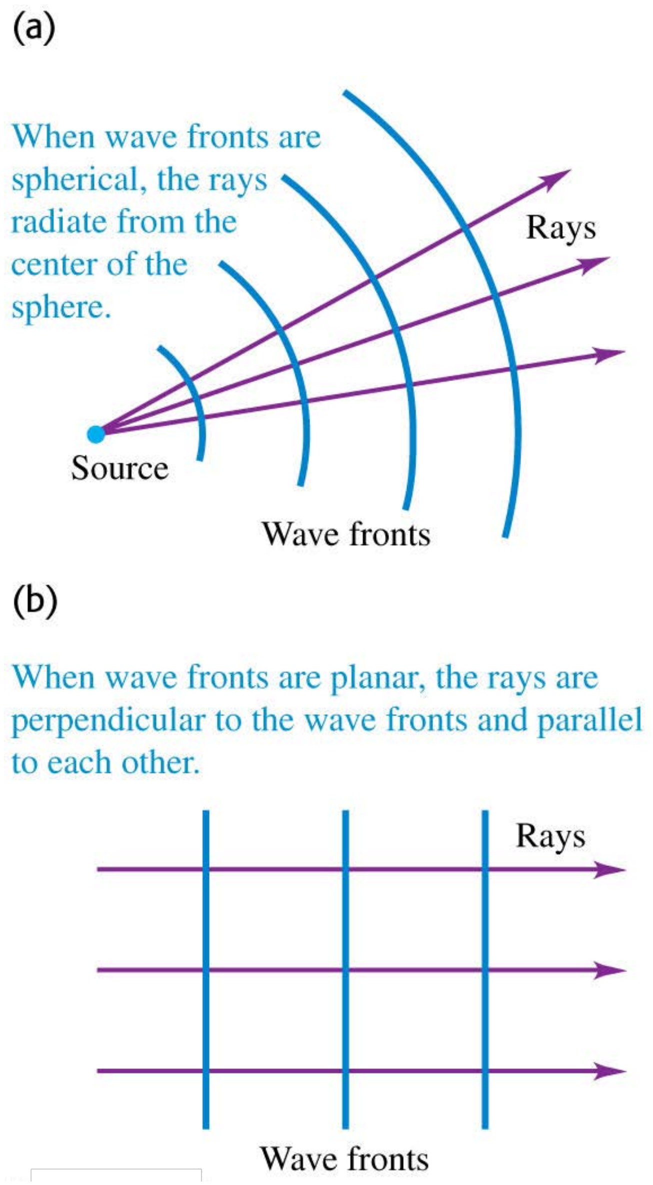
(3)

Hind I. Al-Shaikh

The nature of light

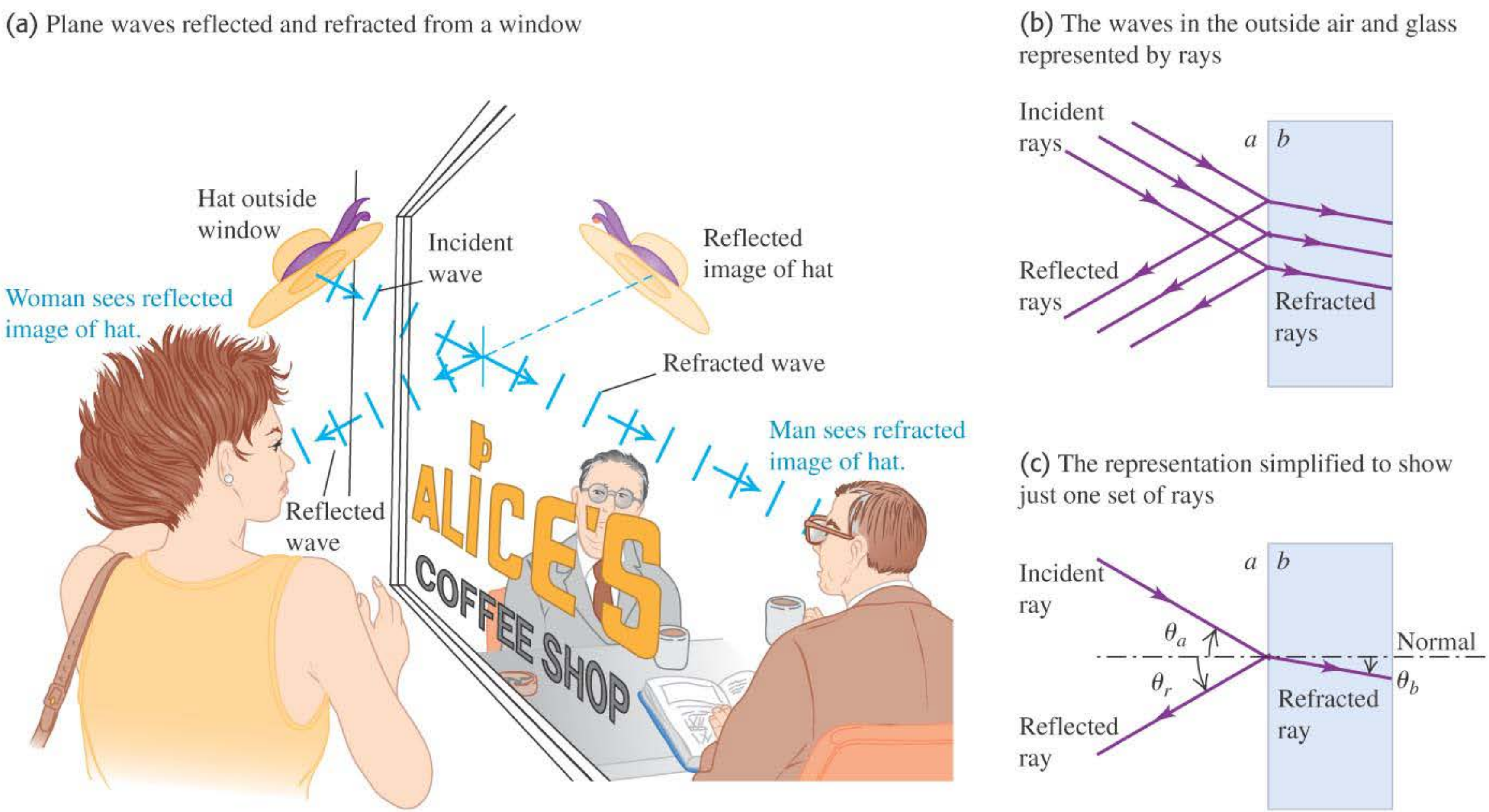
– Light has properties of *both* waves and particles. The wave model is easier for explaining propagation, but some other behavior requires the particle model.

– The *rays* are perpendicular to the *wave fronts*. See Figure 1 at the right.



Reflection and refraction

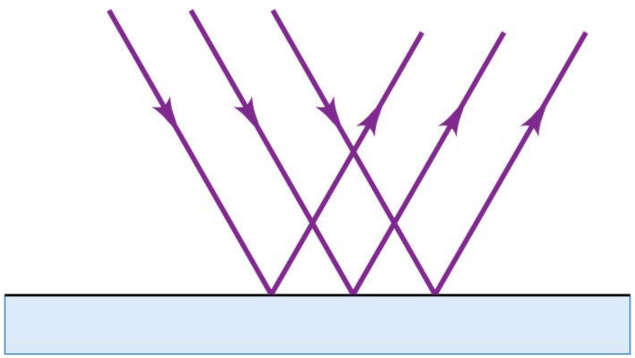
- **In Figure 2 the light is both reflected *and* refracted by the window.**



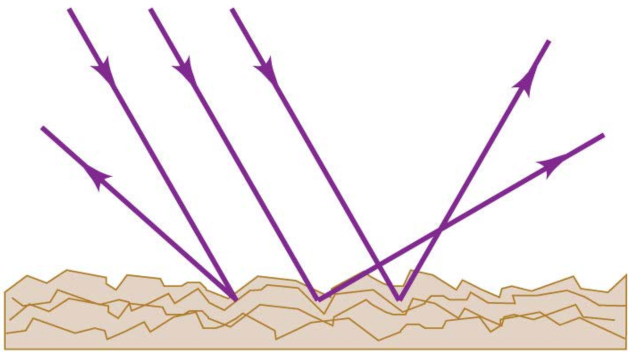
Specular and diffuse reflection

- *Specular reflection* **occurs at a very smooth surface (left figure).**
- *Diffuse reflection* **occurs at a rough surface (right figure).**
- **Our primary concern is with specular reflection.**

(a) Specular reflection



(b) Diffuse reflection



Laws of reflection and refraction

- **The frequency does not change on passing through a surface, but velocity does, and so wavelength.**
- $f = f_0 \Rightarrow v/\lambda = v_0/\lambda_0 \Rightarrow v/c\lambda = v_0/c\lambda_0$
- **The *index of refraction* is**

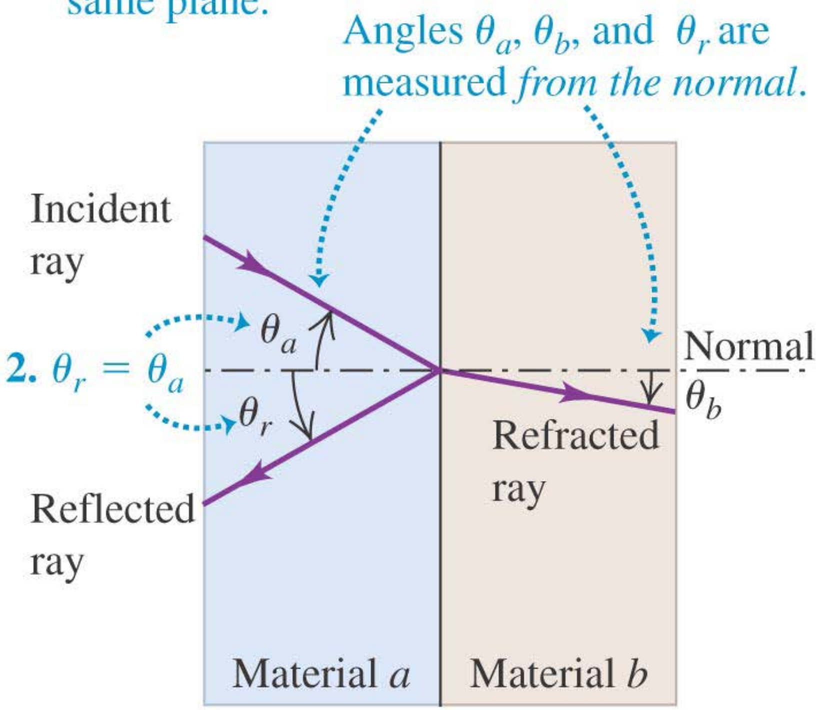
  $n = c/v > 1$
- **Angles are measured with respect to the *normal*.**
- ***Reflection*: The angle of reflection is equal to the angle of incidence.**
- ***Refraction*: Snell's law applies.**

  In a material

  $\lambda = \lambda_0/n$

Figure 3 (right) illustrates the laws of reflection and refraction.

1. The incident, reflected, and refracted rays and the normal to the surface all lie in the same plane.

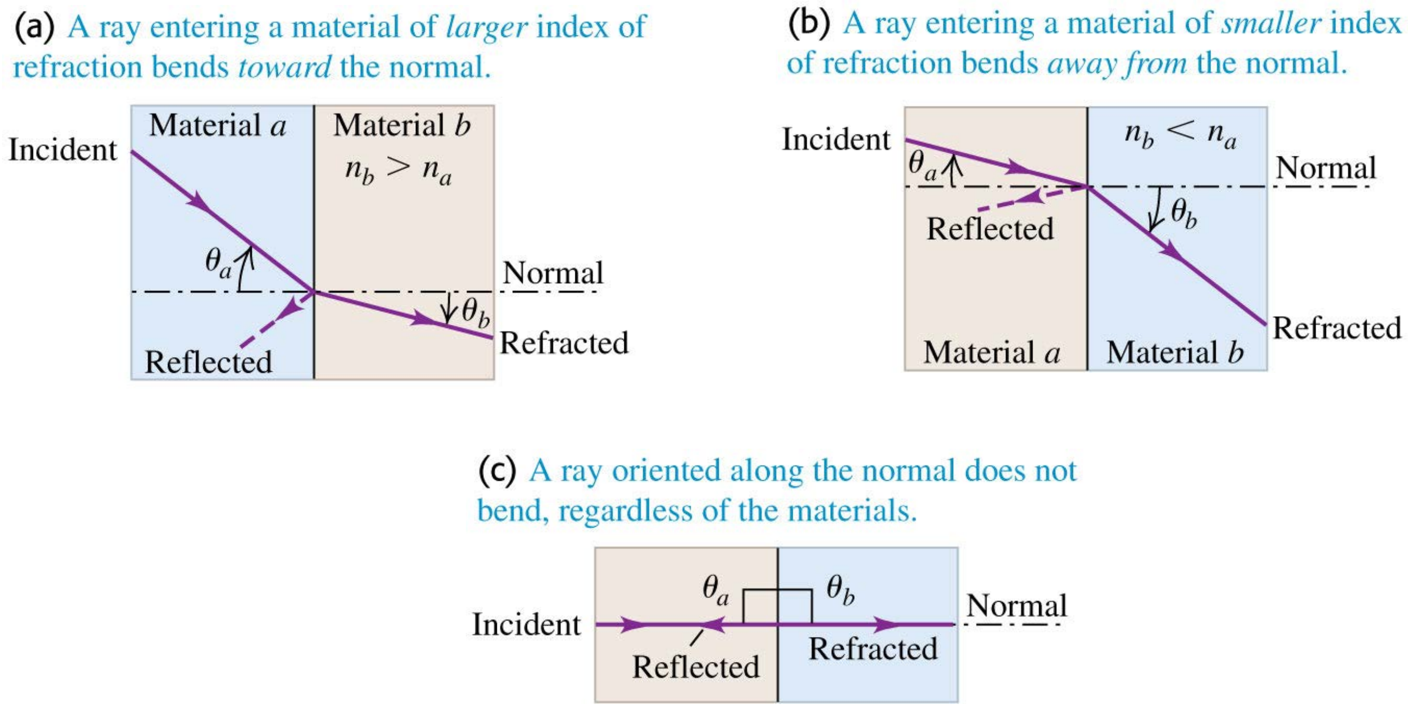


3. When a monochromatic light ray crosses the interface between two given materials *a* and *b*, the angles θ_a and θ_b are related to the indexes of refraction of *a* and *b* by

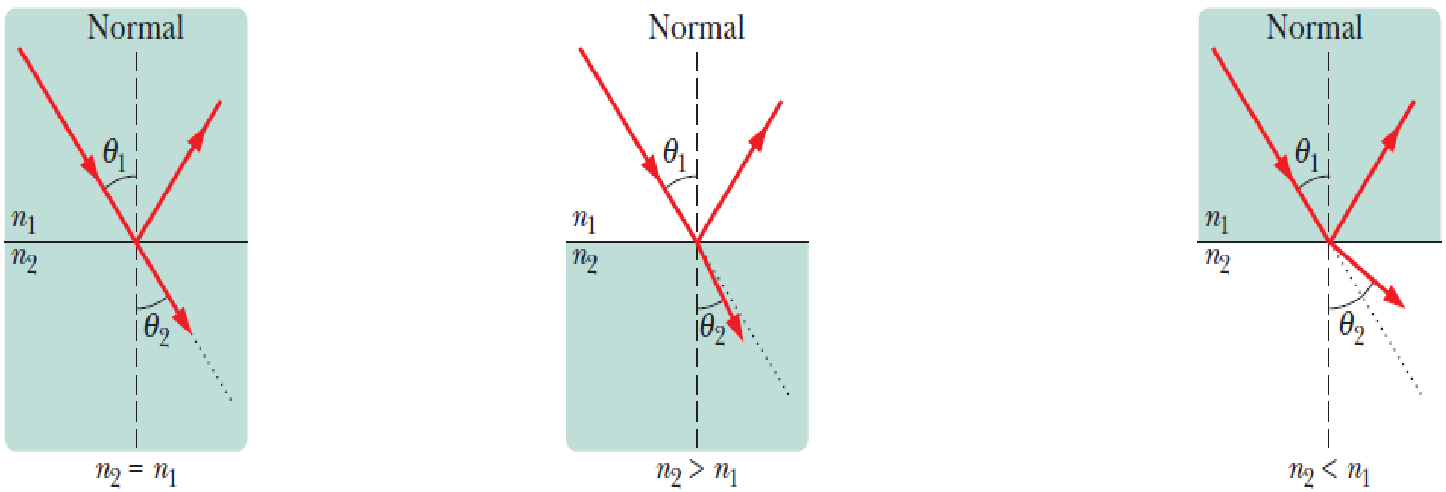
$$\frac{\sin\theta_a}{\sin\theta_b} = \frac{n_b}{n_a}$$

Reflection and refraction in three cases

- **Figure 4 below shows three important cases:**
  - ✓ **If $n_b > n_a$ the refracted ray is bent *toward* the normal.**
  - ✓ **If $n_b < n_a$ the refracted ray is bent *away from* the normal.**
  - ✓ **A ray oriented along the normal never bends.**



Reflection and Refraction



(*a*) If the indexes match, there is no direction change.

(*b*) If the next index is greater, the ray is bent *toward* the normal.

(*c*) If the next index is less, the ray is bent *away from* the normal.

Fig. 33-17 Refraction of light traveling from a medium with an index of refraction n_1 into a medium with an index of refraction n_2. (*a*) The beam does not bend when $n_2 = n_1$; the refracted light then travels in the *undeflected direction* (the dotted line), which is the same as the direction of the incident beam. The beam bends (*b*) toward the normal when $n_2 > n_1$ and (*c*) away from the normal when $n_2 < n_1$.

Why does the ruler appear to be bent?

The straight ruler in Figure 5(a) appears to bend at the surface of the water, why?

a) This ruler is actually straight, but it appears to bend at the surface of the water. (b) Light rays from any submerged object bend away from the normal when they emerge into the air. As seen by an observer above the surface of the water, the object appears to be much closer to the surface than it actually is

$\theta_a = 0$ and $\sin\theta_a = 0$, so from Eq. (33.4) θ_b is also equal to zero, so the transmitted ray is also normal to the interface. Equation (33.2) shows that θ_r, too, is equal to zero, so the reflected ray travels back along the same path as the incident ray.

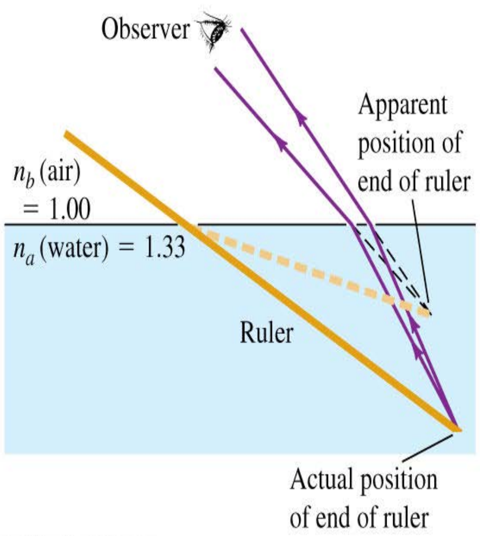
The law of refraction explains why a partially submerged ruler or drinking straw appears bent; light rays coming from below the surface change in direction at the air–water interface, so the rays appear to be coming from a position above their actual point of origin

$$\theta_r = \theta_a \qquad \text{(law of reflection)} \qquad (33.2)$$

$$n_a \sin\theta_a = n_b \sin\theta_b \qquad \text{(law of refraction)} \qquad (33.4)$$

(a) A straight ruler half-immersed in water

(b) Why the ruler appears bent



33.8: Reflection and Refraction:

Table 33-1

Some Indexes of Refraction[a]

Medium	Index	Medium	Index
Vacuum	Exactly 1	Typical crown glass	1.52
Air (STP)[b]	1.00029	Sodium chloride	1.54
Water (20°C)	1.33	Polystyrene	1.55
Acetone	1.36	Carbon disulfide	1.63
Ethyl alcohol	1.36	Heavy flint glass	1.65
Sugar solution (30%)	1.38	Sapphire	1.77
Fused quartz	1.46	Heaviest flint glass	1.89
Sugar solution (80%)	1.49	Diamond	2.42

[a]For a wavelength of 589 nm (yellow sodium light).

[b]STP means "standard temperature (0°C) and pressure (1 atm)."

An example of reflection and refraction

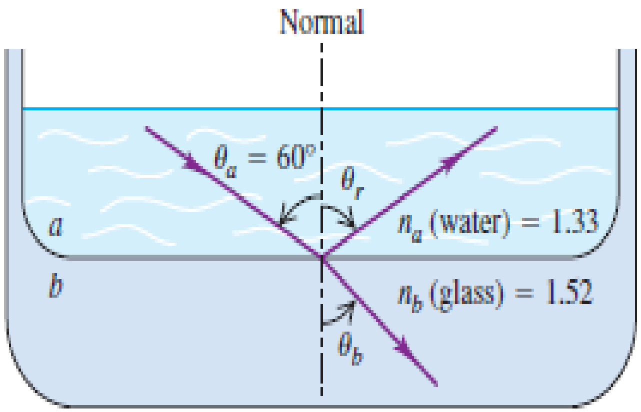
Example 33.1 Reflection and refraction

In Fig. 33.11, material a is water and material b is glass with index of refraction 1.52. The incident ray makes an angle of 60.0° with the normal; find the directions of the reflected and refracted rays.

SOLUTION

IDENTIFY and SET UP: This is a problem in geometric optics. We are given the angle of incidence $\theta_a = 60.0°$ and the indexes of refraction $n_a = 1.33$ and $n_b = 1.52$. We must find the angles of reflection and refraction θ_r and θ_b; to do this we use Eqs. (33.2) and (33.4), respectively. Figure 33.11 shows the rays and angles; n_b is slightly greater than n_a, so by Snell's law [Eq. (33.4)] θ_b is slightly smaller than θ_a, as the figure shows.

33.11 Reflection and refraction of light passing from water to glass.



EXECUTE: According to Eq. (33.2), the angle the reflected ray makes with the normal is the same as that of the incident ray, so $\theta_r = \theta_a = 60.0°$.

To find the direction of the refracted ray we use Snell's law, Eq. (33.4):

$$n_a \sin\theta_a = n_b \sin\theta_b$$

$$\sin\theta_b = \frac{n_a}{n_b}\sin\theta_a = \frac{1.33}{1.52}\sin 60.0° = 0.758$$

$$\theta_b = \arcsin(0.758) = 49.3°$$

EVALUATE: The second material has a larger refractive index than the first, as in Fig. 33.8a. Hence the refracted ray is bent toward the normal and $\theta_b < \theta_a$.

Example 33.2 Index of refraction in the eye

The wavelength of the red light from a helium-neon laser is 633 nm in air but 474 nm in the aqueous humor inside your eyeball. Calculate the index of refraction of the aqueous humor and the speed and frequency of the light in it.

SOLUTION

IDENTIFY and SET UP: The key ideas here are (i) the definition of index of refraction n in terms of the wave speed v in a medium and the speed c in vacuum, and (ii) the relationship between wavelength λ_0 in vacuum and wavelength λ in a medium of index n. We use Eq. (33.1), $n = c/v$; Eq. (33.5), $\lambda = \lambda_0/n$; and $v = \lambda f$.

EXECUTE: The index of refraction of air is very close to unity, so we assume that the wavelength λ_0 in vacuum is the same as that in air, 633 nm. Then from Eq. (33.5),

$$\lambda = \frac{\lambda_0}{n} \qquad n = \frac{\lambda_0}{\lambda} = \frac{633\ \text{nm}}{474\ \text{nm}} = 1.34$$

This is about the same index of refraction as for water. Then, using $n = c/v$ and $v = \lambda f$, we find

$$v = \frac{c}{n} = \frac{3.00 \times 10^8\ \text{m/s}}{1.34} = 2.25 \times 10^8\ \text{m/s}$$

$$f = \frac{v}{\lambda} = \frac{2.25 \times 10^8\ \text{m/s}}{474 \times 10^{-9}\ \text{m}} = 4.74 \times 10^{14}\ \text{Hz}$$

EVALUATE: Note that while the speed and wavelength have different values in air and in the aqueous humor, the *frequency* in air, f_0, is the same as the frequency f in the aqueous humor:

$$f_0 = \frac{c}{\lambda_0} = \frac{3.00 \times 10^8\ \text{m/s}}{633 \times 10^{-9}\ \text{m}} = 4.74 \times 10^{14}\ \text{Hz}$$

When a light wave passes from one material into another, the wave speed and wavelength both change but the wave frequency is unchanged.

Example, Reflection and Refraction of a Monochromatic Beam:

(a) In Fig. 33-22*a*, a beam of monochromatic light reflects and refracts at point A on the interface between material 1 with index of refraction $n_1 = 1.33$ and material 2 with index of refraction $n_2 = 1.77$. The incident beam makes an angle of 50° with the interface. What is the angle of reflection at point A? What is the angle of refraction there?

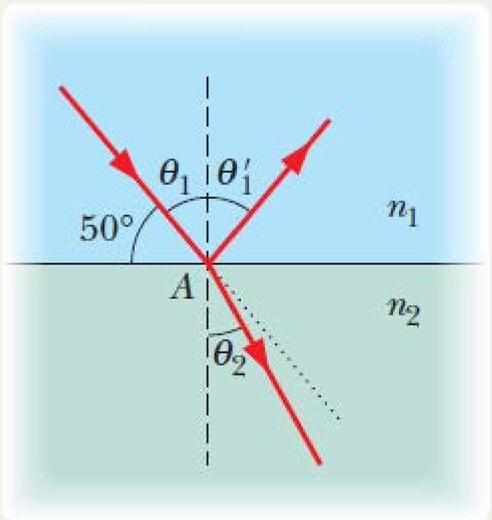


Fig. 33-22 (*a*)

Calculations: In Fig. 33-22*a*, the normal at point A is drawn as a dashed line through the point. Note that the angle of incidence θ_1 is not the given 50° but is 90° − 50° = 40°. Thus, the angle of reflection is

$$\theta'_1 = \theta_1 = 40°. \qquad \text{(Answer)}$$

The light that passes from material 1 into material 2 undergoes refraction at point A on the interface between the two materials. Again we measure angles between light rays and a normal, here at the point of refraction. Thus, in Fig. 33-22*a*, the angle of refraction is the angle marked θ_2. Solving Eq. 33-42 for θ_2 gives us

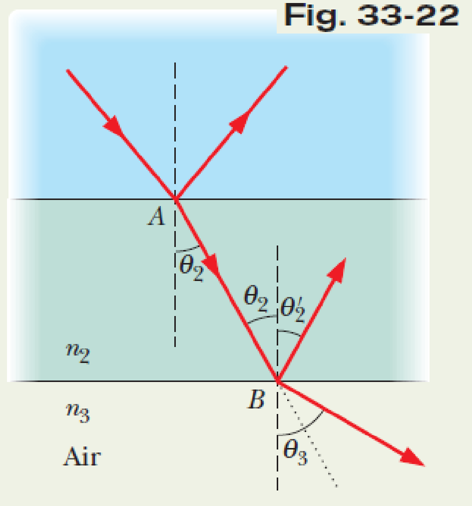
$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1.33}{1.77}\sin 40°\right)$$

$$= 28.88° \approx 29°. \qquad \text{(Answer)}$$

This result means that the beam swings toward the normal (it was at 40° to the normal and is now at 29°). The reason is that when the light travels across the interface, it moves into a material with a greater index of refraction. *Caution:* Note that the beam does *not* swing through the normal so that it appears on the left side of Fig. 33-22*a*.

Example, Reflection and Refraction of a Monochromatic Beam:

Fig. 33-22



(b)

(b) The light that enters material 2 at point A then reaches point B on the interface between material 2 and material 3, which is air, as shown in Fig. 33-22b. The interface through B is parallel to that through A. At B, some of the light reflects and the rest enters the air. What is the angle of reflection? What is the angle of refraction into the air?

Calculations: We first need to relate one of the angles at point B with a known angle at point A. Because the interface through point B is parallel to that through point A, the incident angle at B must be equal to the angle of refraction θ_2, as shown in Fig. 33-22b. Then for reflection, we again use the law of reflection. Thus, the angle of reflection at B is

$$\theta_2' = \theta_2 = 28.88° \approx 29°. \qquad \text{(Answer)}$$

Next, the light that passes from material 2 into the air undergoes refraction at point B, with refraction angle θ_3. Thus, we again apply Snell's law of refraction, but this time we write Eq. 33-40 as

$$n_3 \sin \theta_3 = n_2 \sin \theta_2. \qquad (33\text{-}43)$$

Solving for θ_3 then leads to

$$\theta_3 = \sin^{-1}\left(\frac{n_2}{n_3}\sin \theta_2\right) = \sin^{-1}\left(\frac{1.77}{1.00}\sin 28.88°\right)$$
$$= 58.75° \approx 59°. \qquad \text{(Answer)}$$

This result means that the beam swings away from the normal (it was at 29° to the normal and is now at 59°). The reason is that when the light travels across the interface, it moves into a material (air) with a lower index of refraction.

Total internal reflection

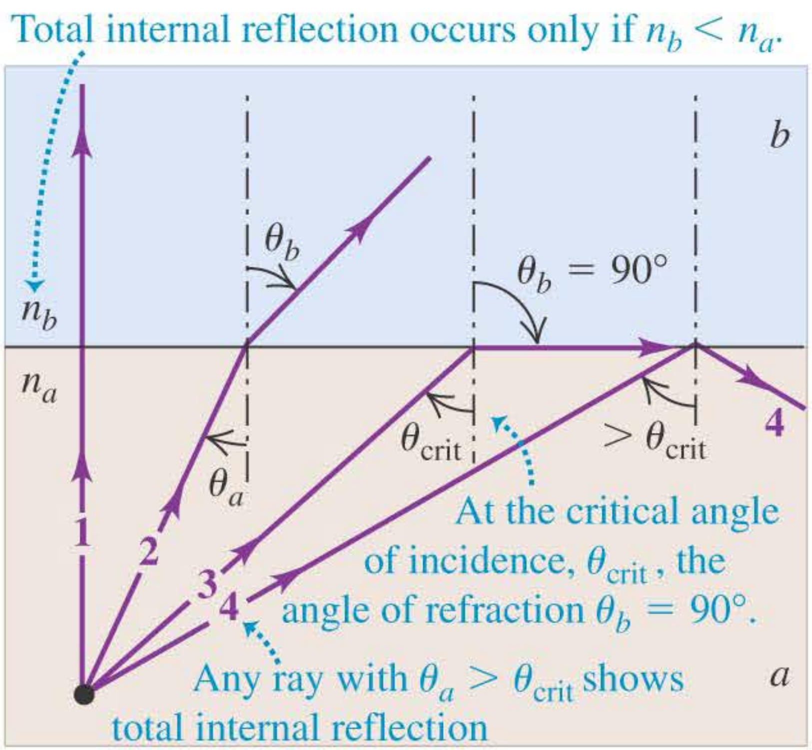
- **Light striking at the critical angle emerges tangent to the surface. (See Figure 33.13 below).**

$$n_a \sin\theta_a = n_b \sin\theta_b = n_b \text{ for total internal reflection}$$

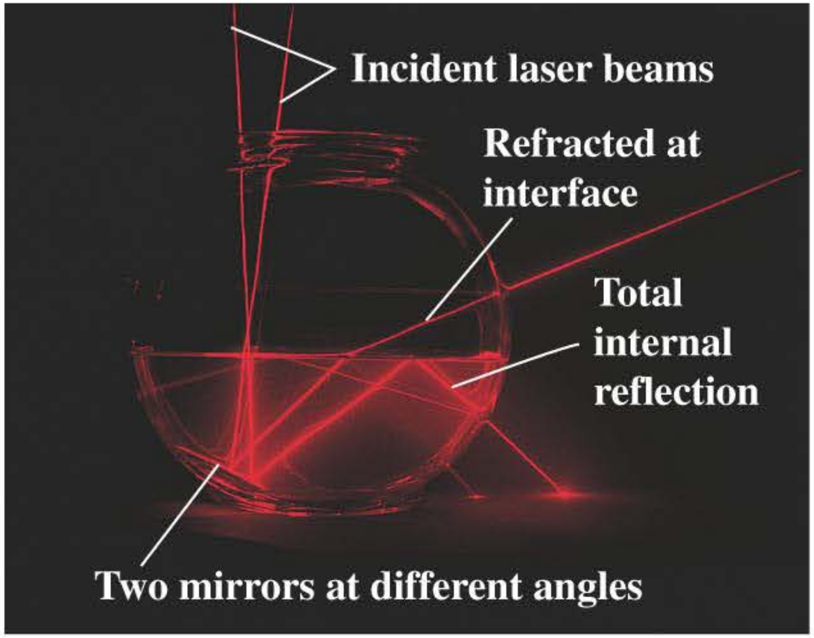
- **If $\theta_a > \theta_{crit}$, the light is undergoes *total internal reflection*.**

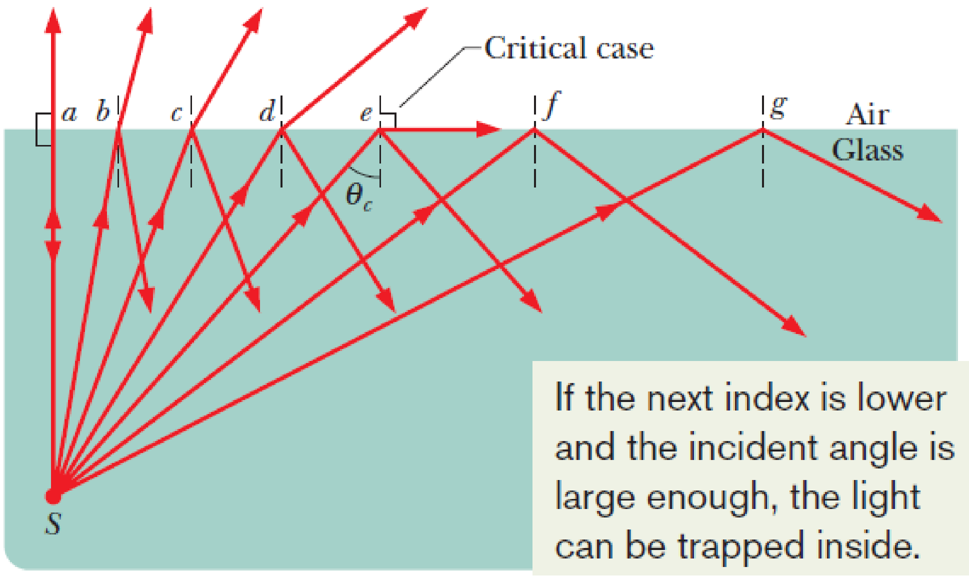
$$\theta_{crit} = \sin^{-1}(n_b / n_a)$$

(a) Total internal reflection



(b) Total internal reflection demonstrated with a laser, mirrors, and water in a fishbowl





(a)

(b)

For angles of incidence larger than θ_a there is no refracted ray and all the light is reflected; this effect is called total internal reflection. For the critical angle.

$$n_1 \sin \theta_c = n_2 \sin 90°,$$

Which means that

$$\theta_c = \sin^{-1} \frac{n_2}{n_1} \qquad \text{(critical angle).}$$

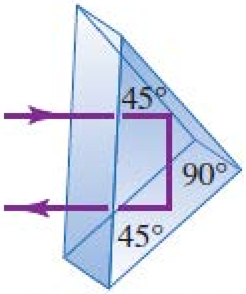
Applications of Total Internal Reflection

Total internal reflection finds numerous uses in optical technology. As an example, consider glass with index of refraction $n = 1.52$. If light propagating within this glass encounters a glass–air interface, the critical angle is

$$\sin\theta_{\text{crit}} = \frac{1}{1.52} = 0.658 \qquad \theta_{\text{crit}} = 41.1^\circ$$

The light will be *totally reflected* if it strikes the glass–air surface at an angle of 41.1° or larger. Because the critical angle is slightly smaller than 45°, it is possible to use a prism with angles of 45°−45°−90° as a totally reflecting surface. As reflectors, totally reflecting prisms have some advantages over metallic surfaces such as ordinary coated-glass mirrors. While no metallic surface reflects 100% of the light incident on it, light can be *totally* reflected by a prism. These reflecting properties of a prism are permanent and unaffected by tarnishing.

(a) Total internal reflection in a Porro prism



If the incident beam is oriented as shown, total internal reflection occurs on the 45° faces (because, for a glass–air interface, $\theta_{\text{crit}} = 41.1$).

Conceptual Example 33.4 A leaky periscope

A submarine periscope uses two totally reflecting 45°–45°–90° prisms with total internal reflection on the sides adjacent to the 45° angles. Explain why the periscope will no longer work if it springs a leak and the bottom prism is covered with water.

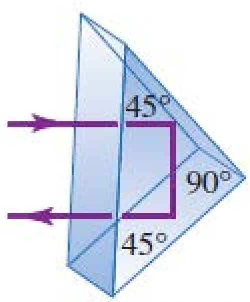
SOLUTION

The critical angle for water ($n_b = 1.33$) on glass ($n_a = 1.52$) is

$$\theta_{\text{crit}} = \arcsin\frac{1.33}{1.52} = 61.0^\circ$$

The 45° angle of incidence for a totally reflecting prism is *smaller* than this new 61° critical angle, so total internal reflection does not occur at the glass–water interface. Most of the light is transmitted into the water, and very little is reflected back into the prism.

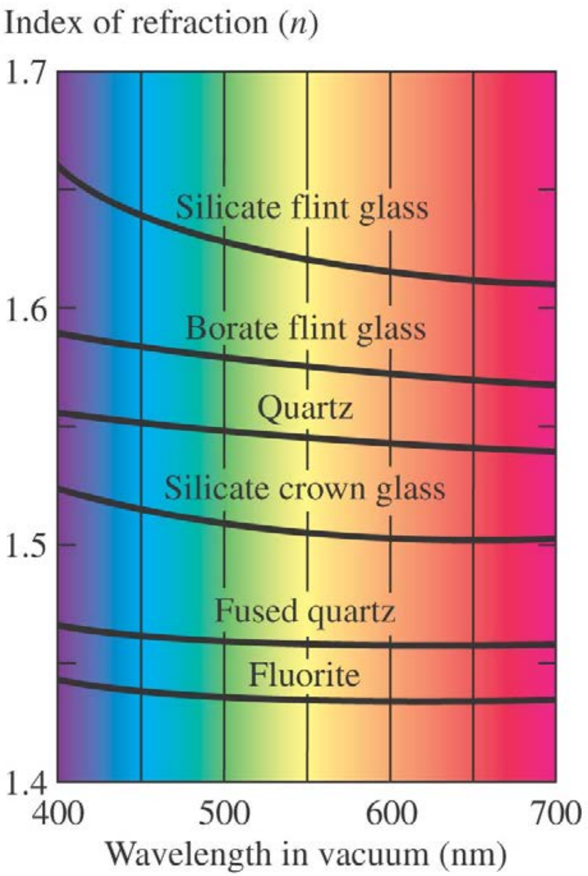
(a) Total internal reflection in a Porro prism

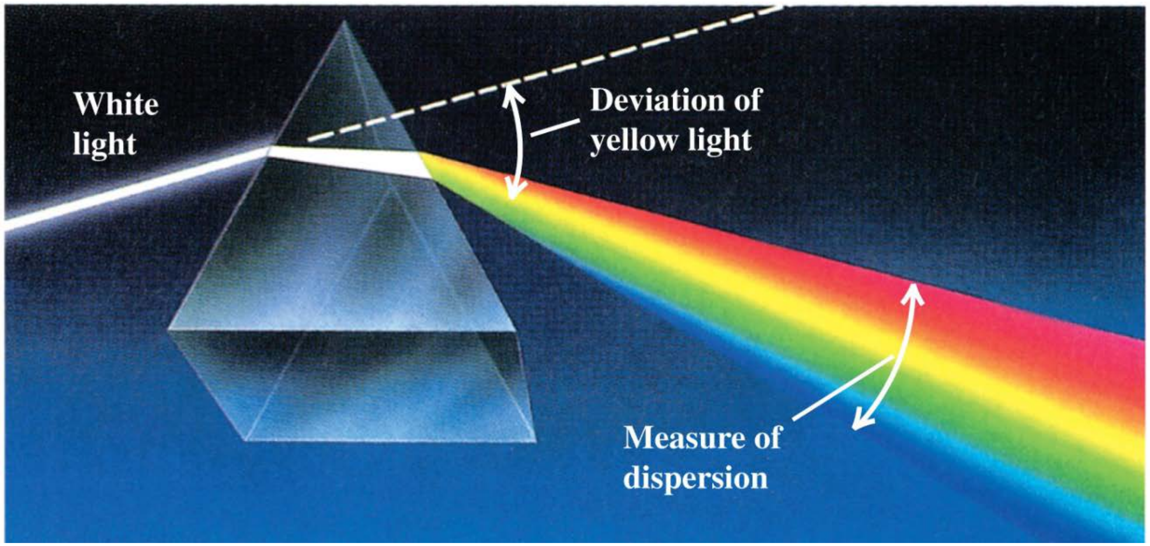


If the incident beam is oriented as shown, total internal reflection occurs on the 45° faces (because, for a glass–air interface, $\theta_{\text{crit}} = 41.1$).

Dispersion

- *Dispersion*: The index of refraction depends on the wavelength of the light. See Figure 6 (right).
- Figure 7 (below) shows dispersion by a prism.

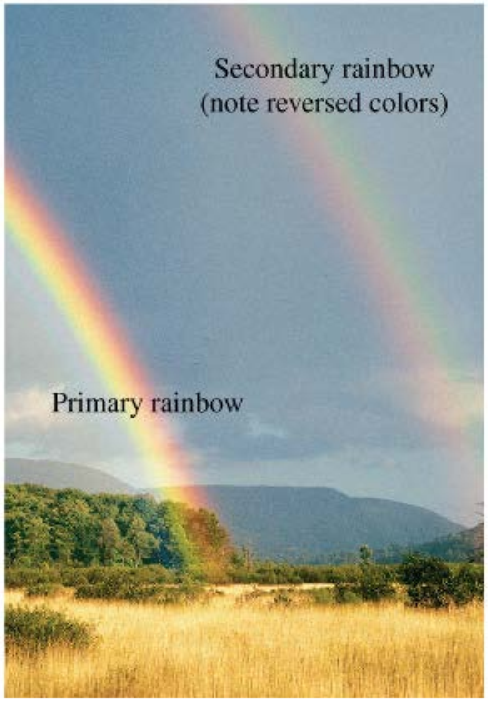




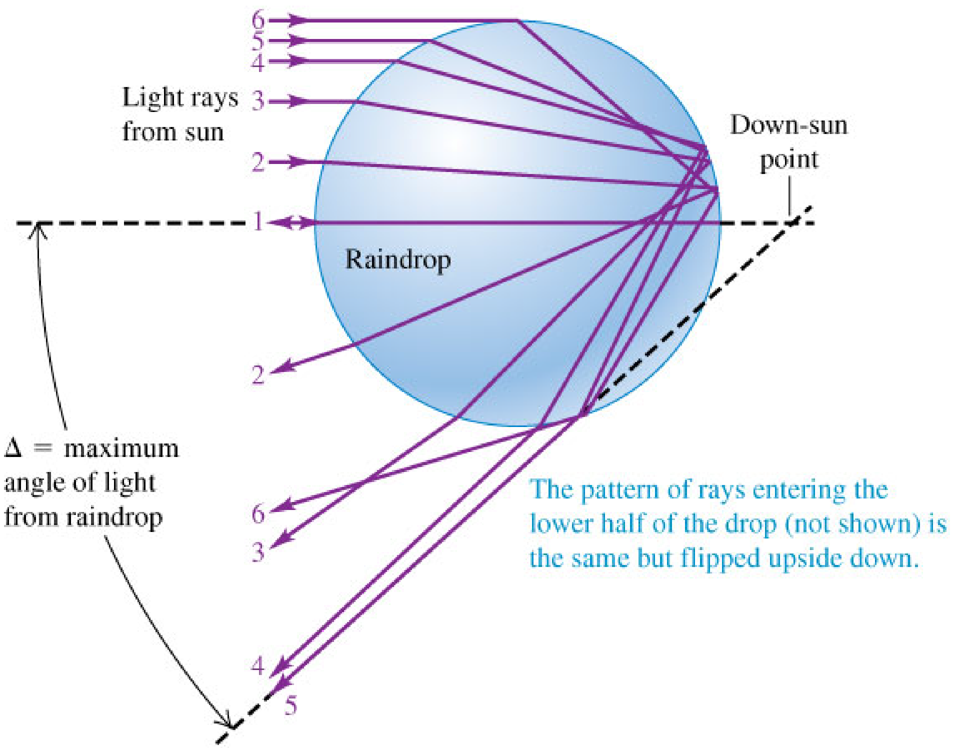
Rainbows—I

- **The formation of a rainbow is due to the combined effects of dispersion, refraction, and reflection. (See Figure 8 below and on the next slide.)**

(a) A double rainbow

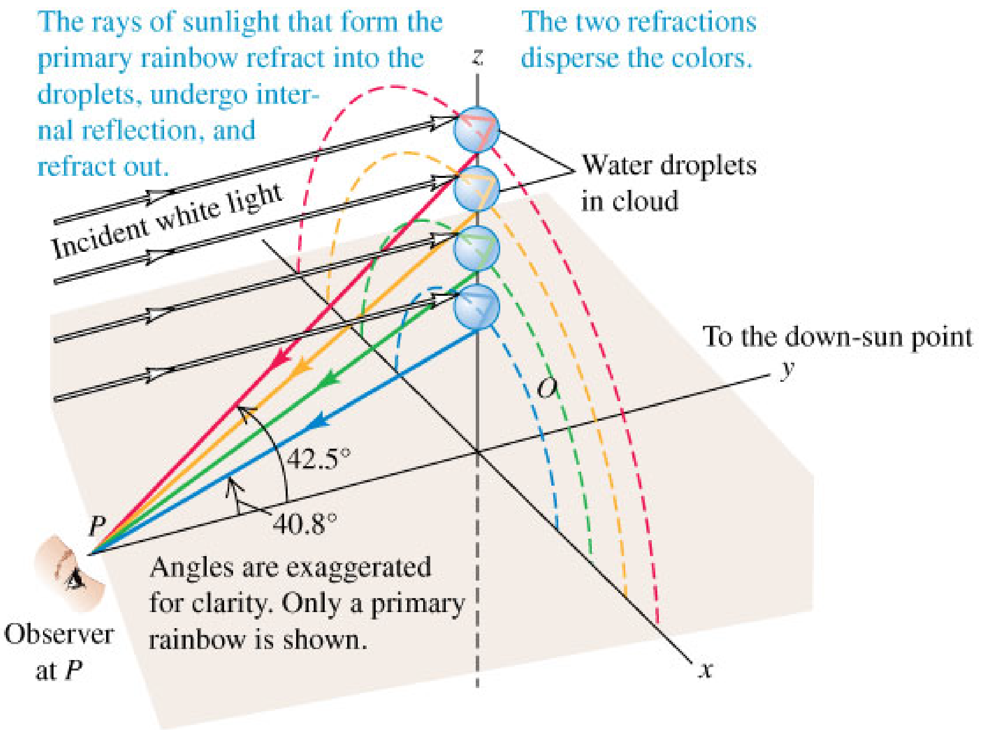


(b) The paths of light rays entering the upper half of a raindrop

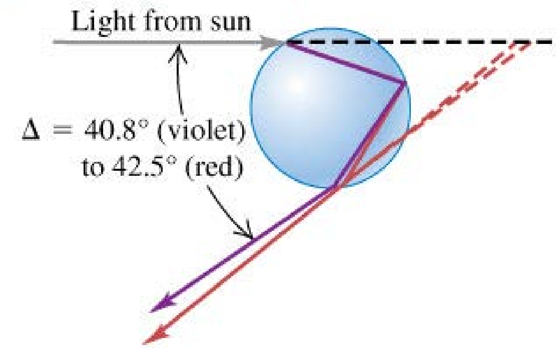


Rainbows—II

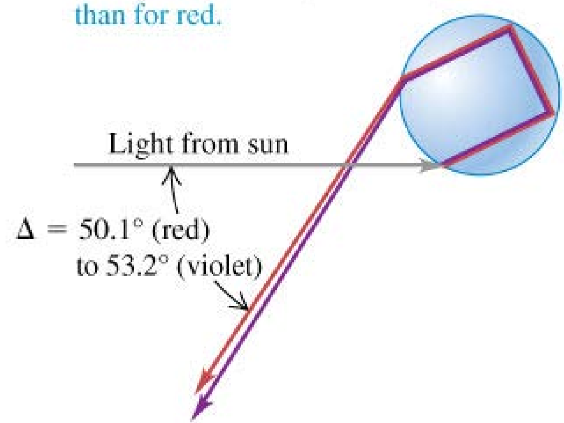
(c) Forming a rainbow. The sun in this illustration is directly behind the observer at P.



(d) A primary rainbow is formed by rays that undergo two refractions and one internal reflection. The angle Δ is larger for red light than for violet.



(e) A secondary rainbow is formed by rays that undergo two refractions and *two* internal reflections. The angle Δ is larger for violet light than for red.



Polarization

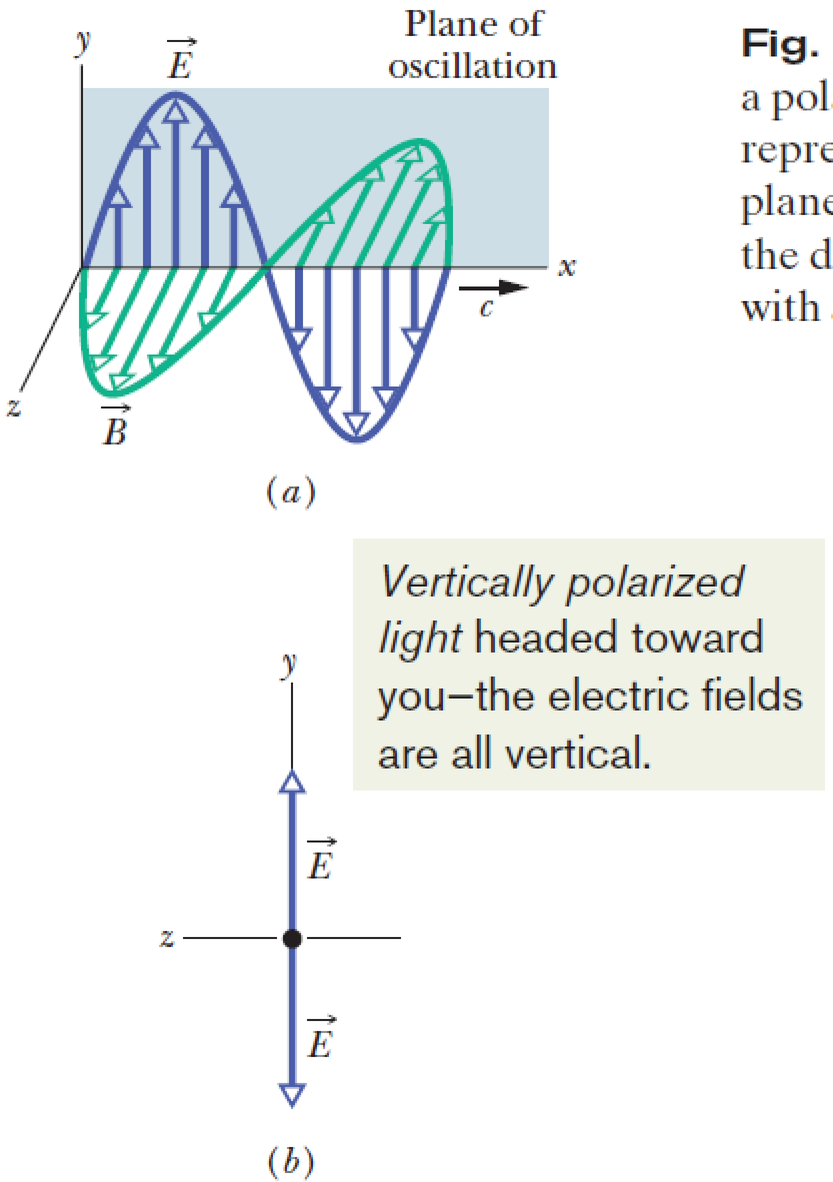


Fig. 33-9 (*a*) The plane of oscillation of a polarized electromagnetic wave. (*b*) To represent the polarization, we view the plane of oscillation head-on and indicate the directions of the oscillating electric field with a double arrow.

Polarization

An electric field component parallel to the polarizing direction is passed (*transmitted*) by a polarizing sheet; a component perpendicular to it is absorbed.

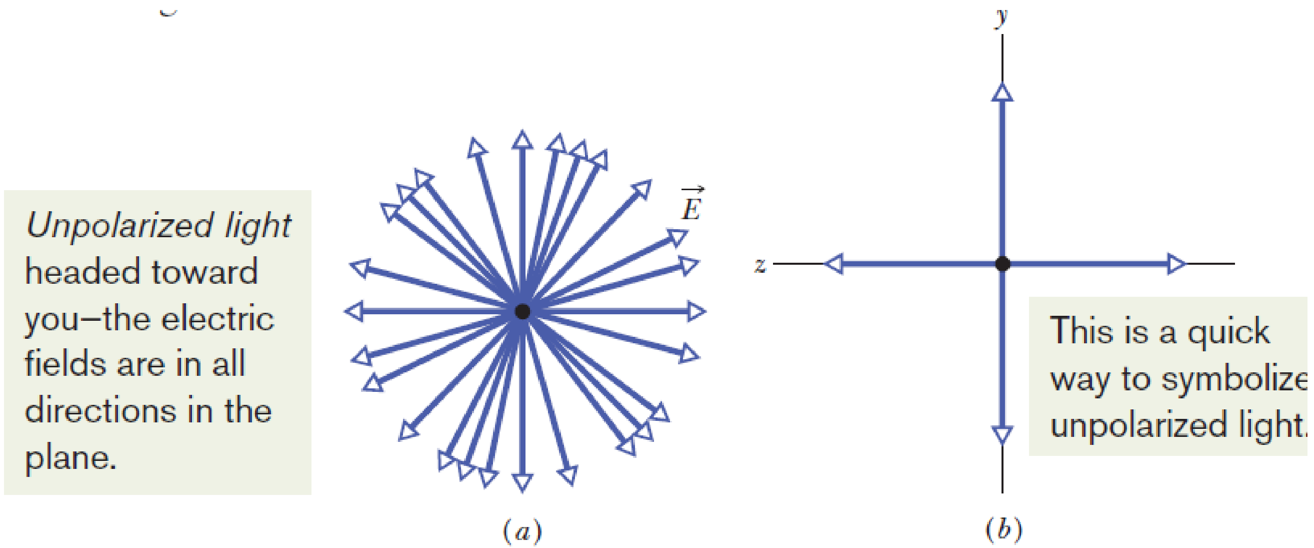


Fig. 33-10 (*a*) Unpolarized light consists of waves with randomly directed electric fields. Here the waves are all traveling along the same axis, directly out of the page, and all have the same amplitude *E*. (*b*) A second way of representing unpolarized light—the light is the superposition of two polarized waves whose planes of oscillation are perpendicular to each other.

If the intensity of original unpolarized light is I_o, then the intensity of the emerging light through the polarizer, I, is half of that.

$$I = \frac{1}{2} I_0.$$

Polarization: Intensity of Polarized Light

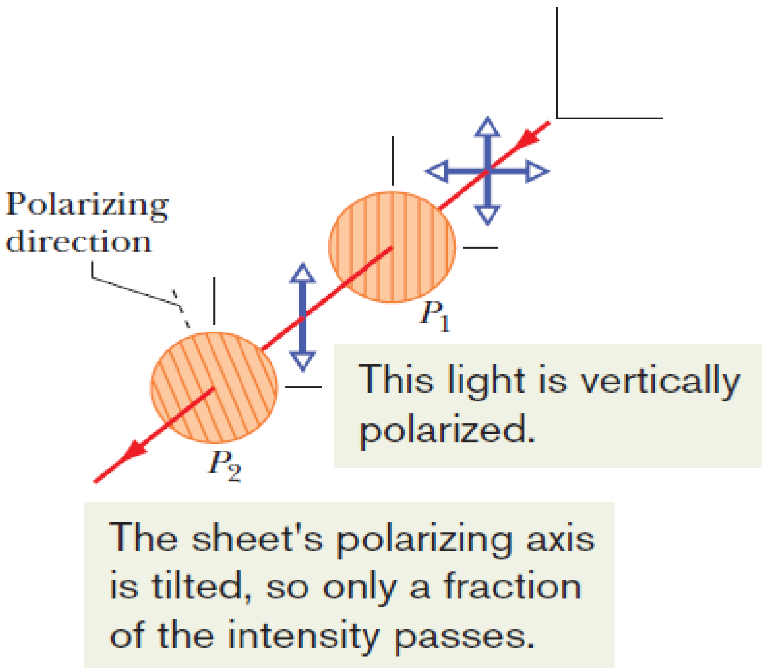


Fig. 33-13 The light transmitted by polarizing sheet P_1 is vertically polarized, as represented by the vertical double arrow. The amount of that light that is then transmitted by polarizing sheet P_2 depends on the angle between the polarization direction of that light and the polarizing direction of P_2 (indicated by the lines drawn in the sheet and by the dashed line).

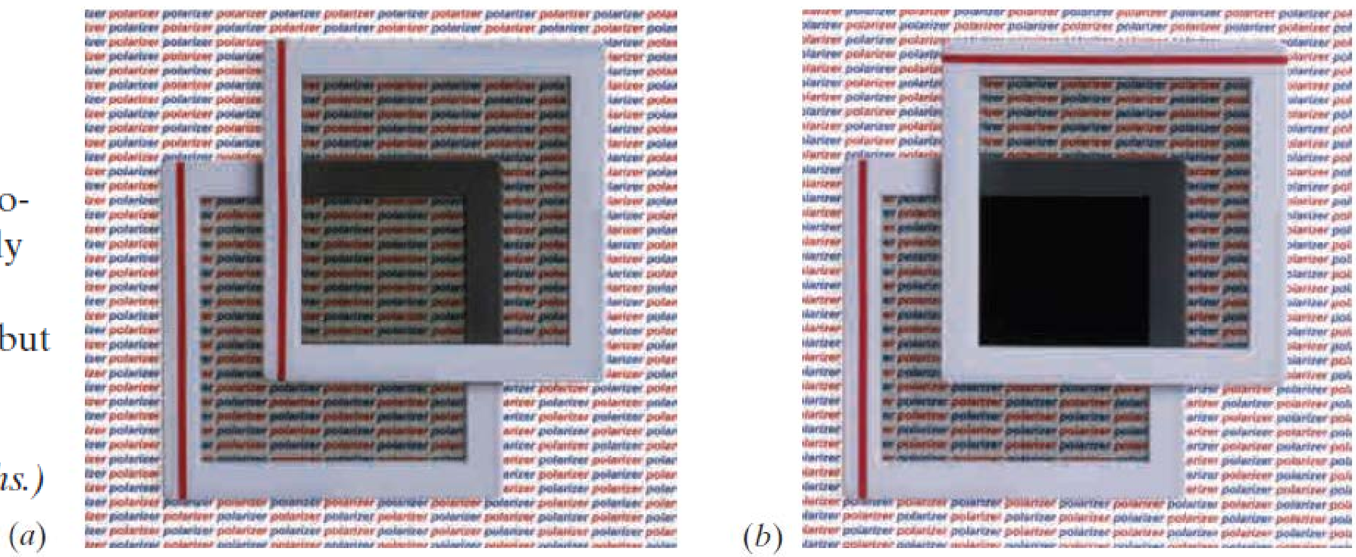
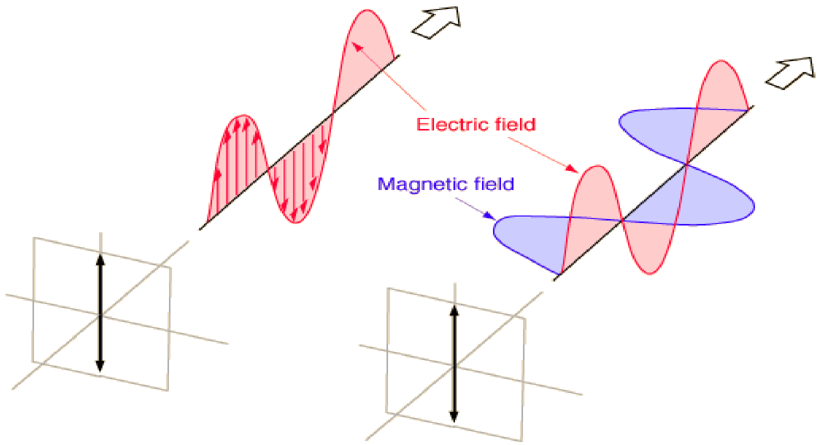


Fig. 33-14 (*a*) Overlapping polarizing sheets transmit light fairly well when their polarizing directions have the same orientation, but (*b*) they block most of the light when they are crossed. *(Richard Megna/Fundamental Photographs.)*

Polarization

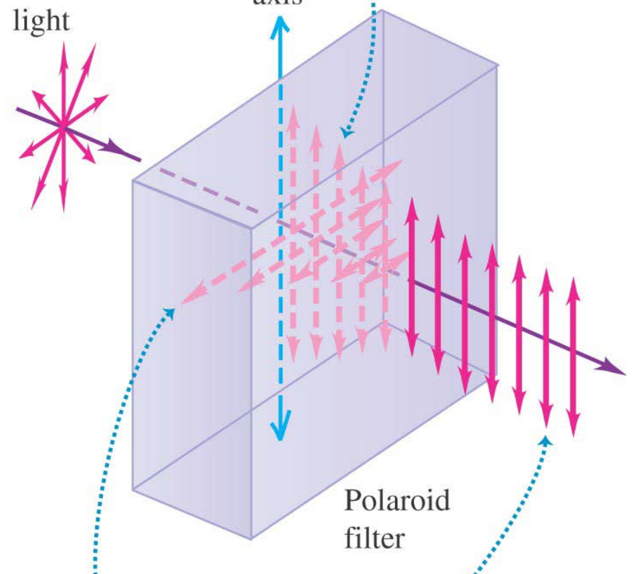
- **An electromagnetic wave is *linearly polarized* if the electric field has only one component.**



- **Figure 9 at the right shows a Polaroid *polarizing filter*.**

$$\mathbf{E}(x,t) = \hat{\mathbf{j}} E_{\max} \cos(kx - \omega t)$$

$$\mathbf{B}(x,t) = \hat{\mathbf{k}} B_{\max} \cos(kx - \omega t)$$



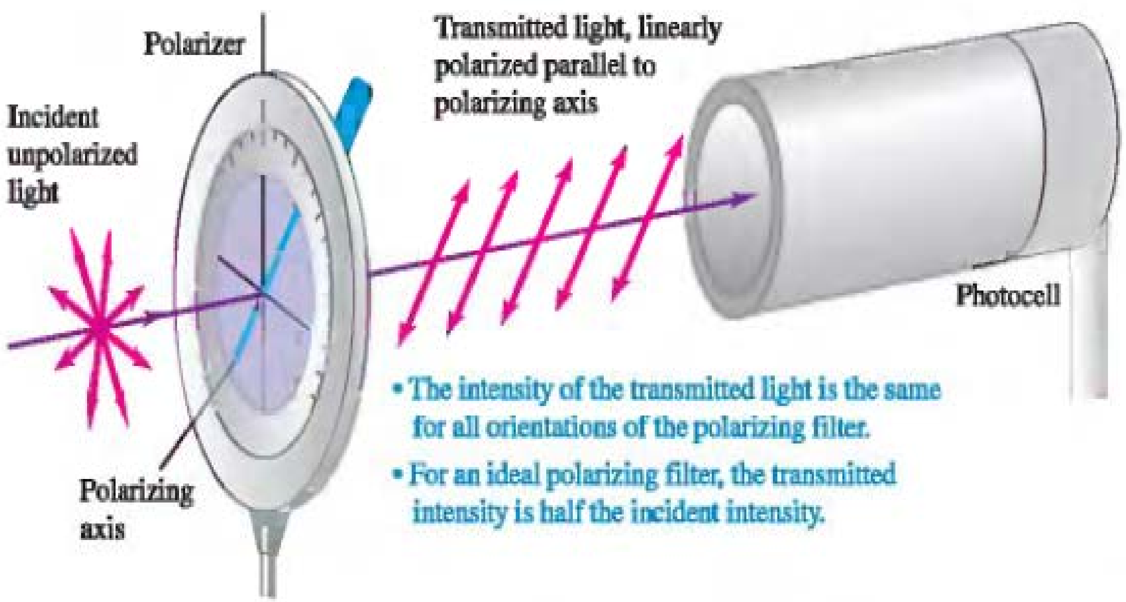
Polarizing Filters

An *ideal* polarizing filter (polarizer) passes 100% of the incident light that is polarized in the direction of the filter's polarizing axis but completely blocks all light that is polarized perpendicular to this axis.

Malus's law

- **Figure 33.25 below shows a polarizer and an analyzer.**
- **A polarizer reduces the intensity of unpolarized light (I_0) by a factor of 2, so the intensity of transmitted light is $I_0/2$.**
- **A second polarizer (the analyzer) at angle f relative to the first further reduces the intensity according to:**

$$\textit{Malus's law:} \quad I = I_{\max}\cos^2\phi.$$



33.24 Unpolarized natural light is incident on the polarizing filter. The photocell measures the intensity of the transmitted linearly polarized light.

Example 33.5 Two polarizers in combination

In Fig. 33.25 the incident unpolarized light has intensity I_0. Find the intensities transmitted by the first and second polarizers if the angle between the axes of the two filters is 30°.

SOLUTION

IDENTIFY: This problem involves a polarizer (a polarizing filter on which unpolarized light shines, producing polarized light) and an analyzer (a second polarizing filter on which the polarized light shines).

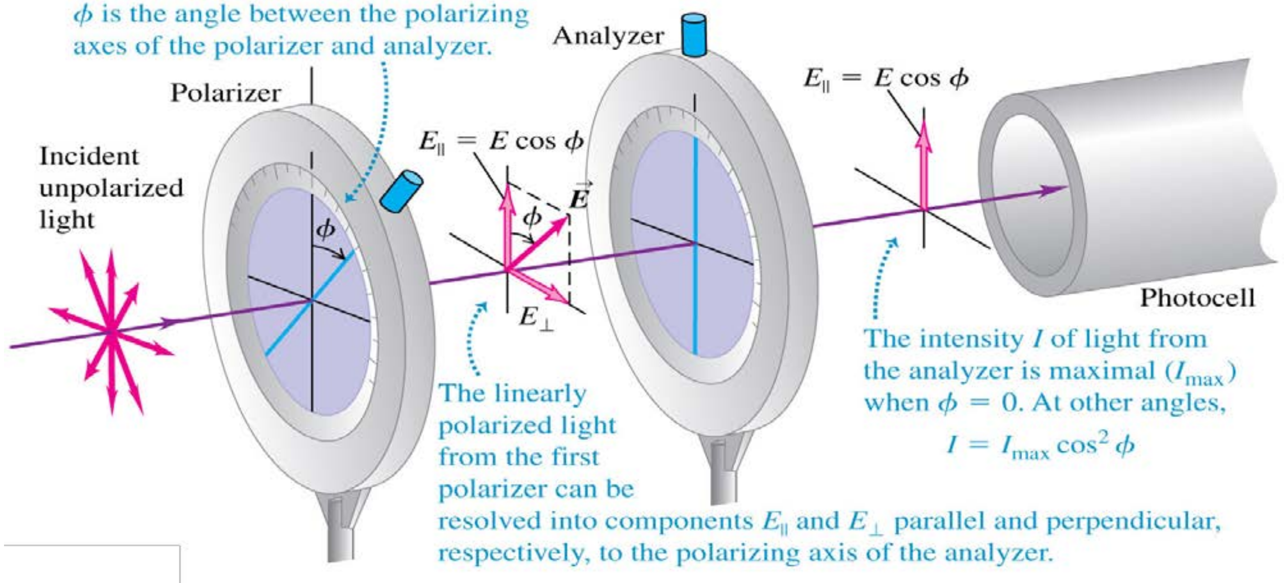
SET UP: The diagram has already been drawn for us in Fig. 33.25. We are given the intensity I_0 of the incident natural light and the angle $\phi = 30°$ between the polarizing axes. Our target variables are the intensities of the light emerging from the first polarizer and of the light emerging from the second polarizer.

EXECUTE: As we explained above, the intensity of the linearly polarized light transmitted by the first filter is $I_0/2$. According to Eq. (33.7) with $\phi = 30°$, the second filter reduces the intensity by a factor of $\cos^2 30° = \frac{3}{4}$. Thus the intensity transmitted by the second polarizer is

$$\left(\frac{I_0}{2}\right)\left(\frac{3}{4}\right) = \frac{3}{8}I_0$$

EVALUATE: Note that the intensity decreases after each passage through a polarizer. The only situation in which the transmitted intensity does *not* decrease is if the polarizer is ideal (so it absorbs none of the light that passes through it) and if the incident light is linearly polarized along the polarizing axis, so $\phi = 0$.

Example polarizer and analyzer with *f* = 30°.



Polarization by reflection

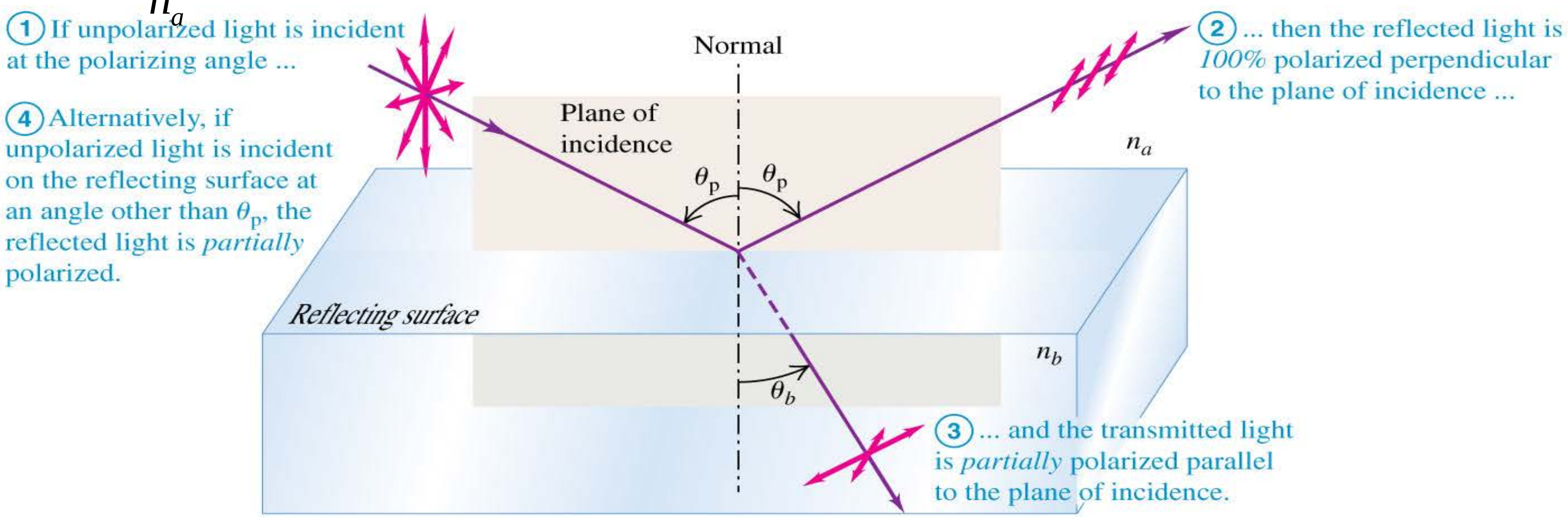
In 1812 the British scientist Sir David Brewster discovered that when the angle of incidence is equal to the polarizing angle θ_p, the reflected ray and the refracted ray are perpendicular to each other (Fig. 33.28). In this case the angle of refraction θ_b becomes the complement of θ_p, so $\theta_b = 90° - \theta_p$. From the law of refraction,

- **When light is reflected at the *polarizing angle* θ_p (Brewster's angle), the reflected light is linearly polarized. See Figure 10 below.**
- **The polarizing angle is θ_p when the reflected and refracted rays are 90° from each other, i.e. when $\theta_p + \theta_b = 90°$.**

$$n_a \sin\theta_p = n_b \sin\theta_b$$

$$n_a \sin\theta_p = n_b \sin(90-\theta_p) = n_b \cos\theta_p$$

$$\tan\theta_p = \frac{n_b}{n_a} \text{ (Brewster's Law)}$$



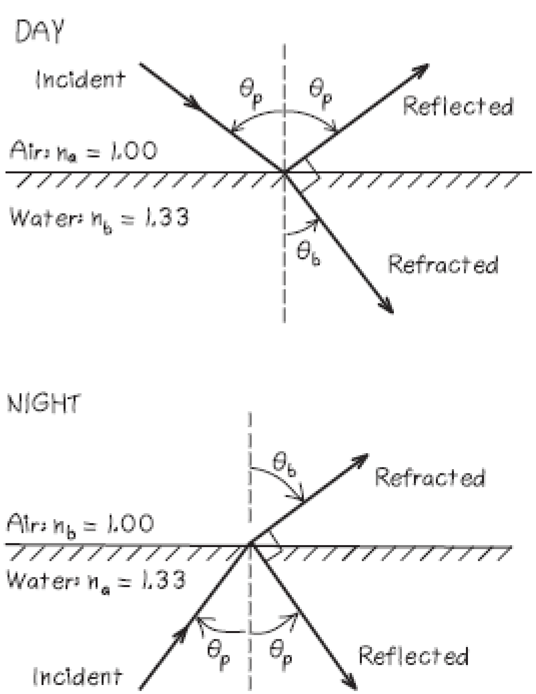
Example 33.6 Reflection from a swimming pool's surface

Sunlight reflects off the smooth surface of a swimming pool. (a) For what angle of reflection is the reflected light completely polarized? (b) What is the corresponding angle of refraction? (c) At night, an underwater floodlight is turned on in the pool. Repeat parts (a) and (b) for rays from the floodlight that strike the surface from below.

SOLUTION

IDENTIFY and SET UP: This problem involves polarization by reflection at an air–water interface in parts (a) and (b) and at a water–air interface in part (c). Figure 33.29 shows our sketches.

33.29 Our sketches for this problem.



For both cases our first target variable is the polarizing angle θ_p, which we find using Brewster's law, Eq. (33.8). For this angle of reflection, the angle of refraction θ_b is the complement of θ_p (that is, $\theta_b = 90° - \theta_p$).

EXECUTE: (a) During the day (shown in the upper part of Fig. 33.29) the light moves in air toward water, so $n_a = 1.00$ (air) and $n_b = 1.33$ (water). From Eq. (33.8),

$$\theta_p = \arctan\frac{n_b}{n_a} = \arctan\frac{1.33}{1.00} = 53.1°$$

(b) The incident light is at the polarizing angle, so the reflected and refracted rays are perpendicular; hence

$$\theta_b = 90° - \theta_p = 90° - 53.1° = 36.9°$$

(c) At night (shown in the lower part of Fig. 33.29) the light moves in water toward air, so now $n_a = 1.33$ and $n_b = 1.00$. Again using Eq. (33.8), we have

$$\theta_p = \arctan\frac{1.00}{1.33} = 36.9°$$

$$\theta_b = 90° - 36.9° = 53.1°$$

EVALUATE: We check our answer in part (b) by using Snell's law, $n_a \sin\theta_a = n_b \sin\theta_b$, to solve for θ_b:

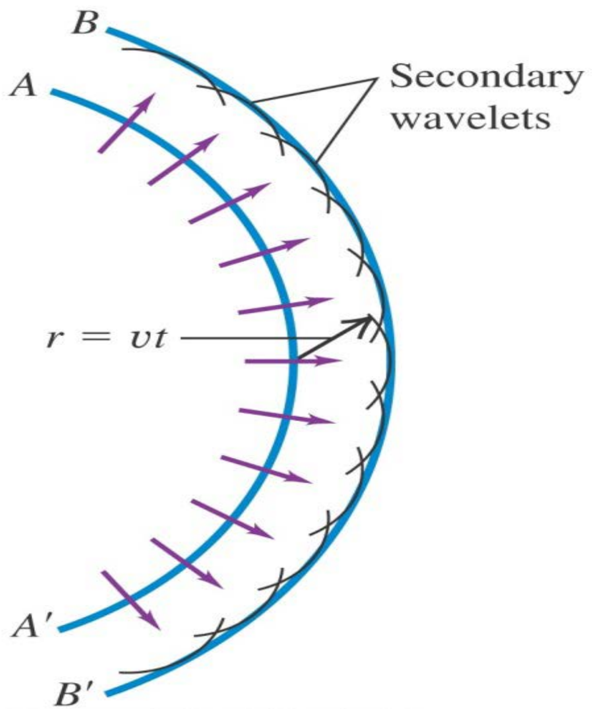
$$\sin\theta_b = \frac{n_a \sin\theta_p}{n_b} = \frac{1.00 \sin 53.1°}{1.33} = 0.600$$

$$\theta_b = \arcsin(0.600) = 36.9°$$

Note that the two polarizing angles found in parts (a) and (c) add to

Huygens's principle

- *Huygens's principle*: Every point of a wave front can be considered to be a source of secondary wavelets that spread out in all directions with a speed equal to the speed of propagation of the wave. See Figure 33.34 at the right.



Summary

Light and its properties: Light is an electromagnetic wave. When emitted or absorbed, it also shows particle properties. It is emitted by accelerated electric charges. The speed of light is a fundamental physical constant.

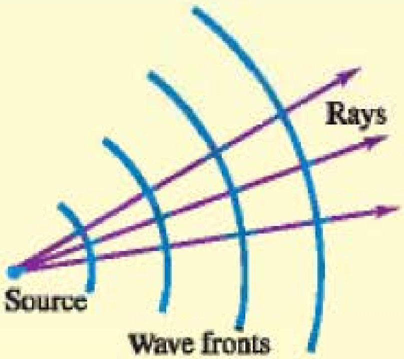
A wave front is a surface of constant phase; wave fronts move with a speed equal to the propagation speed of the wave. A ray is a line along the direction of propagation, perpendicular to the wave fronts. Representation of light by rays is the basis of geometric optics.

When light is transmitted from one material to another, the frequency of the light is unchanged, but the wavelength and wave speed can change. The index of refraction n of a material is the ratio of the speed of light in vacuum c to the speed v in the material. If λ_0 is the wavelength in vacuum, the same wave has a shorter wavelength λ in a medium with index of refraction n. (See Example 33.2.)

The variation of index of refraction n with wavelength λ is called dispersion. Usually n decreases with increasing λ.

$$n = \frac{c}{v} \qquad (33.1)$$

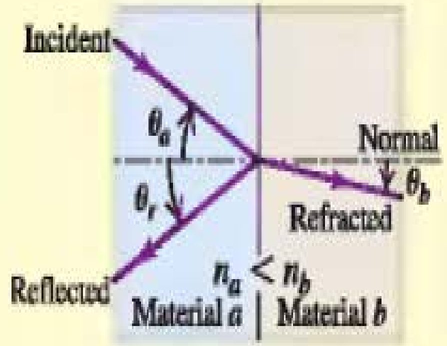
$$\lambda = \frac{\lambda_0}{n} \qquad (33.5)$$



Reflection and refraction: At a smooth interface between two optical materials, the incident, reflected, and refracted rays and the normal to the interface all lie in a single plane called the plane of incidence. The law of reflection states that the angles of incidence and reflection are equal. The law of refraction relates the angles of incidence and refraction to the indexes of refraction of the materials. Angles of incidence, reflection, and refraction are always measured from the normal to the surface. (See Examples 33.1 and 33.3.)

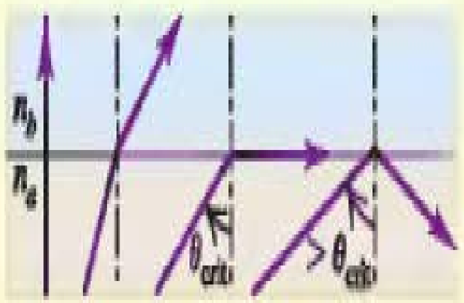
$$\theta_r = \theta_a \quad (33.2)$$
(law of reflection)

$$n_a \sin\theta_a = n_b \sin\theta_b \quad (33.4)$$
(law of refraction)



Total internal reflection: When a ray travels in a material of greater index of refraction n_a toward a material of smaller index n_b, total internal reflection occurs at the interface when the angle of incidence exceeds a critical angle θ_{crit}. (See Example 33.4.)

$$\sin\theta_{crit} = \frac{n_b}{n_a} \quad (33.6)$$

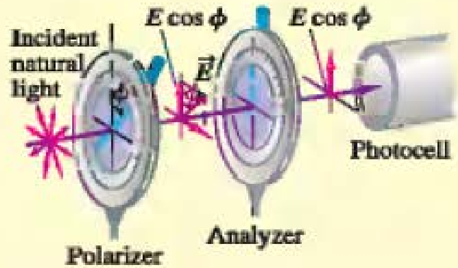


Polarization of light: The direction of polarization of a linearly polarized electromagnetic wave is the direction of the $\vec{E}$ field. A polarizing filter passes waves that are linearly polarized along its polarizing axis and blocks waves polarized perpendicularly to that axis. When polarized light of intensity I_{max} is incident on a polarizing filter used as an analyzer, the intensity I of the light transmitted through the analyzer depends on the angle ϕ between the polarization direction of the incident light and the polarizing axis of the analyzer. (See Example 33.5.) When two linearly polarized waves with a phase difference are superposed, the result is circularly or elliptically polarized light. In this case the $\vec{E}$ vector is not confined to a plane containing the direction of propagation, but rather describes circles or ellipses in planes perpendicular to the propagation direction.

Light is scattered by air molecules. The scattered light is partially polarized.

$$I = I_{max}\cos^2\phi \quad (33.7)$$

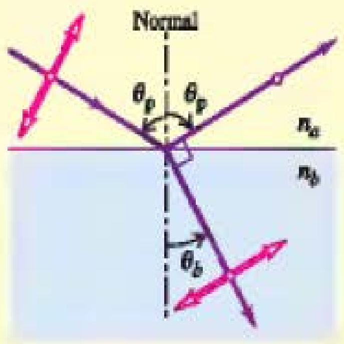
(Malus's law)



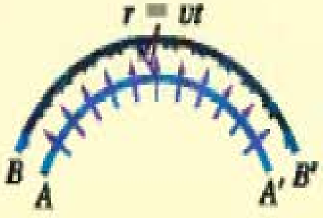
Polarization by reflection: When unpolarized light strikes an interface between two materials, Brewster's law states that the reflected light is completely polarized perpendicular to the plane of incidence (parallel to the interface) if the angle of incidence equals the polarizing angle θ_p. (See Example 33.6.)

$$\tan\theta_p = \frac{n_b}{n_a} \quad (33.8)$$

(Brewster's law)



Huygens's principle: Huygens's principle states that if the position of a wave front at one instant is known, then the position of the front at a later time can be constructed by imagining the front as a source of secondary wavelets. Huygens's principle can be used to derive the laws of reflection and refraction.



Discussion Questions

Q33.1. Light requires about 8 minutes to travel from the sun to the earth. Is it delayed appreciably by the earth's atmosphere? Explain.

Q33.2. Sunlight or starlight passing through the earth's atmosphere is always bent toward the vertical. Why? Does this mean that a star is not really where it appears to be? Explain.

Q33.3. A beam of light goes from one material into another. On *physical* grounds, explain *why* the wavelength changes but the frequency and period do not.

Sources of Radiation (4)

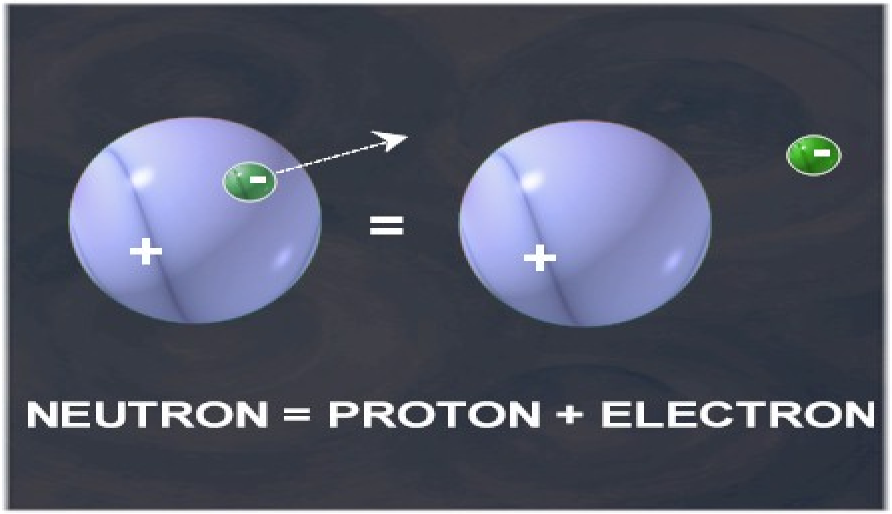
Dr. Hind I. Abdulgafour

Definition of Radiation

- "Radiation is an energy in the form of electro-magnetic waves or particulate matter, traveling in the air."

Radioactivity: Elements & Atoms

- Atoms are composed of smaller particles referred to as:
  - Protons
  - Neutrons
  - Electrons



Basic Model of a Neutral Atom.

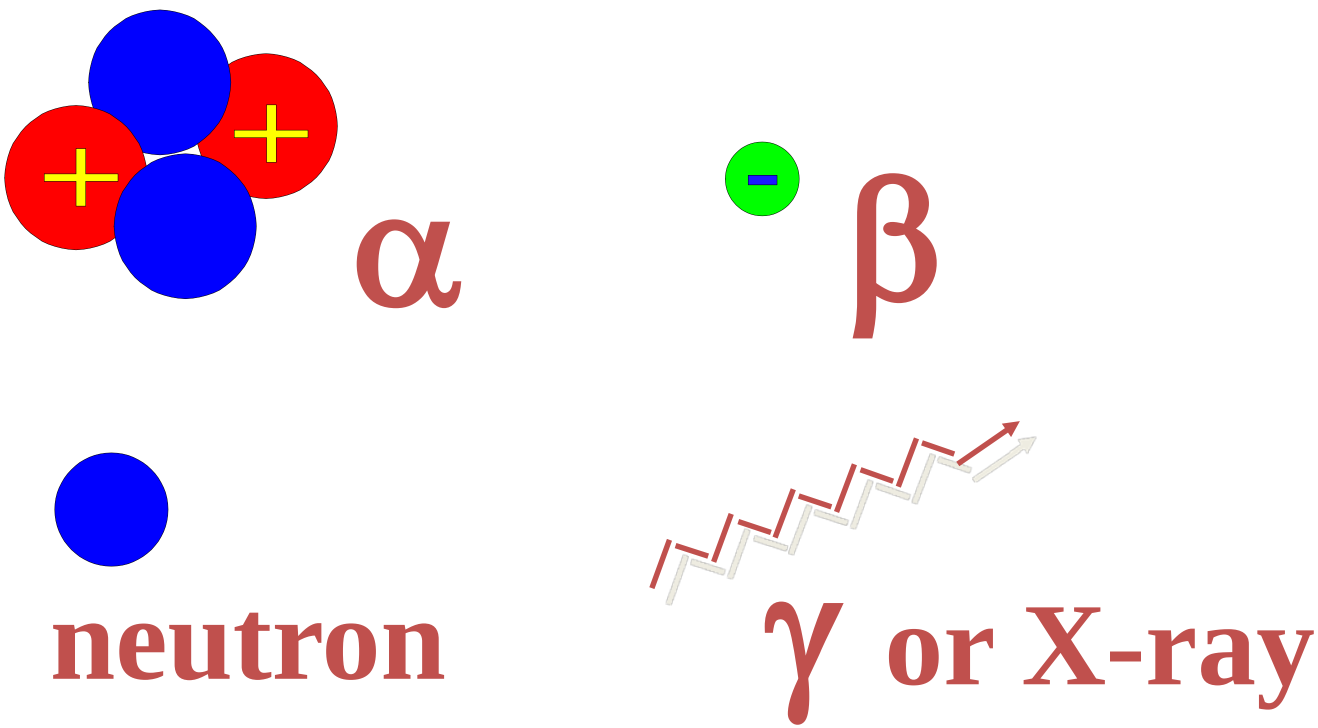
- Electrons (-) orbiting nucleus of protons (+) and neutrons. Same number of electrons as protons; net charge = 0.
- **Atomic number** (number of protons) determines element.
- **Mass number** (protons + neutrons)

Ionization

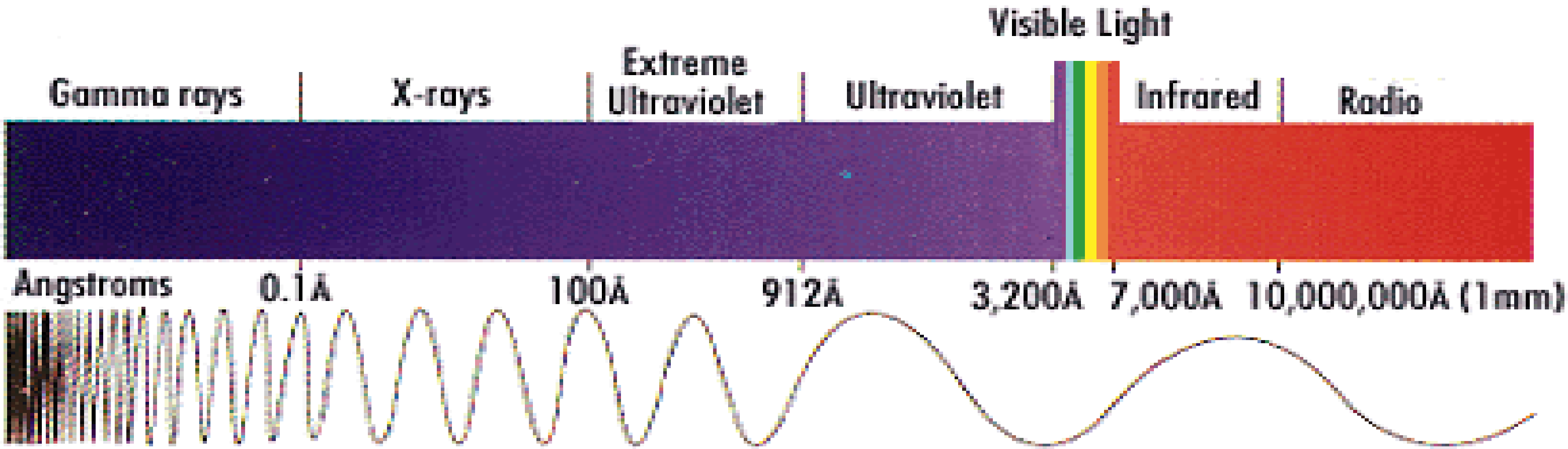
- **Ionizing radiation is produced by unstable atoms. Unstable atoms differ from stable atoms because they have an excess of energy or mass or both.**

- **Unstable atoms are said to be radioactive. In order to reach stability, these atoms give off, or emit, the excess energy or mass. These emissions are called radiation.**

Types or Products of Ionizing Radiation



- **The electro-magnetic waves vary in their length and frequency along a very wide spectrum.**



Types of Radiation

- Radiation is classified into:
  - Ionizing radiation
  - Non-ionizing radiation

Ionizing Versus Non-ionizing Radiation

- **Ionizing Radiation**
  - **Higher energy electromagnetic waves (gamma) or heavy particles (beta and alpha).**
  - **High enough energy to pull electron from orbit.**

- **Non-ionizing Radiation**
  - **Lower energy electromagnetic waves.**
  - **Not enough energy to pull electron from orbit, but can excite the electron.**

Ionizing Radiation

- **Definition:**

“ It is a type of radiation that is able to disrupt atoms and molecules on which they pass through, giving rise to ions and free radicals”.

Types of Ionizing Radiation

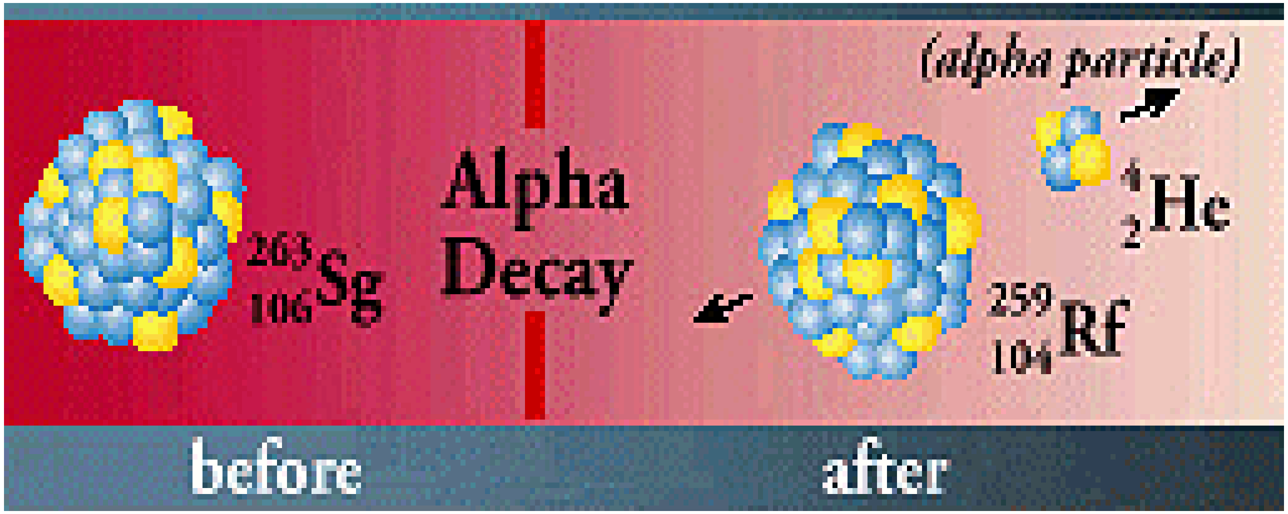
- **Alpha particles**
- **Beta particles**
- **Gamma rays (or photons)**
- **X-Rays (or photons)**
- **Neutrons**

Types and Characteristics of Ionizing Radiation

Alpha Particles

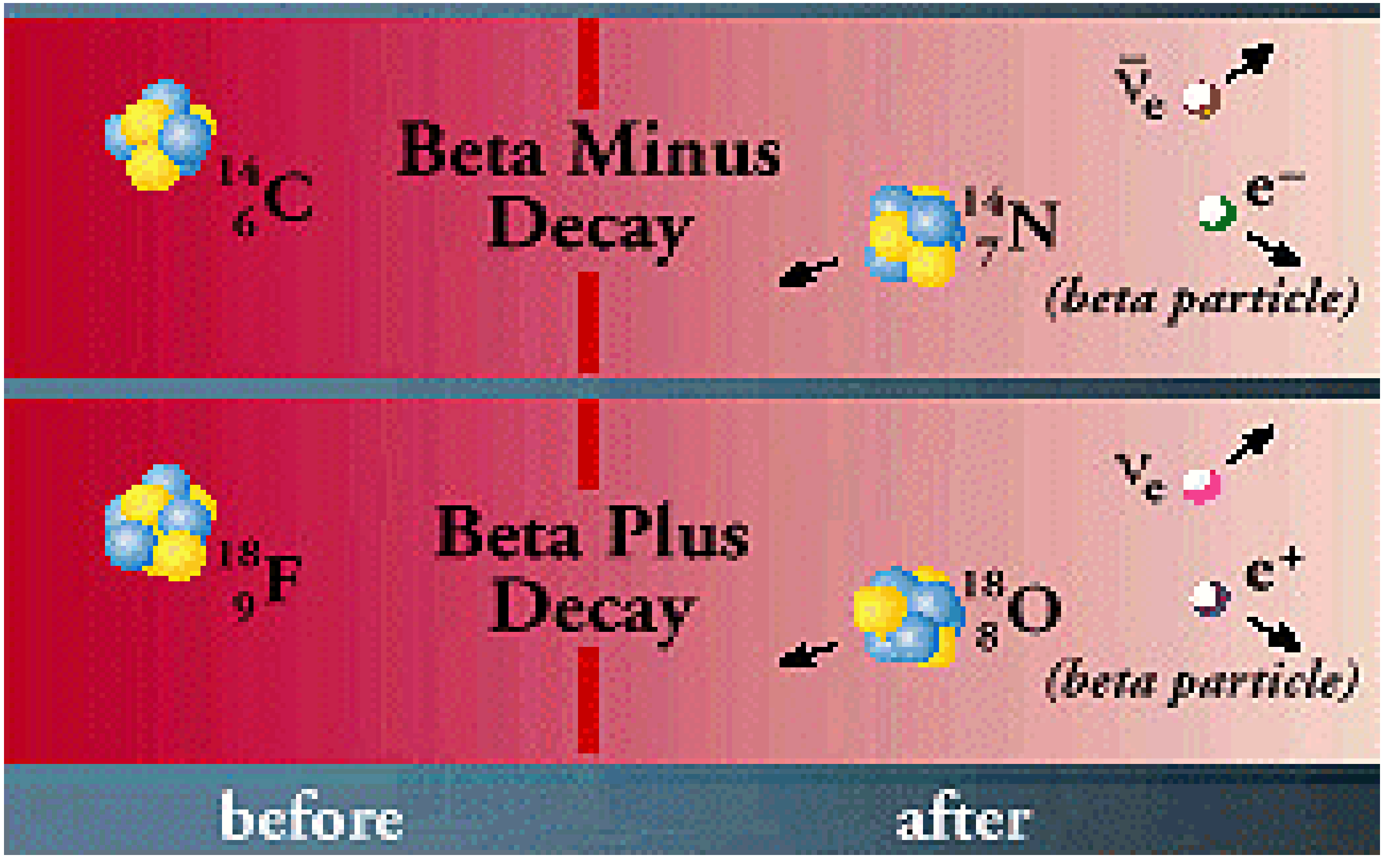
Alpha Particles: 2 neutrons and 2 protons
They travel short distances, have large mass
Only a hazard when inhaled

Alpha Particles (or Alpha Radiation): **Helium nucleus (2 neutrons and 2 protons)**



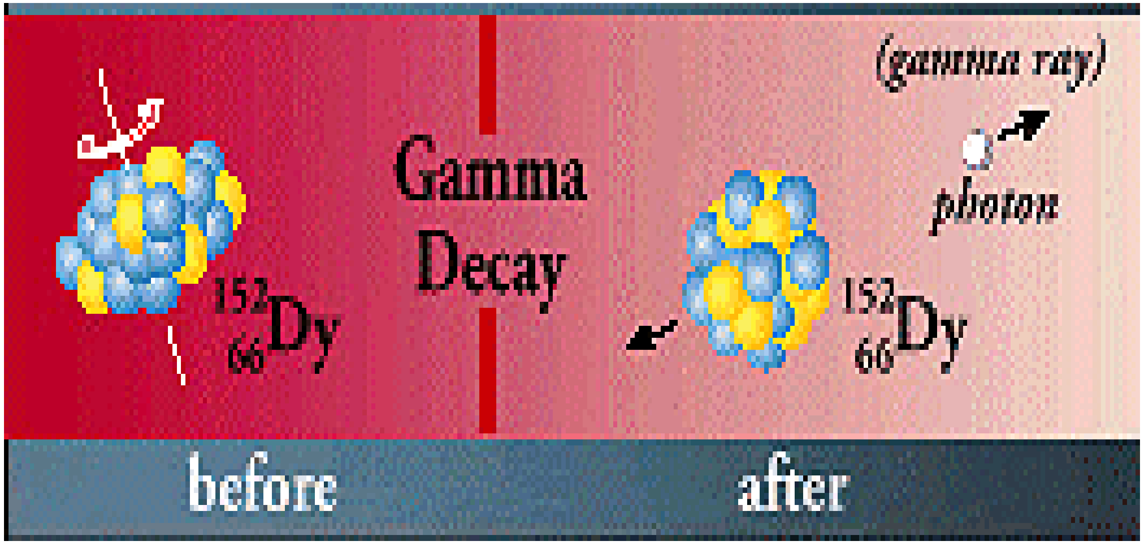
Beta Particles

Beta Particles: Electrons or positrons having small mass and variable energy. Electrons form when a neutron transforms into a proton and an electron.



Gamma Rays

Gamma Rays (or photons): Result when the nucleus releases energy, usually after an alpha, beta or positron transition



X-Rays

X-Rays: Occur whenever an inner shell orbital electron is removed and rearrangement of the atomic electrons results with the release of the elements characteristic X-Ray energy

- ***X-*** *and* ***Gamma Rays***: **X-rays** are photons (Electromagnetic radiations) emitted **from electron orbits**. **Gamma rays** are photons emitted **from the nucleus**, often as part of radioactive decay. Gamma rays typically have higher energy (Mev's) than X-rays (KeV's), but both are unlimited.

Neutrons

Neutrons: Have the same mass as protons but are uncharged

Non-ionizing Radiation

- **Definition:**

“ They are electromagnetic waves incapable of producing ions while passing through matter, due to their lower energy.”

Examples on Nonionizing Radiation Sources

- **Visible light**
- **Microwaves**
- **Radios**
- **Video Display Terminals**
- **Power lines**
- **Radiofrequency**
- **Lasers**

Natural Background Radiation

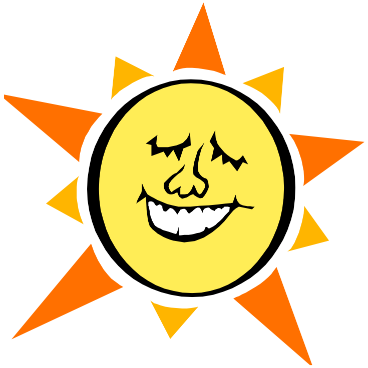
- Cosmic Radiation
- Terrestrial Radiation
- Internal Radiation

Cosmic Radiation

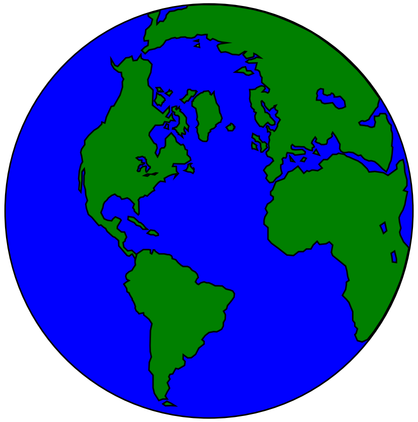
- The earth, and all living things on it, are constantly being bombarded by radiation from outer space <u>(~ 80% protons and 10% alpha particles).</u>

- Charged particles from the sun and stars interact with the earth's atmosphere and magnetic field to produce a shower of radiation.

- The amount of cosmic radiation varies in different parts of the world due to differences in elevation and to the effects of the earth's magnetic field.



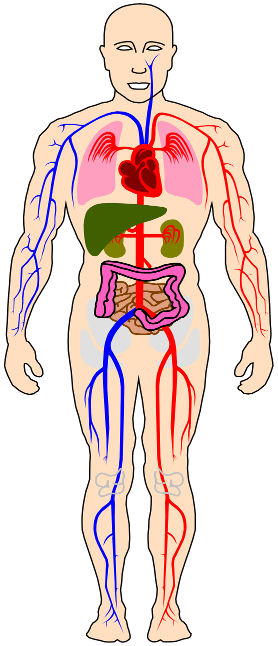
Terrestrial Radiation
(Uranium, Actinium, Thorium decay series)



- Radioactive material is found throughout nature in soil, water, and vegetation.
- Important radioactive elements include uranium and thorium and their radioactive decay products which have been present since the earth was formed billions of years ago.
- Some radioactive material is ingested with food and water. Radon gas, a radioactive decay product of uranium is inhaled.
- The amount of terrestrial radiation varies in different parts of the world due to different concentrations of uranium and thorium in soil.

Internal Radiation

- People are exposed to radiation from radioactive material inside their bodies such as Radon (Ra).



Man-Made Radiation

Radioactive material is used in:

- **Medicine - diagnostic (X-ray, CAT)**
- **Medicine - therapeutic (Co-60, Linac)**
- **Medical research (radio-pharmaceuticals, accel.)**
- **Industry - (X-ray density gauges, well logging)**

Radiation in Medicine

- Radiation used in medicine is the largest source of man-made radiation.

- Most exposure is from diagnostic x-rays.

Elements of Radiation Protection Program

- ***Monitoring of exposures*: Personal, area, and screening measurements; Medical/biologic monitoring.**

- ***Task-Specific Procedures and Controls*: Initial, periodic, and post-maintenance or other non-scheduled events.**

- ***Emergency procedures*: Response, "clean-up", post clean-up testing and spill control.**

- ***Training and Hazard Communications* including signs, warning lights, and information given.**

- ***Material Handling*: Receiving, storage, and disposal.**