AL-Karkh University of Science
College of Geophysics and Remote Sensing
Department of Geophysics

Numerical Methods

Dr. haleema swaidan Ali

AL-Karkh University of Science

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By:

Dr. Haleema Swaidan Ali

College of Remote Sensing and Geophysics
Department of Geophysics
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References:

1- Mathematical Methods Dr. T. K. V. Iyengar , Dr. B Krishna Gandhi (20080

Chapter 7

Curve Fitting

7.1 LEAST - SQUARES CURVE FITTING PROCEDURES :

With an experimetal data, the data is ploted on a graph paper and a straight line is drawn through the plotted points. This is the usual method to fit a mathematical equation to experimental data. The method of least squares is the most systematic procedure to fit a unique curve through the given data points. Its application is wide in practical computations.

Let the set of data points be $(x_i, y_i), i = 1, 2, ..., m$. Suppose the curve $y = f(x)$ is fitted to this data. Let the observed value at $x = x_i$ is y_i and the corresponding value on the curve is $f(x_i)$. Let e_i is the error of approximation at $x = x_i$, then we have,

$$e_i = y_i - f(x_i) \quad ...(1)$$

Consider $S = [y_1 - f(x_1)]^2 + [y_2 - f(x_2)]^2 + ...[y_m - f(x_m)]^2$

$$= e_1^2 + e_2^2 + ... + e_m^2 \quad ...(2)$$

The method of least squares consists of minimising S.

7.2 FITTING A STRAIGHT LINE

Let $y = a_0 + a_1 x$ is a straight line to be fitted to the given data. Then

$$S = [y_1 - (a_0 + a_1 x_1)]^2 + [y_2 - (a_0 + a_1 x_2)]^2 + ... + [y_m - (a_0 + a_1 x_m)]^2 \quad ...(3)$$

If S is to be minimum, we must have

$$\frac{\partial S}{\partial a_0} = 0 \Rightarrow -2\,[y_1 - (a_0 + a_1 x_1)] - 2\,[y_2 - (a_0 + a_1 x_2)]....$$
$$- 2\,[y_m - (a_0 + a_1 x_m)] = 0 \quad ...(4)$$

$$\frac{\partial S}{\partial a_1} = 0 \Rightarrow -2x_1[y_1 - (a_0 + a_1 x_1)] - 2x_2[y_2 - (a_0 + a_1 x_2)]....$$
$$-2x_m[y_m - (a_0 + a_1 x_m)] = 0 \quad ...(5)$$

Simplifying these equations, we get

$$m a_0 + a_1(x_1 + x_2 + ... + x_m) = y_1 + y_2 + ... + y_m$$

and $a_0(x_1 + x_2 + ... + x_m) + a_1(x_1^2 + x_2^2 + ... + x_m^2)$

$$= x_1 y_1 + x_2 y_2 + ... + x_m y_m \quad ...(6)$$

Writing in simplified form, we have

$$m a_0 + a_1 \sum_{i=1}^{m} x_i = \sum_{i=1}^{m} y_i \quad ...(7)$$

and $a_0 \sum_{i=1}^{m} x_i + a_1 \sum_{i=1}^{m} x_i^2 = \sum_{i=1}^{m} x_i y_i \quad ...(8)$

Since we know x_i and y_i equations (4) and (5) can be solved for two unknowns a_0 and a_1. Equations (7) and (8) are called *normal equations.*

These equations can be written as

$$\boxed{\begin{aligned} \Sigma y &= na_0 + a_1 \Sigma x \\ \Sigma xy &= a_0 \Sigma x + a_1 \Sigma x^2 \end{aligned}}$$

Differentiating (4) and (5) once again w.r.t. a_0 and a_1 we get, $\dfrac{\partial^2 S}{\partial a_0{}^2}$ and $\dfrac{\partial^2 S}{\partial a_1{}^2}$.

We find that these two will be positive at a_0 and a_1. Thus S is minimum at a_0 and a_1.

Dividing the equation (7) with m we obtain, $a_0 + a_1 \bar{x} = \bar{y}$ where $(\bar{x}, \bar{y})$ is a centroid of the given data points. Thus we can conclude that the fitted straight line passes through centroid of the data points.

Result : Derive the normal equations to fit the straight line $y = a + bx$ **[JNTU 2006 (Set No.3)]**

Sol. Taking $a_0 = a$ and $a_1 = b$ in the above, we get the result.

7.3 NON - LINEAR CURVE FITTING

We discuss now a power function, a polynomial of nth degree and an exponential function to fit the given data points $(x_i, y_i), i = 1, 2, ,...m$.

1. Power function :

Let $y = ax^c$ is the function to be fitted using the given data.

Taking logarithms on both sides, we get $\log y = \log a + c \log x$ ... (1)

which is of the form $\mathrm{Y} = a_0 + a_1 \mathrm{X}$ where $a_0 = \log a, a_1 = c$ and $\mathrm{Y} = \log y$ and $\mathrm{X} = \log x$. Using the procedure described earlier we can find a_0 and a_1 and hence c and a.

2. Polynomial of nth degree

Consider that the n[th] degree polynomial $\mathrm{Y} = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n$ ...(2)

be fitted to the data points $(x_i, y_i), i = 1, 2, ,...m$. Then we have

$$\begin{aligned} S = [y_1 - (a_0 + a_1 x_1 + ... + a_n x_1{}^n)]^2 + [y_2 - (a_0 + a_1 x_2 ... + a_n x_2{}^n)]^2 \\ + ... + [y_m - (a_0 + a_1 x_m ... + a_n x_m{}^n)]^2 \end{aligned} \quad ...(3)$$

As done earlier we equate the first partial derivatives to zero and after simplification we get the following normal equations.

$$\left.\begin{array}{l} ma_0 + a_1 \sum\limits_{i=1}^{m} x_i + a_2 \sum\limits_{i=1}^{m} x_i{}^2 + ... + a_n \sum\limits_{i=1}^{m} x_i{}^n = \sum\limits_{i=1}^{m} y_i \\ a_0 \sum\limits_{i=1}^{m} x_i + a_1 \sum\limits_{i=1}^{m} x_i{}^2 + ... + a_n \sum\limits_{i=1}^{m} x_i{}^{n+1} = \sum\limits_{i=1}^{m} x_i y_i \\ -------------------------- \\ a_0 \sum\limits_{i=1}^{m} x_i{}^n + a_1 \sum\limits_{i=1}^{m} x_i{}^{n+1} + ... + a_n \sum\limits_{i=1}^{m} x_i{}^{2n} = \sum\limits_{i=1}^{m} x_i{}^n y_i \end{array}\right\} \quad ...(4)$$

These are $(m + 1)$ equations in $(m + 1)$ unknowns. Hence can be solved for $a_0, a_1, ..., a_n$.

Substituting these values in (2) we get the required polynomial of n[th] degree.

3. **Parabola :** Considering $m = 2$, we get the curve to be fitted is parabola $y = a_0 + a_1 x + a_2 x^2$.

The normal equations are

$$\boxed{\begin{aligned} \Sigma y_i &= m a_0 + a_1 \Sigma x_i + a_2 \Sigma x_i^2 , \\ \Sigma x_i y_i &= a_0 \Sigma x_i + a_1 \Sigma x_i^2 + a_2 \Sigma x_i^3 , \\ \text{and } \Sigma x_i^2 y_i &= a_0 \Sigma x_i^2 + a_1 \Sigma x_i^3 + a_2 \Sigma x_i^4 \end{aligned}}$$

4. **Exponential function :**

(i) Suppose the curve to be fitted with the given data is $y = a_0 e^{a_1 x}$ ...(5)

Taking logarithms on both sides we get, $\log y = \log a_0 + a_1 x$ ...(6)

which can be written in the form $Z = A + Bx$ where $Z = \log y$, $A = \log a_0$ and $B = a_1$.

Then the problem reduces to finding a least square straight line.

(ii) Let the exponential curve be $y = ab^x$

Taking logarithms on both sides, we obtain

$$\log_{10} y = \log_{10} a + x \log_{10} b \quad \text{or} \quad Y = A + BX$$

where $Y = \log_{10} y$, $A = \log_{10} a$ and $B = \log_{10} b$.

The normal equations are given by

$$\boxed{\begin{aligned} \Sigma Y &= mA + B\Sigma X, \\ \Sigma XY &= A\Sigma X + B\Sigma X^2 \end{aligned}}$$

EXAMPLES

Ex. 1: By the method of least squares, find the straight line that best fits the following data :

x	1	2	3	4	5
y	14	27	40	55	68

Sol. The values of $\sum x$, $\sum y$, $\sum x^2$ and $\sum xy$ are calculated as follows :

x_i	y_i	x_i^2	$x_i y_i$
1	14	1	14
2	27	4	54
3	40	9	120
4	55	16	220
5	68	25	340

$\sum x_i = 15, \sum y_i = 204, \sum x_i^2 = 55$ and $\sum x_i y_i = 748$

The normal equations are

$m a_0 + a_1 \sum x_i = \sum y_i$ ...(7)

$a_0 \sum x_i + a_1 \sum x_i^2 = \sum x_i y_i$ ...(8)

Substituting the values, we get

$15 a_1 + 5 a_0 = 204$

$55 a_1 + 15 a_0 = 748$

Solving, we get $a_0 = 0$, and $a_1 = 13.6$.

Thus the line of best fitting is $y = 13.6x$.

Ex. 2: The temperatures T (in ^{0}c) and lengths L (in mm) of a heated rod are given below. If L = $a_0 + a_1$T, find the best values for a_0 and a_1.

T	20	30	40	50	60	70
L	800.3	800.4	800.6	800.7	800.9	801.0

Sol. We require $\sum T, \sum L, \sum T^2$ and $\sum TL$

and these are computed as in the following table.

T	L	T2	TL
20	800.3	400	16006
30	800.4	900	24012
40	800.6	1600	32024
50	800.7	2500	40035
60	800.9	3600	48054
70	801.0	4900	56070
ΣT=270	ΣL = 4803.9	$\Sigma T^2 = 113900$	ΣTL = 216201

We have the normal equations

$$ma_0 + a_1 \sum x_i = \sum y_i \quad ...(1); \quad a_0 \sum x_i + a_1 \sum x_i^2 = \sum x_i y_i \quad ...(2)$$

Using (1) and (2), we obtain

$6a_0 + 270a_1 = 4803.9$ and $270a_0 + 13900a_1 = 216201$

Solving these equations, we get $a_0 = 800$ and $a_1 = 0.0146$..

Ex. 3: Certain exprimental values of x and y are given below

x	0	2	5	7
y	-1	5	12	20

If $y = a_0 + a_1 x$ find the approximation values of a_0 and a_1.

Sol. We form the following table values :

x	y	x^2	xy
0	-1	0	0
2	5	4	10
5	12	25	60
7	20	49	140
$\sum x = 14$	$\sum y = 36$	$\sum x^2 = 78$	$\sum xy = 210$

Using normal equations we obtain $4a_0 + 14a_1 = 36$ and $14a_0 + 78a_1 = 210$

Solving the above two equations, we obtain $a_0 = -1.1381$ and $a_1 = 2.8966$.

Ex. 4: A chemical company, wishing to study the effect of extraction time on the efficiency of an extraction operation, obtained the data shown in the following table. **[JNTU 2004 Sept (Set No.1)]**

Extraction time minutes (x)										
	27	45	41	19	3	39	19	49	15	31
Efficiency (y)										
	57	64	80	46	62	72	52	77	57	68

Fit a straight line to the given data by the method of least squares.

Sol. Let the least squares of straight line of Y on X is $Y = a_0 + a_1X$

Its normal equations are

$\Sigma Y = Na_0 + a_1\Sigma X$

and $\Sigma XY = a_0\Sigma + a_1\Sigma X^2$

X	Y	X^2	Y^2	XY
27	57	729	3249	1539
45	64	2025	4096	2800
41	80	1681	6400	3280
19	46	361	2116	874
3	62	9	3844	186
39	72	1521	5184	2808
19	52	361	2704	988
49	77	2401	5929	3773
15	57	225	3249	855
31	68	961	4624	2108
288	635	10274	41395	19211

$\Sigma X = 228$, $\Sigma Y = 635$, $\Sigma X^2 = 10274$, $\Sigma Y^2 = 41395$, $\Sigma XY = 19211$ and $N = 10$

Substituting these values in the above normal equations, we get

$$635 = 10a_0 + 288a_1 \qquad \text{... (1)}$$

$$19211 = 288a_0 + 10274a_1 \qquad \text{... (2)}$$

To find a_0 and a_1, solve these equations.

$$(1) \times 29.8 \Rightarrow 288a_0 + 8294.4a_1 = 18288$$

$$(2) \times 1 \Rightarrow 288a_0 + 10274a_1 = 19211$$

$$-1979.6\, a_1 = -923$$

$$\Rightarrow a_1 = 2.35 \times 10^{-4}$$

$$\text{and } a_0 = 634.93$$

Thus the least square straight line Y on X is $Y = 634.93 + 0.000235X$.

Ex. 5: Fit a second degree polynomial to the following data by the method of least squares :

x	10	12	15	23	20
y	14	17	23	25	21

Sol. Let the required polynomial is $y = a_2x^2 + a_1x + a_0$.

From (4), we get the normal equations as (see page 262)

$$\sum y_i = ma_0 + a_1\sum x_i + a_2\sum x_i^2$$
$$\sum x_iy_i = a_0\sum x_i + a_1\sum x_i^2 + a_2\sum x_i^3$$
$$\sum x_i^2y_i = a_0\sum x^2 + a_1\sum x_i^3 + a_2\sum x_i^4$$

The above values are calculated by means of the following table.

x_i	y_i	x_i^2	x_i^3	x_i^4	x_iy_i	$x_i^2y_i$
10	14	100	1000	10000	140	1400
12	17	144	1728	20736	204	2448
15	23	225	3375	50625	345	5175
23	25	529	12167	279841	575	13225
20	21	400	8000	160000	420	8400
$\sum x_i = 80$	$\sum y_i = 100$	$\sum x_i^2 = 1398$	$\sum x_i^3 = 26270$	$\sum x_i^4 = 521202$	$\sum x_iy_i = 1684$	$\sum x_i^2y_i = 30648$

Substituting these values in normal equations, we obtain

$5a_0 + 80a_1 + 1398a_2 = 100,\ 80a_0 + 1398a_1 + 26270a_2 = 1684;$

$1398a_0 + 26270a_1 + 521202a_2 = 30648$

Solving these equations, we get

$$a_0 = -8.72790114;\ a_1 = 3.009927929 \text{ and } a_2 = -0.069495514$$

$\therefore$ The required equation is $y = -0.695x^2 + 3.001x - 8.728$.

Ex. 6: Fit a second degree polynomial to the following data by the method of least squares :

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

Sol. Let the required polynomial equation is $y = a_2x^2 + a_1x + a_0$. The normal equations are

$$\sum y_i = ma_0 + a_1\sum x_i + a_2\sum x_i^2$$
$$\sum x_iy_i = a_0\sum x_i + a_1\sum x_i^2 + a_2\sum x_i^3$$

$$\sum x_i^2 y_i = a_0 \sum x_i^2 + a_1 \sum x_i^3 + a_2 \sum x_i^4$$

The above values are calculated by means of the following table.

x_i	y_i	x_i^2	x_i^3	x_i^4	$x_i y_i$	$x_i^2 y_i$
0	1	0	0	0	0	0
1	1.8	1	1	1	1.8	1.8
2	1.3	4	8	16	2.6	5.2
3	2.5	9	27	81	7.5	22.5
4	6.3	16	64	256	25.2	100.8
$\sum x_i = 10$	$\sum y_i = 12.9$	$\sum x_i^2 = 30$	$\sum x_i^3 = 100$	$\sum x_i^4 = 354$	$\sum x_i y_i = 37.1$	$\sum x_i^2 y_i = 130.3$

Substituting these values in normal equations, we get

$$5a_0 + 10a_1 + 30a_2 = 12.9,\ 10a_0 + 30a_1 + 100a_2 = 37.1\ ;\ 30a_0 + 100a_1 + 354a_2 = 130.3$$

Solving these equations, we get $a_0 = 1.42, a_1 = -1.07$ and $a_2 = 0.55$.

$\therefore$ The required equation is $y = 0.55x^2 - 1.07x + 1.42$.

Ex. 7: Fit a polynomial of second degree to the data points given in the following table:

x	0	1.0	2.0
y	**1.0**	**6.0**	**17.0**

Sol. We require the quantities $\sum x_i, \sum x_i^2, \sum x_i^3, \sum x_i^4, \sum y_i, \sum x_i y_i$ and $\sum x_i^2 y$. These are computed in the following table.

x_i	y_i	x_i^2	x_i^3	x_i^4	$x_i y_i$	$x_i^2 y_i$
0	1	0	0	0	0	0
1	6	1	1	1	6	6
2	17	4	8	16	34	68
$\Sigma x_i = 3$	$\Sigma y_i = 24$	$\Sigma x_i^2 = 5$	$\Sigma x_i^3 = 9$	$\Sigma x_i^4 = 17$	$\Sigma x_i y_i = 40$	$\Sigma x_i^2 y_i = 74$

Using equations (4), we get the equations (see Ex.5 on page 266)

$$3a_0 + 3a_1 + 5a_2 = 24;\ 3a_0 + 5a_1 + 9a_2 = 40;\ 5a_0 + 9a_1 + 17a_2 = 74$$

Solving these equations, we get $a_0 = 1, a_1 = 2$ and $a_2 = 3$.

The required polynomial is then given by $y = 1 + 2x + 3x^2$. From the given data points it is seen that this polynomial fitting as "exact".

Ex. 8: Determine the constants a and b by the method of least squares such that $y = ae^{bx}$.

x	2	4	6	8	10
y	4.077	11.084	30.128	81.897	222.62

Sol. The given relation is $y = ae^{bx}$

Taking logarithms of both sides, we obtain $\log y = \log a + bx$ ... (1)

Let $\log y = Y$, $x = X$. Then $\log a = a_0$ and $b = a_1$.

The relation (1) takes the form $Y = a_0 + a_1 X$, which is a straight line.

The method of procedure is the same as in fitting a straight line and we form the following table :

$X = x$	$Y = \ln y$	x^2	XY
2	1.405	4	2.810
4	2.405	16	9.620
6	3.405	36	20.430
8	4.405	64	35.240
10	5.405	100	54.050
$\sum X = 30$	$\sum Y = 17.025$	$\sum x^2 = 220$	$\sum XY = 122.150$

The normal equations to fit the straight line gives

$5a_0 + 30a_1 = 17.025$, $30a_0 + 220a_1 = 122.150$

which yield the solution $a_0 = 0.405$ and $a_1 = 0.5$.

Hence $a = e^{a_0} = e^{0.405} = 1.499$ and $b = a_1 = 0.5$.

Hence the required curve is $y = 1.499e^{0.5x}$.

Ex. 9: Find the curve of best fit of the type $y = ae^{bx}$ to the following data by the method of least squares.

x	1	5	7	9	12
y	10	15	12	15	21

Sol. Required curve to be fitted is $y = ae^{bx}$

Taking logarithms of both sides, we obtain $\log y = \log a + bx$ ... (1)

Setting $\log y = Y, x = X$, $\log a = a_0$ and $b = a_1$.

The relation (1) takes the form $Y = a_0 + a_1 X$, which is a straight line.

The method of porcedure is the same as in fitting a straight line and we form the following table

$x = X$	y	$Y = \log_{10} y$	x^2	XY
1	10	1.0000	1	1
5	15	1.1761	25	5.8805
7	12	1.0792	49	7.5544
9	15	1.1761	81	10.5849
12	21	1.3222	144	15.8664
$\sum X = 34$		$\sum Y = 5.7536$	$\sum x^2 = 300$	$\sum XY = 40.8862$

Substituting these values in normal equations, we get

$5a_0 + 34a_1 = 5.7536, \quad 34a_0 + 300a_1 = 40.8862$

On solving, $a_0 = 0.9766$ and $a_1 = 0.02561$

$\therefore \; a = e^{a_0} = e^{0.9766} = 9.4754$ and $b = a_1 = 0.059$.

Hence the required curve is $y = 9.4754 \cdot e^{0.059x}$

Ex. 10: Using the method of least squares determine the constants a and b such that $y = ae^{bx}$ fits the following data.

x	0.0	0.5	1.0	1.5	2.0	2.5
y	0.10	0.45	2.15	9.15	40.35	180.75

Sol. Required curve to be fitted is $y = ae^{bx}$

Taking logarithms of both sides, we obtain $\log y = \log a + bx$

Taking $\log y = Y, x = X$, $\log a = a_0$ and $b = a_1$, the above relation takes the form $Y = a_0 + a_1 X$, which is a straight line.

The method of porcedure is the same as in fitting a straight line and we form the following table

$x = X$	y	$Y = \log_{10} y$	x^2	XY
0	0.10	−1	0	0
0.5	0.45	−0.3468	0.25	−0.1734
1.0	2.15	0.3324	1.0	0.3324
1.5	9.15	0.9614	2.25	1.4421
2.0	40.35	1.6058	4.0	3.2116
2.5	180.75	2.2571	6.25	5.6428
$\sum X = 7.5$		$\sum Y = 3.8099$	$\sum x^2 = 13.75$	$\sum XY = 10.4555$

Substituting these values in normal equations, we obtain

$6a_0 + 7.5a_1 = 3.8099$ and $7.5a_0 + 13.75a_1 = 10.4555$

On solving, $a_0 = -0.9916$ and $a_1 = 1.3013$

$\therefore\ a = e^{a_0} = e^{-0.9916} = 0.37098$ and $b = a_1 = 1.3013$

Hence the required curve is $y = 0.37098\, e^{1.3013x}$.

Ex. 11: Obtain a relation of the form $y = ab^x$ for the following data by the method of least squares.

x	2	3	4	5	6
y	8.3	15.4	33.1	65.2	127.4

Sol. The curve to be fitted is $y = a(b^x)$

Taking logarithms on both sides, $\log y = \log a + x \log b$

Consider $\log y = Y, x = X$ $\log a = a_0$ and $a_1 = \log b$

We get the curve to be fitted as $Y = a_0 + a_1X$.

To fit the above curve we form the table as follows :

$x = X$	y	$Y = \log y$	X^2	XY
2	8.3	0.9191	4	1.8382
3	15.4	1.1875	9	3.5625
4	33.1	1.5198	16	6.0792
5	65.2	1.8142	25	9.0710
6	127.4	2.1052	36	12.631
$\sum X = 20$		$\sum Y = 7.5455$	$\sum X^2 = 90$	$\sum XY = 33.1819$

Substituting these values in normal equations, we get

$5a_0 + 20a_1 = 7.5455$, $20a_0 + 90a_1 = 33.1819$

On solving, $a_0 = 0.31$ and $a_1 = 0.3$

$\therefore\ a =$ antilog of $a_0 = 2.04$, $b =$ antilog of $a_1 = 1.995$

Hence the required curve is $y = 2.04\,(1.995)^x$.

Ex. 12: Fit the curve $y = ae^{bx}$ to the following data.

x :	0	1	2	3	4	5	6	7	8
y :	20	30	52	77	135	211	326	550	1052

[JNTU Dec 2004 (Set No.2)]

Sol. Given curve is $Y = ae^{bx}$

Taking logarithm on both sides, we get

$$\log Y = \log a + bx \log e$$

Put $\log y = y^*$, $a^* = \log a$, $b^* = b$.

Then $y^* = a^* + b^* x$. This is a linear equation is X.

Its normal equations are

$$\Sigma Y^* = Na^* + b^* \ \Sigma X$$

$$\Sigma XY^* = a^* \Sigma X \ + \Sigma X^2$$

X	Y	$Y^* = \log Y$	X^2	XY^*
0	20	2.995	0	0
1	30	3.401	1	3.401
2	52	3.951	4	7.902
3	77	4.343	9	13.029
4	135	4.905	16	19.62
5	211	5.351	25	26.755
6	320	5.768	36	34.608
7	550	6.309	49	44.163
8	1052	6.958	64	55.664
36		43.981	204	205.142

$\Sigma X = 36$, $\Sigma Y^* = 43.981$, $\Sigma X^2 = 204$, $\Sigma XY^* = 205.142$ and $N = 9$

Now the normal equations are

$$43.981 = 9a^* + 36b^* \quad \ldots (1)$$

$$205.142 = 36a^* + 204b^* \quad \ldots (2)$$

Solving (1) & (2) we get, $a^* = 2.942$, $b^* = 0.486$

$\because a^* = \log a \Rightarrow a = 18.95$ and $b^* = b = 0.486$

The least square exponential function is $Y = 18.95e^{0.486X}$.

Ex. 13: Fit $y = a(b^x)$ by the method of least squares to the data given below.

x	0	1	2	3	4	5	6	7
y	10	21	35	59	92	200	400	610

Sol. The curve to be fitted is $y = a(b^x)$

Taking logarithms on both sides, $\log y = \log a + x \log b$

Consider $Y = \log y, x = X \cdot a_0 = \log a$ and $a_1 = \log b$.

We get the curve to be fitted as $Y = a_0 + a_1 X$.