10

A Transformational View of Cartography

10.1 A FRAMEWORK FOR TRANSFORMATIONS

Much of the focus of the previous chapters has been on the tools of mapmaking and on the structure of cartographic data at each stage in the mapping process. The approach has concentrated upon the *states* of the cartographic data as the data move from cartographic entity to cartographic object to cartographic symbol. But this is only half the story. It is time to ask how we move between states, to focus on the cartographic transformation, and to take *a transformational view of cartography*. In this approach we will concentrate on *analytical* rather than computer cartography. Cartographic transformations are central to what analytical cartography is all about. In fact, analytical cartography could be called a discipline in transformations.

Cartographic transformations come in many forms. There are transformations of attribute data, transformations of the locational properties of maps, graphic transformations, transformations of the information content of maps, and the scale transformations of generalization and selection. As will be shown, it is particularly interesting to ask if any given transformation is invertible,that is, whether or not a cartographic transformation can be undone or reversed to produce the initial starting conditions. Recent advances in mathematical theory have focused attention on transformation inversions that are unstable, that is, the inverse transformation produces chaos. A thorough knowledge of stable versus unstable cartographic transformations would go a long way toward providing a theory of cartographic transformations. Stable transformations are controllable and therefore are effectively programmed and modeled, especially with respect to the error introduced. The analytical cartography introduced in this part is the core of what stands behind much of the computer cartographic software developed to date. This means that it will also be reflected in the computer cartography of the future.

Many of the ideas in this chapter have been adapted from the paper titled "A Transformational View of Cartography," in which the cartographer Waldo Tobler stated his

Content scaling level	Defining relations	FORM OF CARTOGRAPHIC SYMBOL		
		POINT	**LINE**	**AREA**
Nominal	Equivalence	Wholesale and retail establishments	A, B, C, D, E Highway connectivity	A, B, C Land ownership
Ordinal	Equivalence Greater than	Medium, Small, Large Population centers	Roads by degree of improvement	Low, Med, High Crop yield
Interval	Equivalence Greater than Ratio of intervals	+ 147 + 210 + 132 + 122 Spot elevations	Graticule	1824, 1845, 1862 Date of settlement
Ratio	Equivalence Greater than Ratio of intervals Ratio of scale values	Area proportional to population	20, 30, 20, 10 Population density isopleths	Value proportional to population density

Figure 10.1 Classification by scaling and dimension. (After Robinson and Sale, *Elements of Cartography, 3d ed.* ,© 1969, by John Wiley & Sons, Inc. Used with permission.)

conceptual viewpoint on analytical cartography (Tobler, 1979). Tobler's idea of transformations originated with a single set, the transformations of map space caused by map projections, that is, point-to-point transformations. This was expanded to cover other dimensions, and transformations between dimensions. To these can be added transformations of scale and the mapping transformation that actually produces the map, known as the symbolization transformation. In this and the next chapter, these four types of transformations will be examined in detail and used to present a transformational view as a unifying theme in analytical cartography.The idea of transformations in cartography has roots in the work of the cartographer Arthur Robinson, who devised a way of classifying the types of cartographic symbolization and map types in common use (Robinson and Sale, 1969). Robinson placed methods into the framework of a set of states classified by level of measurement and dimension (Figure 10.1). Robinson proceeded to place most of the methods used to make maps into one or other of the categories. David Unwin expanded upon Robinson's classification to add the distinction between map types and data types. Data properties define data types, while the type of symbolization defines map types. This implies that cartography involves a data type-to- map type *symbolization* transformation. Analytical cartography is the scientific treatment of the properties of the data to map type transformation.

To start with Robinson's classification, the primary concern was with the symbolization transformation. The idea was to break down the ways in which symbolization takes place by two factors: the *type of data* and the *level of attribute measurement.* In this context, the symbolization method means those actual symbols used on a given map; blue tints, brown lines, dot patterns, and so forth. The map type is the cartographic technique used to produce the map. As an example, a map with areas shaded red or blue depicting the winning political party within counties has a data type that is area/nominal and a map type that is colored areal. Using red and blue and shading the areas within the boundaries of the counties are the results of the choice of symbolization technique. Under the Robinson/Unwin system, the *states* in the mapping process could be summarized as area/nominal data to colored area map to, say, color ink-jet print from a particular choropleth mapping package. The sequence is *data type* to *map type* to *symbolization* to *map*.

The framework for cartographic transformations proposed here is that the mapping process is the sum of a series of state transformations and their interactions. At any stage in the process, the cartographic information with which we are working has a data structure, and the transformations operate upon this structure. By using Robinson's types of symbolization and Unwin's data and map types, we can lay out a finite set of transformations, each of which can then be examined in more detail. Before we go into the *how* however, we should be fully aware *why* we are performing these important transformations.

10.2 REASONS FOR TRANSFORMING CARTOGRAPHIC DATA

Three of the most important reasons for transforming cartographic data are generalization, converting the geometry of the map base, and changing the data structure. The first reason, *cartographic generalization,* underlies much of analytical cartography. A good subtitle for the discipline of cartography would be a "discipline in reduction" because we are always changing map scales from larger to smaller as we move from data to map and attempting to understand what happens in the process. We may wish to have maps, which have been collected in different ways, transformed to a common basis. It may be a common statistical basis, where we have directly measurable quantities of the same values: where the same statistic, for example, needs to be compared across different countries. Or we may want maps put onto the same geographic reference base. For example, we may want maps with different map projections transformed to one common coordinate system.

Another reason for transformations is the need for converting the geometry of the map base. Cartographic analysis, modeling, different map types, and symbolization methods all favor a particular cartographic data type. If we want to do automated contouring, we somehow have to convert our data into integers in a grid data structure, because this is how about half of the automated contouring packages work. The problems of transforming cartographic data are usually as important as the problems associated with making the final map.

Another major reason for cartographic transformations is to *change data structure.* The different phases that we go through when we handle cartographic data, such as data input or output and data manipulation of various different data types, all have their best structures. The best way to digitize a polygon map for choropleth mapping, unless you

want to duplicate information, and assuming that one wants the capability to do some editing of the data after initial entry, is to enter the data as connected line segments forming chains. Then we deal with the problem of transforming them into the format that we really want, which in this case is the polygon list, so that we can shade the areas as required by our symbolization method. Transformation between data structures is worthwhile because the real difficulty is digitizing, and we want to make this stage as easy as possible.

Data structure transformations, however, are for the purpose of suiting the cartographic data for different types of analyses or symbolization and are not an end in themselves. Many other processes involve exactly the same issue; for example, certain analytical operations are far better performed on a grid than on a vector data structure, and vice versa. We almost always find ourselves in need of transforming between data structures, so that the data structure transformation is usually the first, if not the most important transformation, we may apply during the mapping process.

10.3 TYPES OF DATA

The basis of Robinson's classification of symbolization and map types was the division by data dimensions: the familiar division of cartographic objects into the types—*point*, *line*, and *area*—to which can be added *volume.* Point data can be described simply by a location with an attribute. Line data are usually thought of as being strings of connected straight-line segments, but can also be represented by a mathematical function such as a polynomial or a spline. Area data are regions or polygons, predefined geographic areas with a boundary, a given topology, and an attribute, such as the state of New York. Volumes can represent either continuous statistical distributions, such as measurements of air pollution or terrain elevations.

The second division of cartographic objects under Robinson's classification is by the level of measurement of the data. *Level of measurement* means the classification of the attribute data by its complexity. This division was defined by Stevens (1946). The normal categories we use are *nominal*, *ordinal*, *interval*, and *ratio.*

Nominal simply means assigning a name or label. A nominal cartographic feature may be a place name associated with a point object, such as the label **New York** assigned to the location 40 degrees 40 minutes north latitude, 73 degrees 58 minutes west longitude on a small-scale map. Such objects may have rules or methods associated with them, especially in relation to how they are cartographically symbolized and what map types can be used with them.

Ordinal data has a sequence or ranking associated with it, so relations like "larger," "smaller," or "greater than" can be used.

Interval data has a measured numerical value as an associated attribute, such as a measured field elevation. The scale, however, is not absolute but pegged to an arbitrary unit or false zero. Thus a geological map may classify rock types by age before present (BP), representing an age that cannot be precisely determined, yet it may give a relative history to the rock sequences that is less arbitrary than a set of younger than/older than references.

A *ratio* value is a measured value on a scale with a meaningful zero on which absolute mathematical operations can be performed. The Kelvin temperature scale, for example,

has a zero determined by the lowest possible temperature rather than the temperature at which ice melts. Examples of ratio values are population density, change rates, and percentages. Transformations between levels of measurement are not necessarily map data transformations, but they do affect the types of maps we can use to portray the transformed data.

A good example, within a normal cartographic representational procedure, is in the making of a choropleth map. We may collect data on population (ratio) and the areas of states (ratio). From these two ratio values we compute a third, in this case population density in persons per square kilometer. This is a simple numerical, mathematical transformation, and it is invertible on the map because we will be showing the outline of the states, and therefore allowing the computation of their areas (if we show a scale). Next we represent our ratio value using an area technique known as choropleth mapping, which first reduces the ratio data to ordinal by classification and then assigns ordinal shade categories and patterns to the regions on the map. The entire set of level-of-measurement transformations is depicted in Figure 10.2. Normally, the symbolization transformation is not an invertible transformation, and frequently it is far from simply performed, because there are many possible ways to subdivide the data into categories.

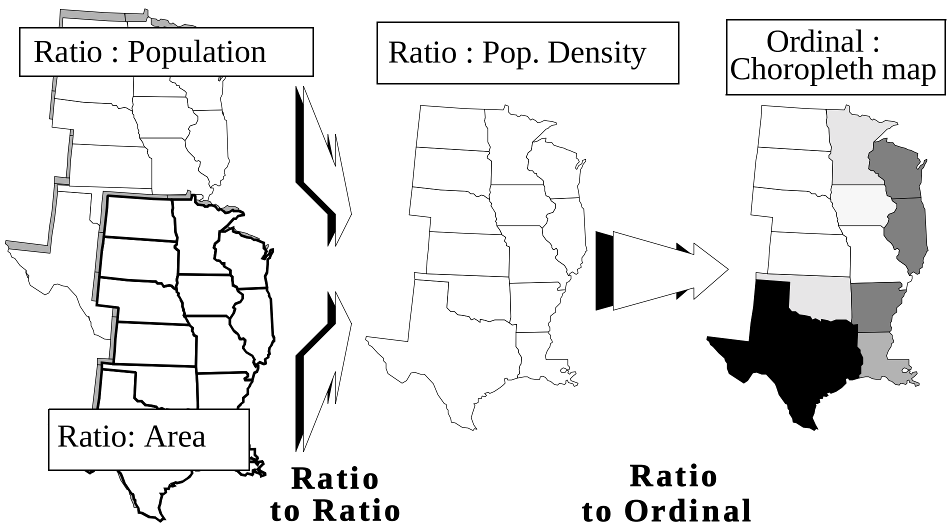

Figure 10.2 Level of measurement transformations for choropleth mapping.

Jenks and Caspall (1971) provided a means of measuring the error involved in this last data transformation, allowing an objective evaluation of the various classification methods. In fact, Jenks and Caspall presented four measures of the accuracy of this transformation. One of these was a composite of the remaining three, which were all based on comparison with an idealized model of a choropleth map with enough classes to have one class per data value. The error was computed using the concept of a three-dimensional solid raised to the height of the choroplethic data value and the standard deviation.

With only one choropleth value (the worst case), the entire map would be represented

by the data mean. The overview error would then be the sum of the absolute values of the data values minus the mean, in each case, times the areas represented on the map. Similarly, the tabular error is simply the height differences, without the transformation to volumes. The two versions of this measure are the absolute and relative values of this number. The error indices provide an objective function to maximize, and Jenks and Caspall presented a worked example of a classification optimization for the choropleth data they used. An optimal set of choropleth mapping transformations would minimize this error.

For a choropleth map we can transform the data into a map and simultaneously make definitive statements about the error or information loss in the transformation, which defines the invertibility of the transformation. These types of transformations, their characteristics, how they can be performed, their invertibility, and the error both spatial and aspatial that they embody form the bulk of the subject area for analytical cartography. So familiar are these transformations that we often perform them without thinking, or even perform then mentally while looking at a table of numbers or a map.

10.4 TYPES OF MAPS

David Unwin presented a division of the Robinson classification into data types and map types. Where Robinson was interested in cartographic symbolization, Unwin was interested in the spatial analytic aspects of the state descriptions. Unwin presented two three-by-four tables, shown as Figure 10.3. The classification was the same as Robinson's, with the exception that volumes (surfaces) were added as a data type. The division into map type and data type was based on the distinctions between cartographic entities and their symbolization methods.

Under Unwin's schema, a nominal linear data type was a road, and a nominal linear map type was a network map. The road could be described as the cartographic entity. The network map, however, is both a type of *symbolization* and a type of *cartographic object*. In the following discussion, we will distinguish between the transformation of a cartographic entity to a cartographic object via geocoding and the adoption of a cartographic data structure, and the eventual symbolization transformation of the object required to make a map. The emphasis is not, therefore, on map types or data types, but on the transformations between the states into which cartographic entities, objects, and symbols fall.

The domain of interest of the analytical cartographer is the full set of state transformations possible, yet it is also obvious that there will be an optimal set or pathway of transformations to make a particular map. The symbolization transformation, in which the map is realized and finds a physical description and a tangible reality, forms the topic of computer cartography, being by definition technology dependent. Under the transformational approach, four major cartographic transformations seem to shape the way in which mapping takes place. These are the following: the geocoding transformation between cartographic entities and cartographic objects, involving transitions between levels of measurement, changes in dimension, and changes in data structure; changes in map scale; changes in the locational attributes of the data, that is, transformations of the map base itself; and the transformation resulting in symbolization and a real map.

DATA TYPES

	Point	Line	Area	Volume
Nominal	City	Road	Name of unit	Precipitation or soil type
Ordinal	Large city	Major road	Rich county	Heavy precipitation Good soil
Interval / Ratio	Total population	Traffic flow	Per capita income	Precip. in mm Cation exchange

MAP TYPES

	Point	Line	Area	Volume
Nominal	Dot map	Network map	Colored area map	Freely colored map
Ordinal	Symbol map	Ordered network map	Ordered colored map	Ordered chromatic map
Interval / Ratio	Graduated symbol map	Flow map	Choropleth map	Contour map

Figure 10.3 Map data and map types. (After Unwin, 1981.)

Finally, we should note that transformations of cartographic objects can yield objects with fewer dimensions or even no dimension at all (scalars or just simply, measurements). The intersection of two lines produces a point, and the measurement of area of a polygon produces a scalar. Cartographic measurement, part of cartometry, is itself simply a transformation to a lesser dimension object or a simpler geometry.

10.5 TRANSFORMATIONS OF THE MAP SCALE

To cartographers, the range of interest is the map scales between about 1:1,000 up to about 1:400,000,000. Remember that *large scale* means a representative fraction with a smaller number as the denominator and covers a small area in a great deal of detail. A 1:1,000 map is approaching a *plan* and is of as much interest to the surveyor or the engineer as to the cartographer. Similarly, a *small-scale* map has a large number in the denominator of the representative fraction and shows a large area in less detail, perhaps a whole state, country, or even a continent.

To provide some examples of these scales, at the large-scale end 1 meter on the ground is 1 millimeter on the map; at the small- scale end, the entire earth map would fit onto the surface of a gum ball. We have different names for the disciplines concerned with scales beyond these ranges. At extremely small scales, the concerned disciplines are astronomy and planetary physics; at extremely large scales, the fields include planning, surveying, and engineering. The scale of one-to-one is called *reality*, and scales beyond this are the enlargement disciplines of chemistry, microbiology, and particle physics. We use different instruments at different scales, and the scales affect the collection of data with those instruments. We cannot, for example, measure interplanetary distances with a ruler; neither can we observe distant galaxies with an electron scanning microscope.

Cartographically, transformations between scales are almost always from larger to smaller scale and proceed by a process of *generalization*, involving the selection of features to survive the scale change, the deletion of detail, the smoothing or rounding of the data, and sometimes the change in symbolization necessary to deal with the new scale, which can even involve moving objects to facilitate interpretation. Some automated methods for scale modification are discussed in Leberl et al. (1986). Figure 10.4 summarizes the types of scale change discussed by Weibel (1987) in the specific context of digital elevation models but applicable to most types of cartographic data. Weibel noted that cartographers perform four basic operations while generalizing cartographic data. These are elimination (selection), simplification, combination, and occasionally displacement.

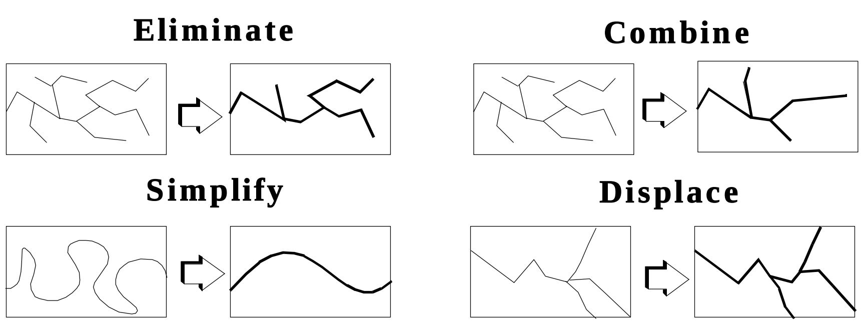

Figure 10.4 Scale change transformations

Elimination consists of dropping cartographic features as we move from large- to small-scale maps. *Simplification* involves classical generalization, such as placing a dot for a city on a small-scale map or showing a river as a smoother and smoother line at smaller and smaller scales. *Combination* involves joining features together at smaller scales, such as joining a river with its tributaries or combining island groups into a smaller number of "representative islands." *Displacement* is occasionally necessary to reveal structure or to allow space for labels. This is often the case when many features are clustered together in a single region on the map.

Our scientific ideal is always to collect data at scales that are larger than we need for a particular map so that we can eliminate, simplify, and generalize using known rules, methods, and criteria. Much of manual cartography, and a great deal of computer cartography, has to do with sets of rules for determining the generalization transformation. Rules and algorithms for scale-related generalization have been researched in considerable detail in recent years (Buttenfield and McMaster, 1991; McMaster and Shea, 1992).

Most algorithms perform the simplification stage, that is, they reduce the number of points in a digital cartographic line. Algorithms include retaining only every *n*th. point, selecting points based on the line's interior angle, selecting points outside a buffer of given width following the line (epsilon filtering), and selecting points recursively based on the maximum orthogonal distance from the line above a threshold.

Different map lines have distinctly different properties, because they reflect different processes on the ground. Map administrative lines are straight or follow meridians and parallels. Contours are smooth but have sharp breaks at streams, and coastlines sometimes show scale-independence or fractal characteristics. Probably the best defined are the rules for generalizing a coastline. We generalize through scales, gently smoothing the coastlines, but we also make jumps at certain scales. When an island becomes a dot at a particular scale, we may choose to eliminate it. Similarly, whole inlets or river estuaries may disappear on small-scale maps. Figure 10.5 shows this phenomenon. The lake in question actually appears to change shape as it moves though the scales at which it is mapped.

Only recently have cartographers thought about the inverse transformation between scales. This is probably because we used to call this process "cartographic license" or sometimes "witchcraft." A better name would be *enhancement,* and it is important to note that not all enhancement is witchcraft, for we may have good information at some places on the map about patterns at larger scales.

Extending the characteristics of this variation uniformly over the map artificially may be termed *emphatic enhancement.* On the other hand, using a model to generate variation, such as fractal enhancement of coastlines, is *synthetic enhancement.* Dutton (1981) (Figure 10.6) was a pioneer in this work and his ideas have since been extended to other types of data (Clarke et al., 1993).

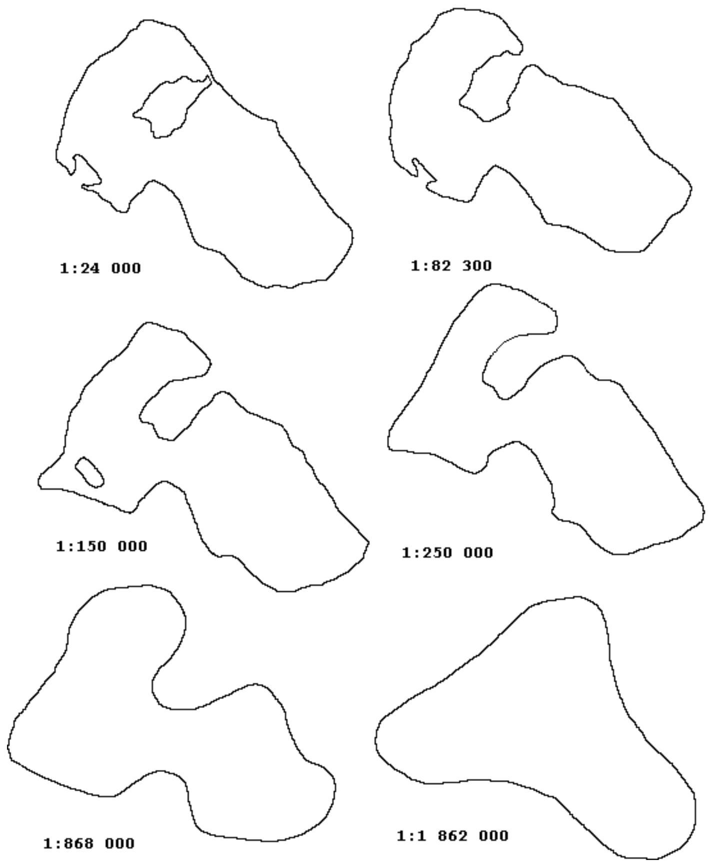

Figure 10.5 Copake Lake (in New York State) at different map scales.

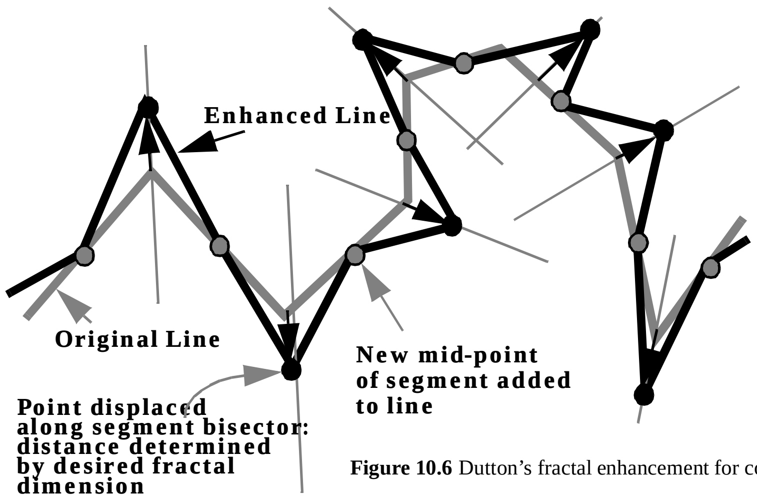

Figure 10.6 Dutton's fractal enhancement for coastlines.

10.6 TRANSFORMATIONS AND ALGORITHMS

The most elegant statement of the characteristics of a transformation is in a mathematical equation. Invertibility of equations can be proven by algebra or other methods, so that we can rearrange them to find other variables given a different set of starting conditions. When dealing with cartographic transformations, we do indeed have many transformations that are expressible in the language of mathematics. Others, however, are less amenable to such simple description.

In computer science, an *algorithm* is a special method of solving a problem. An algorithm may be stated either as a simple formula or as a set of sequential instructions to be followed to arrive at a solution. Church's theorem implies that if a process can be so stated, then it can be automated, in this case programmed. The set of algorithms for cartographic transformations operates on cartographic data that have some preexisting data structure, and results in a form of the data that is closer to the map required. A good summary, with apologies to Wirth, would be that

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data structures + transformational algorithms = maps
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When you dig down far enough within any piece of cartographic software, however complex, cartographic transformational algorithms are the very nuts and bolts with which that software is constructed. Similarly, the cartographic data structures involved are the materials that the transformations hold together to produce maps. An understanding of the data, the problems, the data structures, and the algorithms is really all the analytical

cartographer needs, except of course a real desire to make *better* maps. In the following chapters of Part III on analytical cartography, we will go closer into the algorithms that perform cartographic transformations, and show how some simple computer programs can be written to implement the transformational approach taken here.

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