AL-Karkh University of Science
College of Geophysics and Remote Sensing
Department of Geophysics

PHYSICS II

For Geophysics / Second Semester

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Rotations (8)

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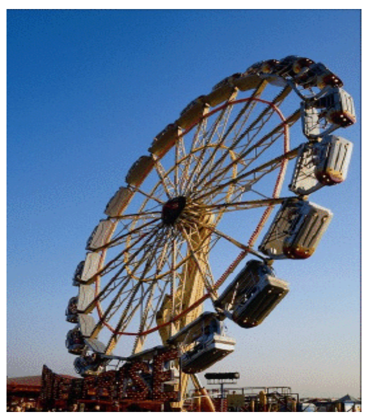
Rotational Motion

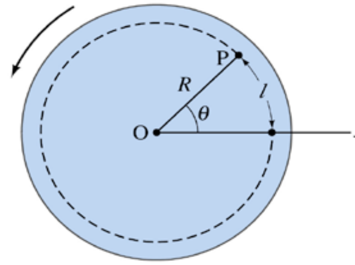
- Angular position and radian
- Angular displacement
- Angular velocity
- Angular acceleration
- Rotational motion under constant angular acceleration
- Relations between angular and linear quantities
- Circular motion



Rotational motion

When the body rotated about a axis, the motion is called rotation motion.

- In our discussion of rotational motion we will first focus on the rotation of rigid objects around a fixed axis.
- The variables that are used to describe this type of motion are similar to those we use to describe linear motion:
  - Angular position
  - Angular velocity
  - Angular acceleration



Angle and Radian

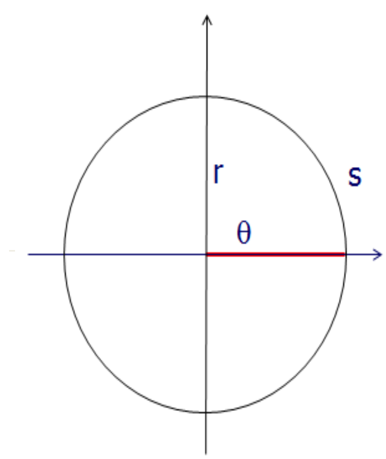
- What is the circumference S?

$$s = (2\pi)r \qquad 2\pi = \frac{s}{r}$$

- θ can be defined as the arc length s along a circle divided by the radius r:

$$\theta = \frac{s}{r}$$

- θ is a pure number, but commonly is given the artificial unit, radian ("rad")



Conversions

- **Comparing degrees and radians**

$$2\pi(rad) = 360^\circ \qquad \pi(rad) = 180^\circ$$

- **Converting from degrees to radians**

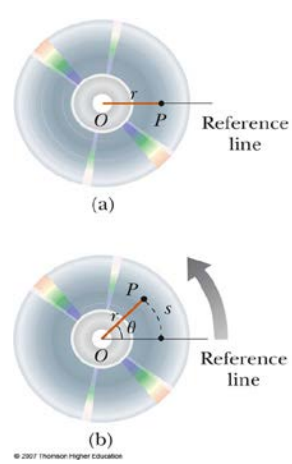
$$\theta\,(rad) = \frac{\pi}{180^\circ}\,\theta\,(degrees)$$

- **Converting from radians to degrees**

$$\theta(\deg rees) = \frac{180^\circ}{\pi}\theta(rad) \qquad 1\,rad = \frac{360^\circ}{2\pi} = 57.3^\circ$$

Angular Position θ

- **Axis of rotation is the center of the disc**
- **Choose a fixed reference line**
- **Point P is at a fixed distance r from the origin**
- **As the particle moves, the only coordinate that changes is θ**
- **As the particle moves through θ, it moves though an arc length s.**
- **The angle θ, measured in radians, is called the angular position.**

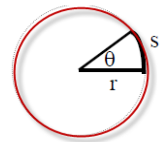


➢ Angular Position θ

The angle θ is the natural quantity to represent the position of a solid object rotating about a fixed axis. This position variable has units of radians and is defined as the ratio of two lengths involving a circle: arc length over the radius r.

Note that θ = 1 radian is defined by s = r.

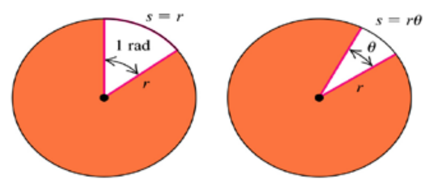
$\theta \equiv s/r$



- **The angular displacement will be defined as a change $\Delta\theta = \theta_2 - \theta_1$ position,**

, during a time interval $\Delta t = t_2 - t_1$

$$1\ rad : s = r$$



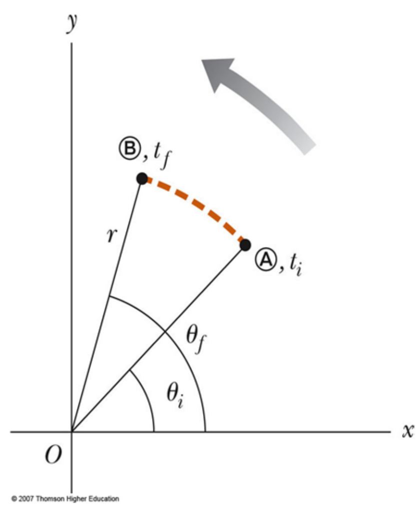
$$\theta = \frac{s}{r}, \quad s = r\theta$$

Angular Displacement

- **The angular displacement is defined as the angle of the object rotates through during some time interval**

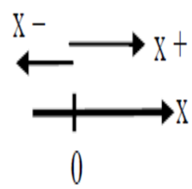
$$\Delta\theta = \theta_f - \theta_i$$

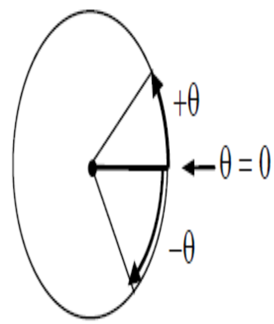
- **SI unit: radian (rad)**
- **This is the angle that the reference line of length *r* sweeps out**



Angle θ of a rigid object is measured relative to some reference orientation, just like 1D position x is measured relative to some reference position (the origin).

Angle θ is the "rotational position". Like position x in 1D, rotational position θ has a sign convention. Positive angles are CCW (counter-clockwise).





Velocity

- **Velocity is the rate of change of position.**
- **Velocity is a vector quantity.**
- **Velocity has both magnitude and direction.**
- **Velocity has a unit of [length/time]: meter/second.**
- **Definition:**
  - **Average velocity** $v_{avg} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_f - x_i}{\Delta t}$

Average and Instantaneous Angular Speed

- **The average angular speed, ω_{avg}, of a rotating rigid object is the ratio of the angular displacement to the time interval**

$$\omega_{avg} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

- **The instantaneous angular speed is defined as the limit of the average speed as the time interval approaches zero**

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

- **SI unit: radian per second (rad/s)**
- **Angular speed positive if rotating in counterclockwise**
- **Angular speed will be negative if rotating in clockwise**

Angular Velocity

$$\bar{v} = \frac{\Delta x}{\Delta t} \rightarrow translational\ velocity$$

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} \rightarrow rotational\ velocity$$

$$v = \frac{dx}{dt}, \omega = \frac{d\theta}{dt}$$

$$1 revolution = 2\pi\ radians = 360^{\circ}$$

Definitions

Angular Position: $\vec{\theta}$ (**normally in radians**)

Angular Displacement: $\Delta\vec{\theta} = \vec{\theta} - \vec{\theta}_0$

Average or mean angular velocity is defined as follows:

$$\vec{\omega}_{avg} \equiv \frac{\vec{\theta} - \vec{\theta}_0}{t - t_0} \equiv \frac{\Delta\vec{\theta}}{\Delta t}$$

Instantaneous angular velocity or just " angular velocity":

$$\vec{\omega} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{\theta}}{\Delta t} \equiv \frac{d\vec{\theta}}{dt}$$

Average Acceleration

- **Changing velocity (non-uniform) means an acceleration is present.**
- **Acceleration is the rate of change of velocity.**
- **Acceleration is a vector quantity.**
- **Acceleration has both magnitude and direction.**
- **Acceleration has a unit of [length/time^2]: m/s^2.**
- **Definition:**
  - **Average acceleration** $a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$
  - **Instantaneous acceleration** $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\frac{dx}{dt} = \frac{d^2 v}{dt^2}$

Angular Acceleration

$$\bar{a} = \frac{\Delta v}{\Delta t} \rightarrow \text{translational acceleration}$$

$$\bar{\alpha} = \frac{\Delta \omega}{\Delta t} \rightarrow \text{rotational acceleration}$$

$$a = \frac{dv}{dt}, \alpha = \frac{d\omega}{dt}$$

$$x = \int v\, dt, \quad \theta = \int \omega\, dt$$

$$v = \int a\, dt, \quad \omega = \int \alpha\, dt$$

Instantaneous Angular Acceleration

- **The instantaneous angular acceleration is defined as the limit of the average angular acceleration as the time goes to 0**

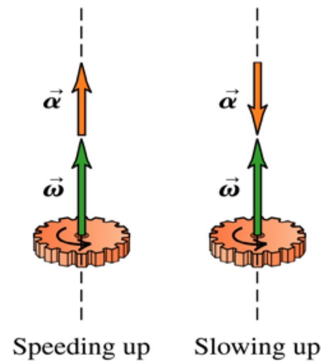
$$\alpha \equiv \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt}$$

- **SI Units of angular acceleration: rad/s²**
- **Positive angular acceleration is in the counterclockwise.**
  - **if an object rotating counterclockwise is speeding up**
  - **if an object rotating clockwise is slowing down**
- **Negative angular acceleration is in the clockwise.**
  - **if an object rotating counterclockwise is slowing down**
  - **if an object rotating clockwise is speeding up**

Angular acceleration

Just as was the case for linear motion:

- the object will be "***speeding up***" if the angular acceleration is in the same direction as the angular velocity, and
- the object will be "***slowing down***" if the angular acceleration is in the opposite direction of the angular velocity.



Translation vs. Rotation

There are 2 types of pure unmixed motion:

- **Translational - linear motion**
- **Rotational - motion involving a rotation or revolution around a fixed chosen axis(** ***an axis which does not move*****).**

We need a system that defines BOTH types of motion working together on a system. Rotational quantities are usually defined with units involving a ***radian*** **measure.**

- **Rotational kinematics: describes rotational motion.**
- **Rigid body: idealized model of a body which has a perfectly definite and unchanging shape and size.**

Homework

For every quantity in linear (1D translational) motion, there is corresponding quantity in rotational motion:

Translation	$\leftrightarrow$	**Rotation**
x	$\leftrightarrow$	θ
$v = \frac{dx}{dt}$	$\leftrightarrow$	$\omega = \frac{d\theta}{dt}$
$a = \frac{dv}{dt}$	$\leftrightarrow$	$\alpha = \frac{d\omega}{dt}$
F	$\leftrightarrow$	(?)
M	$\leftrightarrow$	(?)
$F = Ma$	$\leftrightarrow$	$(?) = (?)\ \alpha$
$KE = (1/2)\ m\ v^2$	$\leftrightarrow$	$KE = (1/2)\ (?)\ \omega^2$

The rotational analogue of force is *torque*.

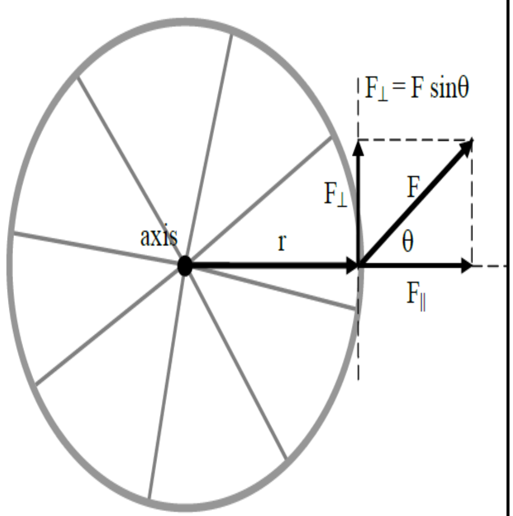
Force F causes acceleration a $\leftrightarrow$

Torque τ causes angular acceleration α .

The torque (pronounced "tork") is a kind of "rotational force".

magnitude of torque: $\boxed{|\tau| \equiv r \cdot F_\perp = r F \sin\theta}$

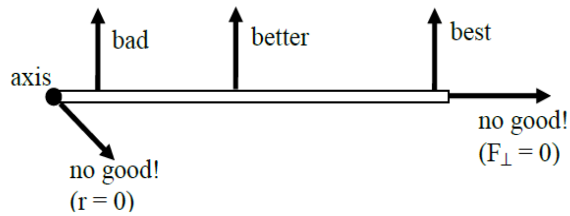
$$[\tau] = [r][F] = \mathrm{m\,N}$$



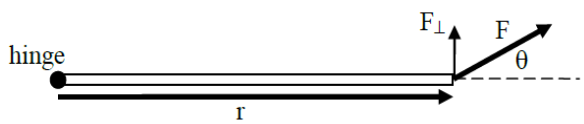
r = "lever arm" = distance from axis to point of application of force

$F_\perp$ = component of force perpendicular to lever arm

Example: Wheel on a fixed axis:
Notice that only the perpendicular component of the force **F** will rotate the wheel. The component of the force parallel to the lever arm ($F_{\parallel}$) has no effect on the rotation of the wheel. If you want to easily rotate an object about an axis, you want a large lever arm r and a large perpendicular force $F_{\perp}$:



Example: Pull on a door handle a distance r = 0.8 m from the hinge with a force of magnitude F = 20 N at an angle θ = 30° from the plane of the door, like so:

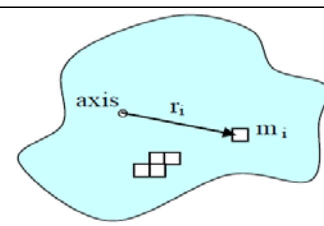


$\tau = r F_{\perp} = r F \sin\theta =$

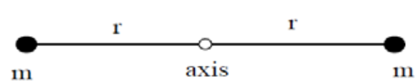
$(0.8 \text{ m})(20 \text{ N})(\sin 30^{o}) = 8.0 \text{ m·N}$

Definition of *moment of inertia* of an extended object about an axis of rotation:

$$\boxed{I \;=\; \sum_i m_i r_i^2 \;=\; m_1 r_1^2 \;+\; m_2 r_2^2 \;+\; \ldots}$$

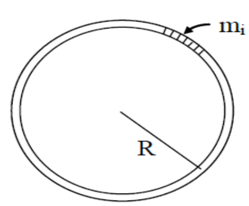


Examples:
• 2 small masses on rods of length r:



$I = 2\, m\, r^2$

• A hoop of total mass M, radius R, with axis through the center, has $I_{hoop} = M R^2$



$$I \;=\; \sum_i m_i\, r_i^2 \;=\; \left(\sum_i m_i\right) R^2 \;=\; M R^2 \quad (\text{since } r_i = R \text{ for all i})$$

In detail:
$$I \;=\; m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots = m_1 R^2 + m_2 R^2 + m_3 R^2 + \ldots$$
$$= (m_1 + m_2 + m_3 + \ldots) R^2 \;=\; M R^2$$

Example 1

The angular position θ of a 0.36-m-diameter flywheel is given by

$$\theta = (2.0\ \text{rad/s}^3)t^3$$

(a) Find θ, in radians and in degrees, at $t_1 = 2.0$ s and $t_2 = 5.0$ s. (b) Find the distance that a particle on the flywheel rim moves over the time interval from $t_1 = 2.0$ s to $t_2 = 5.0$ s. (c) Find the average angular velocity, in rad/s and in rev/min, over that interval. (d) Find the instantaneous angular velocities at $t_1 = 2.0$ s and $t_2 = 5.0$ s.

IDENTIFY and SET UP: We can find the target variables θ_1 (the angular position at time t_1), θ_2 (the angular position at time t_2), and the angular displacement $\Delta\theta = \theta_2 - \theta_1$ from the given expression. Knowing $\Delta\theta$, we'll find the distance traveled and the average angular velocity between t_1 and t_2 using Eqs. (9.1) and (9.2), respectively. To find the instantaneous angular velocities ω_{1z} (at time t_1) and ω_{2z} (at time t_2), we'll take the derivative of the given equation for θ with respect to time, as in Eq.

$$\omega_z = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \qquad \text{(definition of angular velocity)}$$

EXECUTE: (a) We substitute the values of t into the equation for θ:

$$\begin{aligned}
\theta_1 &= (2.0\ \text{rad/s}^3)(2.0\ \text{s})^3 = 16\ \text{rad} \\
&= (16\ \text{rad})\frac{360^\circ}{2\pi\ \text{rad}} = 920^\circ \\
\theta_2 &= (2.0\ \text{rad/s}^3)(5.0\ \text{s})^3 = 250\ \text{rad} \\
&= (250\ \text{rad})\frac{360^\circ}{2\pi\ \text{rad}} = 14{,}000^\circ
\end{aligned}$$

(b) During the interval from t_1 to t_2 the flywheel's angular displacement is $\Delta\theta = \theta_2 - \theta_1 = 250\ \text{rad} - 16\ \text{rad} = 234\ \text{rad}$.

The radius r is half the diameter, or 0.18 m. To use Eq. (9.1), the angles *must* be expressed in radians:

$$s = r\theta_2 - r\theta_1 = r\Delta\theta = (0.18\ \text{m})(234\ \text{rad}) = 42\ \text{m}$$

We can drop "radians" from the unit for s because θ is a pure, dimensionless number; the distance s is measured in meters, the same as r.

(c) From Eq. (9.2),

$$\begin{aligned}
\omega_{\text{av-}z} &= \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{250\ \text{rad} - 16\ \text{rad}}{5.0\ \text{s} - 2.0\ \text{s}} = 78\ \text{rad/s} \\
&= \left(78\frac{\text{rad}}{\text{s}}\right)\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = 740\ \text{rev/min}
\end{aligned}$$

(d) From Eq. (9.3),

$$\begin{aligned}
\omega_z &= \frac{d\theta}{dt} = \frac{d}{dt}[(2.0\ \text{rad/s}^3)t^3] = (2.0\ \text{rad/s}^3)(3t^2) \\
&= (6.0\ \text{rad/s}^3)t^2
\end{aligned}$$

At times $t_1 = 2.0$ s and $t_2 = 5.0$ s we have

$$\begin{aligned}
\omega_{1z} &= (6.0\ \text{rad/s}^3)(2.0\ \text{s})^2 = 24\ \text{rad/s} \\
\omega_{2z} &= (6.0\ \text{rad/s}^3)(5.0\ \text{s})^2 = 150\ \text{rad/s}
\end{aligned}$$

EVALUATE: The angular velocity $\omega_z = (6.0\ \text{rad/s}^3)t^2$ increases with time. Our results are consistent with this; the instantaneous angular velocity at the end of the interval ($\omega_{2z} = 150$ rad/s) is greater than at the beginning ($\omega_{1z} = 24$ rad/s), and the average angular velocity $\omega_{\text{av-}z} = 78$ rad/s over the interval is intermediate between these two values.

Relationship Between Angular and Linear Quantities

- Every point on the rotating object has the same angular motion
- Every point on the rotating object does not have the same linear motion
- Displacement

$$s = \theta r$$

- Speeds

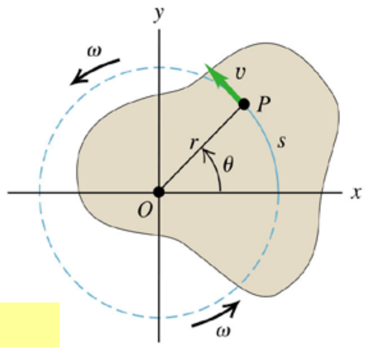
$$v = \omega r$$

- Accelerations

$$a = \alpha r$$

Relating Linear and Angular Kinematics

- Consider a point **P** on a rotating object that is a distance ***r*** away from the axis of rotation. As the object turns through an angle θ the point covers a distance given by $s = r\theta$
  - In the above expression the angle θ must be in radians
- If this expression is differentiated with respect to time then the left hand side will become the *linear speed* of particle
- This speed corresponds to the velocity of the point P which is tangential to the circular arc traced out by the point. When differentiating the right hand side, we notice that *r* is constant and the rate of change of angular position is the angular velocity. This gives:



$$\left|\frac{ds}{dt}\right| = r\left|\frac{d\theta}{dt}\right|$$

$$v = r\omega$$

Relation betwee
angular speed

Relating Linear and Angular Kinematics

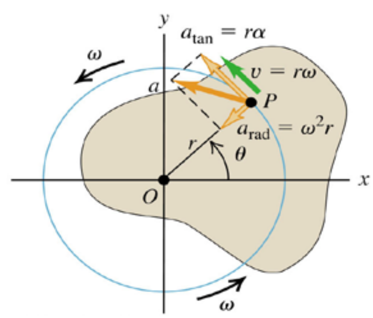
- **Differentiating once again gives a relationship between the tangential acceleration of the point, $\mathbf{a}_{tan}$, and the angular acceleration of the rotation object:**

$$a_{\tan} = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha$$

nally, recall that any obj dergoing circular motio inwardly directed radia

$$a_{rad} = \frac{v^2}{r} = \omega^2 r$$

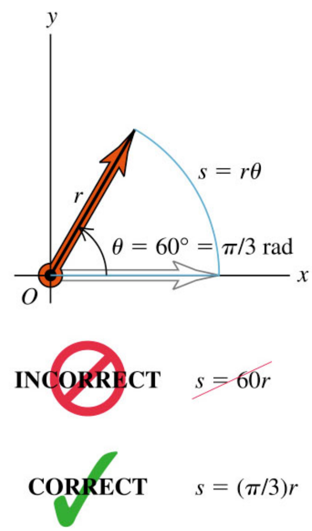
ntripetal acceleration of oint on a rotating body



$$s = r\theta \qquad v = r\omega$$

$$a_{\tan} = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha$$

hese equations apply to *a* as *the same* tangential vel a rotating rigid body



Combining motions –Tangential velocity

$$\Delta s = R\Delta\theta$$

$$\frac{\Delta s}{\Delta t} = R\frac{\Delta\theta}{\Delta t}$$

$$v_T = \frac{\Delta s}{\Delta t}$$

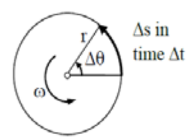
$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$v_T = r\omega$$

First we take our equation for the radian measure and divide BOTH sides by a change in time.

The left side is simply the equation for **LINEAR** velocity. **BUT** in this case the velocity is **TANGENT** to the circle (*according to Newton's first law*). Therefore we call it **TANGENTIAL VELOCITY**.

Inspecting the right side we discover the formula for **ANGULAR VELOCITY.**



Therefore, substituting the appropriate symbols we have a formula that relates Translational velocity to Rotational velocity.

Tangential acceleration and rotational kinematics

$$v_t = r\omega \rightarrow \frac{v_t}{\Delta t} = \frac{r\omega}{\Delta t}$$

$$a_t = r\alpha$$

$$v = v_o + at \rightarrow \quad \omega = \omega_o + \alpha t$$

$$x = x_o + v_o t + \tfrac{1}{2}at^2 \rightarrow \quad \theta = \theta_o + \omega_o t + \tfrac{1}{2}\alpha t^2$$

$$v^2 = v_o^2 + 2a\Delta x \rightarrow \quad \omega^2 = \omega_o^2 + 2\alpha\Delta\theta$$

Example

A turntable capable of angularly accelerating at 12 rad/s^2 needs to be given an initial angular velocity if it is to rotate through a net 400 radians in 6 seconds. What must its initial angular velocity be?

$$\alpha = 12\ rad / s^2$$
$$\Delta\theta = 400\ rad$$
$$t = 6\ s$$
$$\omega_o = ?$$
$$\Delta\theta = \omega_o 2t + \tfrac{1}{2}\alpha t^2$$
$$400 = \omega_o (6) + (0.5)(12)(6)^2$$
$$\omega_o =$$

Rotational Inertia (Moment of Inertia)

- The *rotational inertia* of an object is a measure of the <u>resistance</u> of the object to <u>changes</u> in its <u>rotational</u> <u>motion</u>
- For a system of particles of masses m_i at distances r_i from an axis passing through a point P the rotational inertia of the system about the axis is given by:

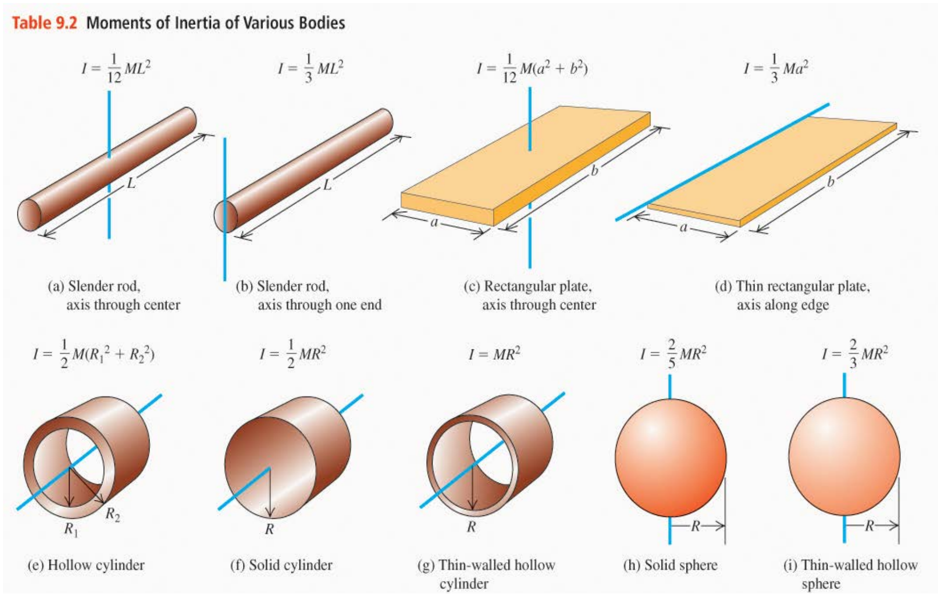
$$I = m_1 r_1^2 + m_2 r_2^2 + \ldots = \sum_i m_i r_i^2$$

I unit of moment of iner

or a solid object the rotat

; we will see later

Rotational Inertia (Moment of Inertia)



Rotational kinetic energy.

- **Since the components of a rotating object have a non-zero (linear) velocity we can associate a kinetic energy with the rotational motion:**

$$K = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i (\omega r_i)^2 = \frac{1}{2}\left\{\sum_i m_i r_i^2\right\}\omega^2$$

- **The kinetic energy is proportional to the square of the rotational velocity w. Note: the equation is similar to the translational kinetic energy (1/2 mv^2) except that instead of being proportional to the the mass m of the object , the rotational kinetic energy is proportional to the moment of inertia I of the object:**

$$I = \sum_i m_i r_i^2 \quad \Rightarrow \quad K = \frac{1}{2} I \omega^2$$

Note: units of I: kg m^2

Rotational Kinetic Energy and Inertia

Just like massive bodies tend to resist changes in their motion (AKA - "Inertia") . Rotating bodies also tend to resist changes in their motion. We call this **ROTATIONAL INERTIA.** We can determine its expression by looking at Kinetic Energy.

$$K = \tfrac{1}{2}mv^2, \quad v_t = r\omega$$

$$K = \tfrac{1}{2}m(r\omega)^2$$

$$K = \tfrac{1}{2}\boxed{mr^2}\omega^2, \quad I = \sum mr^2$$

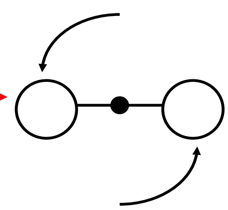
$$K_{rot} = \tfrac{1}{2}I\omega^2$$

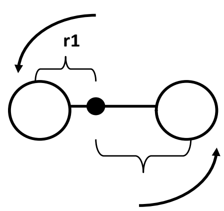
Moment of Inertia, I

$$I = \sum mr^2$$

Tl

ıt we would see if m1 = m2. Suppose





Since it is a ri

$$v_t = r\omega$$

Moment of Inertia, I

Since both masses are moving they have kinetic energy or rotational kinetic in this case.

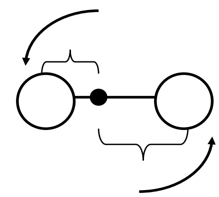
$$K = K_1 + K_2$$

$$K = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2, \quad v_t = r\omega$$

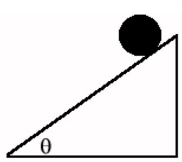
$$K = \tfrac{1}{2} m_1 (r_1^2 \omega^2) + \tfrac{1}{2} m_2 (r_2^2 \omega^2)$$

$$K = \tfrac{1}{2} (m_1 r_1^2 + m_2 r_2^2) \omega^2$$

$$K = \tfrac{1}{2} \left(\sum_{i=1}^{N} m_i r_i^2\right) \omega^2 \rightarrow K = \tfrac{1}{2} I \omega^2$$



Example



work out this problem they way you used to. In the past, everything was SLIDING. Now the object is rolling and thus has MORE energy than normal. So let's assume the ball is like a thin spherical shell and was released from a position 5 m above the ground. Calculate the velocity at the bottom of the incline.

$$E_{before} = E_{after}$$

$$U_g = K_T + K_R$$

$$mgh = \tfrac{1}{2} mv^2 + \tfrac{1}{2} I\omega^2$$

$$v = R\omega \quad I_{sphere@cm} = \tfrac{2}{3} mR^2$$

$$mgh = \tfrac{1}{2} mv^2 + \tfrac{1}{2} (\tfrac{2}{3} mR^2)(\tfrac{v^2}{R^2})$$

$$gh = \tfrac{1}{2} v^2 + \tfrac{1}{3} v^2$$

$$v = \sqrt{\tfrac{6}{5} gh} = \sqrt{\tfrac{6}{5}(9.8)(5)} = \text{7.67 m/s}$$

$$E_{before} = E_{after}$$

$$U_g = K_T$$

$$mgh = \tfrac{1}{2} mv^2$$

$$mgh = \tfrac{1}{2} mv^2$$

$$gh = \tfrac{1}{2} v^2$$

$$v = \sqrt{2gh} = \sqrt{2(9.8)(5)} = 9.90\ m/s$$

If y
inc
rot
ene
ans

Parallel Axis Theorem

This theorem will allow us to calculate the moment of inertia of any rotating body around any axis, provided we know the moment of inertia about the center of mass.

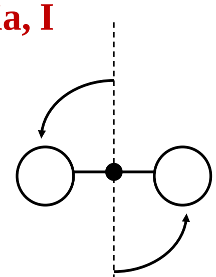
$$I_p = I_{cm} + Md^2$$

ıent of Inertia (I_p) around any axis "P" is inertia (I_{cm}) about some center of mass plus M imes the square of "d" (the distance between

ɜe the parallel axis theorem to calculate the tating around one end and 2m from a fixed

Example: Moment of Inertia, I

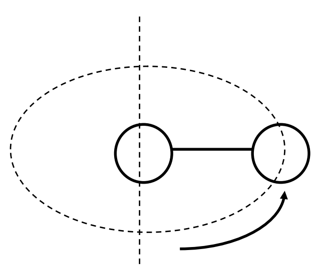
$$I = \sum mr^2$$



$$I = \sum mr^2 = mr^2 + mr^2$$

$$I = (3)(2)^2 + (3)(2)^2 =$$

ıment if Inertia rotating

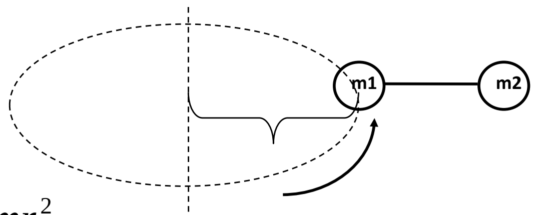


$$I = \sum mr^2 = mr^2 + mr^2$$

$$I = (3)(0)^2 + (3)(4)^2 =$$

Example cont'

Now let's calculate the moment of Inertia rotating at a point 2 meters from one end of the rod.



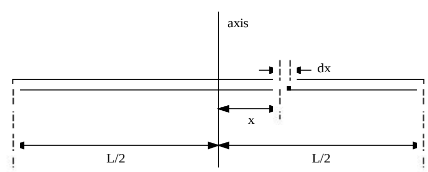
$$I = \sum mr^2 = mr^2 + mr^2$$

$$I = (3)(2)^2 + (3)(6)^2 =$$

As you can see, the FARTI
mass, the moment of inert

Moment of inertia. Sample problem.

- **Consider a rod of length *L* and mass *m*. What is the moment of inertia with respect to an axis through its center of mass?**

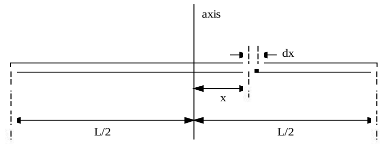


- **Consider a slice of the rod, with width *dx*, located a distance *x* from the rotation axis. The mass *dm* of this slice is equal to**

$$dm = \frac{m}{L}dx$$

Moment of inertia.
Sample problem.

- The moment of inertia *dI* of this slice is equal to



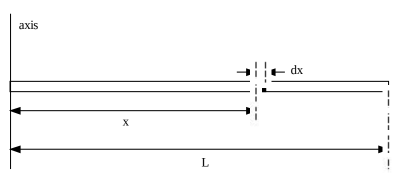
$$dI = x^2 dm = \frac{m}{L} x^2 dx$$

- The moment of inertia of the rod can be found by adding the contributions of all of the slices that make up the rod:

$$I = \int_{-L/2}^{L/2} dI = \int_{-L/2}^{L/2} \frac{m}{L} x^2 dx = \frac{m}{L} \left\{ \frac{1}{3}\left(\frac{L}{2}\right)^3 - \frac{1}{3}\left(-\frac{L}{2}\right)^3 \right\} = \frac{1}{12} mL^2$$

Moment of inertia
Sample problem

- **Consider a rod of length *L* and mass *m*. What is the moment of inertia with respect to an axis through its left corner?**



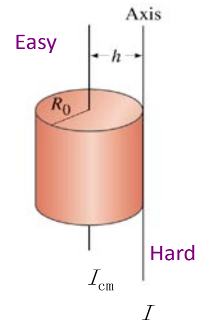
- **We have determined the moment of inertia of this rod with respect to an axis through its center of mass. We use the parallel-axis theorem to determine the moment of inertia with respect to the current axis:**

$$I = I_{cm} + m\left(\frac{L}{2}\right)^2 = \frac{1}{12} mL^2 + \frac{1}{4} mL^2 = \frac{1}{3} mL^2$$

Moment of inertia.
Parallel-axis theorem.

- **Calculating the moment of inertial with respect to a symmetry axis of the object is in general easy.**
- **It is much harder to calculate the moment of inertia with respect to an axis that is not a symmetry axis.**
- **However, we can make a hard problem easier by using the parallel-axis theorem:**

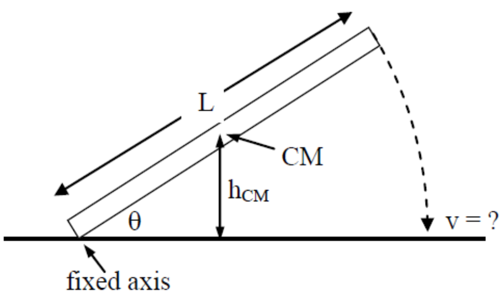
$$I = I_{cm} + Mh^2$$



Another conservation of rotational energy problem:

Rod of mass M, length L, one end stationary on ground, starts from rest at angle θ and falls.

What is speed v of end of stick, when stick hits ground?



$$I = \frac{1}{3} M L^2 \quad \text{(axis at end)}$$

Plan: Use conservation of energy to get ω,

then $v = \omega r = \omega L$

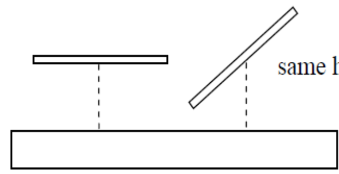
$$E_i = E_f \quad \Rightarrow \quad M g h_{CM} = \tfrac{1}{2} I \omega^2$$

Important point:

$PE_{grav} = Mgh$ where h = height of center-of-mass, independent of the orientation of the stick.

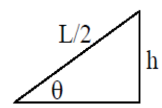
Proof: $PE_{grav} = \sum_i m_i g h_i = g \sum_i m_i h_i = g M Y_{CM} = M g h_{CM}$

(Have used definition of center-of-mass: $M Y_{CM} = \sum_i m_i y_i$)



same h_{CM}, same PE_{grav}

Back to the problem: $M g h_{CM} = \frac{1}{2} I \omega^2, \quad h = \frac{1}{2} L \sin\theta, \quad I = \frac{1}{3} M L^2$



$\cancel{M} g \frac{1}{\cancel{2}} \cancel{L} \sin\theta = \frac{1}{\cancel{2}} \frac{1}{3} \cancel{M} L^{\cancel{2}} \omega^2 \Rightarrow g \sin\theta = \frac{1}{3} L \omega^2$

Use $\omega = v/r = v/L$ to get: $3 g \sin\theta = L \frac{v^2}{L^2} = \frac{v^2}{L} \Rightarrow v = \sqrt{3 g L \sin\theta}$ Done.

Let's Review: **Translation** $\leftrightarrow$ **Rotation**

x	$\leftrightarrow$	θ
$v = \frac{dx}{dt}$	$\leftrightarrow$	$\omega = \frac{d\theta}{dt}$
$a = \frac{dv}{dt}$	$\leftrightarrow$	$\alpha = \frac{d\omega}{dt}$
F	$\leftrightarrow$	τ
M	$\leftrightarrow$	I
$F = Ma$	$\leftrightarrow$	$\tau = I\alpha$
$KE = (1/2)\, m v^2$	$\leftrightarrow$	$KE = (1/2)\, I \omega^2$

Table Some Corresponding Relations for Translational and Rotational Motion

Pure Translation (Fixed Direction)		Pure Rotation (Fixed Axis)	
Position	x	Angular position	θ
Velocity	$v = dx/dt$	Angular velocity	$\omega = d\theta/dt$
Acceleration	$a = dv/dt$	Angular acceleration	$\alpha = d\omega/dt$
Mass	m	Rotational inertia	I
Newton's second law	$F_{net} = ma$	Newton's second law	$\tau_{net} = I\alpha$
Work	$W = \int F\,dx$	Work	$W = \int \tau\,d\theta$
Kinetic energy	$K = \frac{1}{2}mv^2$	Kinetic energy	$K = \frac{1}{2}I\omega^2$
Power (constant force)	$P = Fv$	Power (constant torque)	$P = \tau\omega$
Work–kinetic energy theorem	$W = \Delta K$	Work–kinetic energy theorem	$W = \Delta K$

Sample Problem **Angular velocity derived from angular position**

The disk in Fig. 10-5*a* is rotating about its central axis like a merry-go-round. The angular position $\theta(t)$ of a reference line on the disk is given by

$$\theta = -1.00 - 0.600t + 0.250t^2,$$

with t in seconds, θ in radians, and the zero angular position as indicated in the figure. (If you like, you can translate all this into Chapter 2 notation by momentarily dropping the word "angular" from "angular position" and replacing the symbol θ with the symbol x. What you then have is an equation that gives the position as a function of time, for the one-dimensional motion of Chapter 2.)

(a) Graph the angular position of the disk versus time from $t = -3.0$ s to $t = 5.4$ s. Sketch the disk and its angular position reference line at $t = -2.0$ s, 0 s, and 4.0 s, and when the curve crosses the t axis.

KEY IDEA

The angular position of the disk is the angular position $\theta(t)$ of its reference line, which is given by Eq. 10-9 as a function of time t. So we graph Eq. 10-9; the result is shown in Fig. 10-5*b*.

Calculations: To sketch the disk and its reference line at a particular time, we need to determine θ for that time. To do so, we substitute the time into Eq. 10-9. For $t = -2.0$ s, we get

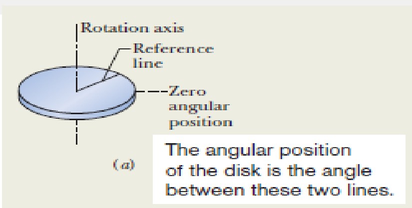
$$\theta = -1.00 - (0.600)(-2.0) + (0.250)(-2.0)^2$$

$$= 1.2 \text{ rad} = 1.2 \text{ rad}\frac{360°}{2\pi \text{ rad}} = 69°.$$

This means that at $t = -2.0$ s the reference line on the disk is rotated counterclockwise from the zero position by angle 1.2 rad = 69° (counterclockwise because θ is positive). Sketch 1 in Fig. 10-5*b* shows this position of the reference line.

Similarly, for $t = 0$, we find $\theta = -1.00$ rad $= -57°$, which means that the reference line is rotated clockwise from the zero angular position by 1.0 rad, or 57°, as shown in sketch 3. For $t = 4.0$ s, we find $\theta = 0.60$ rad $= 34°$ (sketch 5). Drawing sketches for when the curve crosses the t axis is easy, because then $\theta = 0$ and the reference line is momentarily aligned with the zero angular position (sketches 2 and 4).

(b) At what time t_{min} does $\theta(t)$ reach the minimum value shown in Fig. 10-5*b*? What is that minimum value?



Example:

A child's top is spun with angular acceleration

$$\alpha = 5t^3 - 4t,$$

with t in seconds and α in radians per second-squared. At $t = 0$, the top has angular velocity 5 rad/s, and a reference line on it is at angular position $\theta = 2$ rad.

(a) Obtain an expression for the angular velocity $\omega(t)$ of the top. That is, find an expression that explicitly indicates how the angular velocity depends on time. (We can tell that there *is* such a dependence because the top is undergoing an angular acceleration, which means that its angular velocity *is* changing.)

KEY IDEA

By definition, $\alpha(t)$ is the derivative of $\omega(t)$ with respect to time. Thus, we can find $\omega(t)$ by integrating $\alpha(t)$ with respect to time.

Calculations: Equation 10-8 tells us

$$d\omega = \alpha\, dt,$$

so

$$\int d\omega = \int \alpha\, dt.$$

From this we find

$$\omega = \int (5t^3 - 4t)\, dt = \tfrac{5}{4}t^4 - \tfrac{4}{2}t^2 + C.$$

To evaluate the constant of integration C, we note that $\omega = 5$ rad/s at $t = 0$. Substituting these values in our expression for ω yields

$$5 \text{ rad/s} = 0 - 0 + C,$$

so $C = 5$ rad/s. Then

$$\omega = \tfrac{5}{4}t^4 - 2t^2 + 5. \qquad \text{(Answer)}$$

(b) Obtain an expression for the angular position $\theta(t)$ of the top.

KEY IDEA

By definition, $\omega(t)$ is the derivative of $\theta(t)$ with respect to time. Therefore, we can find $\theta(t)$ by integrating $\omega(t)$ with respect to time.

Calculations: Since Eq. 10-6 tells us that

$$d\theta = \omega\, dt,$$

we can write

$$\theta = \int \omega\, dt = \int (\tfrac{5}{4}t^4 - 2t^2 + 5)\, dt$$
$$= \tfrac{1}{4}t^5 - \tfrac{2}{3}t^3 + 5t + C'$$
$$= \tfrac{1}{4}t^5 - \tfrac{2}{3}t^3 + 5t + 2, \qquad \text{(Answer)}$$

where C' has been evaluated by noting that $\theta = 2$ rad at $t = 0$.

Example 3

An athlete whirls a discus in a circle of radius 80.0 cm. At a certain instant, the athlete is rotating at 10.0 rad/s and the angular speed is increasing at 50.0 rad/s^2. At this instant, find the tangential and centripetal components of the acceleration of the discus and the magnitude of the acceleration.

SOLUTION

IDENTIFY and SET UP: We treat the discus as a particle traveling in a circular path (Fig. 2a), so we can use the ideas developed in this section. We are given $r = 0.800$ m, $\omega = 10.0$ rad/s, and $\alpha = 50.0$ rad/s^2 (Fig. 2b).

$$a_{tan} = r\alpha = (0.800 \text{ m})(50.0 \text{ rad/s}^2) = 40.0 \text{ m/s}^2$$
$$a_{rad} = \omega^2 r = (10.0 \text{ rad/s})^2(0.800 \text{ m}) = 80.0 \text{ m/s}^2$$

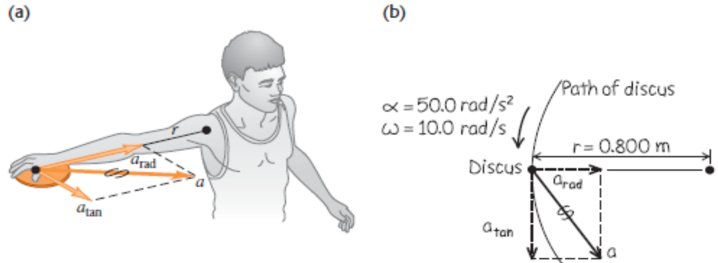
Then

$$a = \sqrt{a_{tan}^2 + a_{rad}^2} = 89.4 \text{ m/s}^2$$

EVALUATE: Note that we dropped the unit "radian" from our results for a_{tan}, a_{rad}, and a. We can do this because "radian" is a dimensionless quantity. Can you show that if the angular speed doubles to 20.0 rad/s while α remains the same, the acceleration magnitude a increases to 322 m/s^2?

(a) Whirling a discus in a circle. (b) Our sketch showing the acceleration components for the discus.

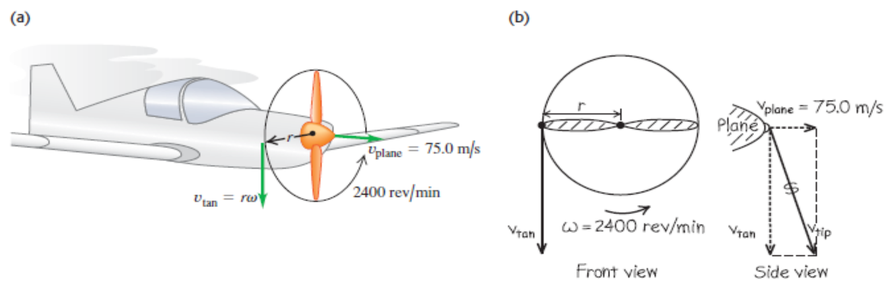
(a) (b)



Example

You are designing an airplane propeller that is to turn at 2400 rpm (Fig. 9.13a). The forward airspeed of the plane is to be 75.0 m/s, and the speed of the tips of the propeller blades through the air must not exceed 270 m/s. (This is about 80% of the speed of sound in air. If the speed of the propeller tips were greater than this, they would produce a lot of noise.) (a) What is the maximum possible propeller radius? (b) With this radius, what is the acceleration of the propeller tip?

9.13 (a) A propeller-driven airplane in flight. (b) Our sketch showing the velocity components for the propeller tip.



Questions

1 Figure 10-20 is a graph of the angular velocity versus time for a disk rotating like a merry-go-round. For a point on the disk rim, rank the instants a, b, c, and d according to the magnitude of the (a) tangential and (b) radial acceleration, greatest first.

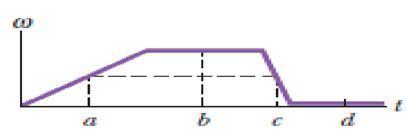


Figure 10-20 Question 1.

2 Figure 10-21 shows plots of angular position θ versus time t for three cases in which a disk is rotated like a merry-go-round. In each case, the rotation direction changes at a certain angular position θ_{change}. (a) For each case, determine whether θ_{change} is clockwise or counterclockwise from $\theta = 0$, or whether it is at $\theta = 0$. For each case, determine (b) whether ω is zero before, after, or at $t = 0$ and (c) whether α is positive, negative, or zero.

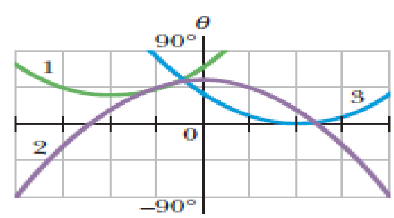


Figure 10-21 Question 2.

3 A force is applied to the rim of a disk that can rotate like a merry-go-round, so as to change its angular velocity. Its initial and final angular velocities, respectively, for four situations are: (a) −2 rad/s, 5 rad/s; (b) 2 rad/s, 5 rad/s; (c) −2 rad/s, −5 rad/s; and (d) 2 rad/s, −5 rad/s. Rank the situations according to the work done by the torque due to the force, greatest first.

Equilibrium and Elasticity (9)

Dr. Hind I. Abdulgafour

Static Equilibrium

- Equilibrium implies the object is at rest (static) or its center of mass moves with a constant velocity (dynamic).
- Static equilibrium is a common situation in engineering.
- Principles involved are of particular interest to civil engineers, architects, and mechanical engineers.

Equilibrium

The state of (mechanical) equilibrium:

1. **The linear momentum of the system is constant.**

2. Its angular momentum is also constant.

Static equilibrium;
dynamic equilibrium

Stable equilibrium;
unstable equilibrium

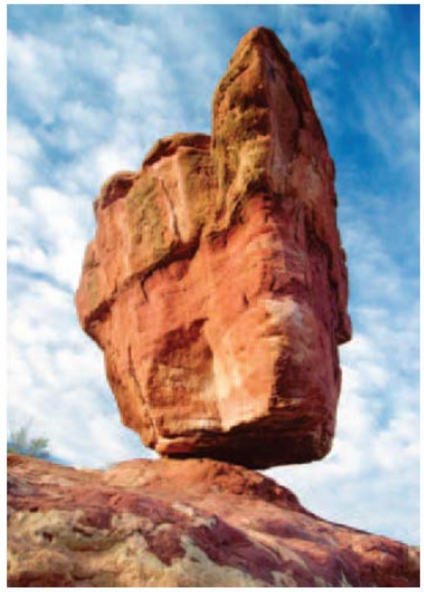


Fig. 1 A balancing rock. Although its perch seems precarious, the rock is in static equilibrium.

The Requirements of Equilibrium

1. The vector sum of all the external forces that act on the body must be zero.
2. The vector sum of all external torques that act on the body, measured about *any* possible point, must also be zero.

$$\vec{F}_{net} = 0 \quad \text{(balance of forces).}$$

+

$$\vec{\tau}_{net} = 0 \quad \text{(balance of torques).}$$

Balance of forces	Balance of torques
$F_{net,x} = 0$	$\tau_{net,x} = 0$
$F_{net,y} = 0$	$\tau_{net,y} = 0$
$F_{net,z} = 0$	$\tau_{net,z} = 0$

1. The linear momentum $\vec{P}$ of its center of mass is constant.
2. Its angular momentum $\vec{L}$ about its center of mass, or about any other point, is also constant.

The translational motion of a body is governed by Newton's second law in its linear momentum form, given by Eq. 9-27 as

$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt}. \qquad (12\text{-}2)$$

If the body is in translational equilibrium—that is, if $\vec{P}$ is a constant—then $d\vec{P}/dt = 0$ and we must have

$$\vec{F}_{\text{net}} = 0 \quad \text{(balance of forces).} \qquad (12\text{-}3)$$

The rotational motion of a body is governed by Newton's second law in its angular momentum form, given by Eq. 11-29 as

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}. \qquad (12\text{-}4)$$

If the body is in rotational equilibrium—that is, if $\vec{L}$ is a constant—then $d\vec{L}/dt = 0$ and we must have

$$\vec{\tau}_{\text{net}} = 0 \quad \text{(balance of torques).} \qquad (12\text{-}5)$$

Equilibrium Summary

- **There are two necessary conditions for equilibrium**
- **The resultant external force must equal zero:**

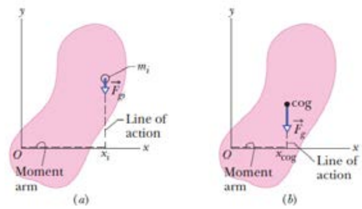
  $\sum \mathbf{F} = 0$
  - This is a statement of translational equilibrium
  - The acceleration of the center of mass of the object must be zero when viewed from an inertial frame of reference
- **The resultant external torque about *any* axis must be zero: $\sum \tau = 0$**
  - This is a statement of rotational equilibrium
  - The angular acceleration must equal zero

The Center of Gravity

The gravitational force $\vec{F}_g$ on a body effectively acts at a single point, called the **center of gravity** (cog) of the body.

If $\vec{g}$ is the same for all elements of a body, then the body's center of gravity (cog) is coincident with the body's center of mass (com).

Fig. 12-4 (*a*) An element of mass m_i in an extended body. The gravitational force $\vec{F}_{gi}$ on the element has moment arm x_i about the origin O of the coordinate system. (*b*) The gravitational force $\vec{F}_g$ on a body is said to act at the center of gravity (cog) of the body. Here $\vec{F}_g$ has moment arm x_{cog} about origin O.



Proof

First, we consider the individual elements of the body. Figure 12-4*a* shows an extended body, of mass M, and one of its elements, of mass m_i. A gravitational force $\vec{F}_{gi}$ acts on each such element and is equal to $m_i\vec{g}_i$. The subscript on $\vec{g}_i$ means $\vec{g}_i$ is the gravitational acceleration *at the location of the element i* (it can be different for other elements).

For the body in Fig. 12-4*a*, each force $\vec{F}_{gi}$ acting on an element produces a torque τ_i on the element about the origin O, with a moment arm x_i. Using Eq. 10-41 ($\tau = r_\perp F$) as a guide, we can write each torque τ_i as

$$\tau_i = x_i F_{gi}. \tag{12-10}$$

The net torque on all the elements of the body is then

$$\tau_{net} = \sum \tau_i = \sum x_i F_{gi}. \tag{12-11}$$

Next, we consider the body as a whole. Figure 12-4*b* shows the gravitational force $\vec{F}_g$ acting at the body's center of gravity. This force produces a torque τ on the body about O, with moment arm x_{cog}. Again using Eq. 10-41, we can write this torque as

$$\tau = x_{cog} F_g. \tag{12-12}$$

The gravitational force $\vec{F}_g$ on the body is equal to the sum of the gravitational forces $\vec{F}_{gi}$ on all its elements, so we can substitute ΣF_{gi} for F_g in Eq. 12-12 to write

$$\tau = x_{cog} \sum F_{gi}. \tag{12-13}$$

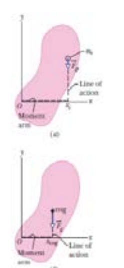


Figure 12-4 (*a*) An element of mass m_i in an extended body. The gravitational force $\vec{F}_{gi}$ on the element has moment arm x_i about the origin O of the coordinate system. (*b*) The gravitational force $\vec{F}_g$ on a body is said to act at the center of gravity (cog) of the body. Here $\vec{F}_g$ has moment arm x_{cog} about origin O.

Now recall that the torque due to force $\vec{F}_g$ acting at the center of gravity is equal to the net torque due to all the forces $\vec{F}_{gi}$ acting on all the elements of the body. (That is how we defined the center of gravity.) Thus, τ in Eq. 12-13 is equal to τ_{net} in Eq. 12-11. Putting those two equations together, we can write

$$x_{cog}\sum F_{gi} = \sum x_i F_{gi}.$$

Substituting $m_i g_i$ for F_{gi} gives us

$$x_{cog}\sum m_i g_i = \sum x_i m_i g_i. \tag{12-14}$$

Now here is a key idea: If the accelerations g_i at all the locations of the elements are the same, we can cancel g_i from this equation to write

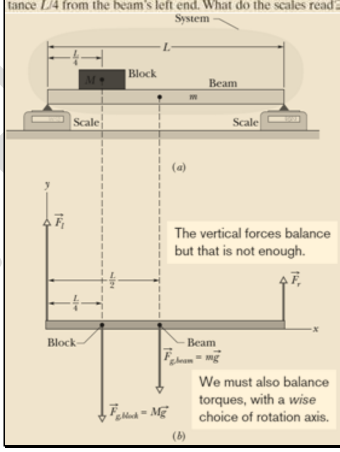
$$x_{cog}\sum m_i = \sum x_i m_i. \tag{12-15}$$

The sum Σm_i of the masses of all the elements is the mass M of the body. Therefore, we can rewrite Eq. 12-15 as

$$x_{cog} = \frac{1}{M}\sum x_i m_i. \tag{12-16}$$

Example, static equilibrium:

In Fig. 12-7*a*, a uniform beam, of length L and mass $m = 1.8$ kg, is at rest on two scales. A uniform block, with mass $M = 2.7$ kg, is at rest on the beam, with its center a distance $L/4$ from the beam's left end. What do the scales read?



$$(F_{net,x} = 0) \longrightarrow F_l + F_r - Mg - mg = 0.$$

$$\tau_{net,z} = 0$$

$$\downarrow$$

$$(0)(F_l) - (L/4)(Mg) - (L/2)(mg) + (L)(F_r) = 0,$$

$$\begin{aligned} F_r &= \tfrac{1}{4}Mg + \tfrac{1}{2}mg \\ &= \tfrac{1}{4}(2.7\ \text{kg})(9.8\ \text{m/s}^2) + \tfrac{1}{2}(1.8\ \text{kg})(9.8\ \text{m/s}^2) \\ &= 15.44\ \text{N} \approx 15\ \text{N}. \qquad \text{(Answer)} \end{aligned}$$

$$\begin{aligned} F_l &= (M + m)g - F_r \\ &= (2.7\ \text{kg} + 1.8\ \text{kg})(9.8\ \text{m/s}^2) - 15.44\ \text{N} \\ &= 28.66\ \text{N} \approx 29\ \text{N}. \qquad \text{(Answer)} \end{aligned}$$

Example, static equilibrium:

In Fig. 12-5*a*, a ladder of length $L = 12$ m and mass $m = 45$ kg leans against a slick wall (that is, there is no friction between the ladder and the wall). The ladder's upper end is at height $h = 9.3$ m above the pavement on which the lower end is supported (the pavement is not frictionless). The ladder's center of mass is $L/3$ from the lower end, along the length of the ladder. A firefighter of mass $M = 72$ kg climbs the ladder until her center of mass is $L/2$ from the lower end. What then are the magnitudes of the forces on the ladder from the wall and the pavement?

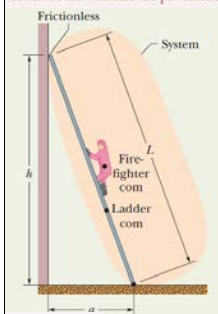


Fig. 12-5 (*a*)

$$\tau_{net,z} = 0$$

$$-(h)(F_w) + (a/2)(Mg) + (a/3)(mg) + (0)(F_{px}) + (0)(F_{py}) = 0. \quad (12\text{-}18)$$

$$a = \sqrt{L^2 - h^2} = 7.58 \text{ m}.$$

$$F_w = \frac{ga(M/2 + m/3)}{h} = \frac{(9.8 \text{ m/s}^2)(7.58 \text{ m})(72/2 \text{ kg} + 45/3 \text{ kg})}{9.3 \text{ m}} = 407 \text{ N} \approx 410 \text{ N}. \quad \text{(Answer)}$$

$$F_{net,x} = 0 \Rightarrow F_w - F_{px} = 0, \quad F_{px} = F_w = 410 \text{ N}.$$

$$F_{net,y} = 0 \Rightarrow F_{py} - Mg - mg = 0,$$

$$F_{py} = (M + m)g = (72 \text{ kg} + 45 \text{ kg})(9.8 \text{ m/s}^2) = 1146.6 \text{ N} \approx 1100 \text{ N}.$$

Example, static equilibrium:

Figure 12-6*a* shows a safe (mass $M = 430$ kg), hanging by a rope (negligible mass) from a boom ($a = 1.9$ m and $b = 2.5$ m) that consists of a uniform hinged beam ($m = 85$ kg) and horizontal cable (negligible mass).

(a) What is the tension T_c in the cable? In other words, what is the magnitude of the force $\vec{T}_c$ on the beam from the cable?

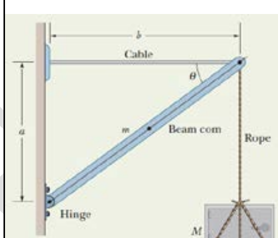
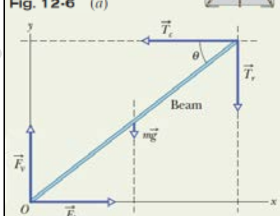


Fig. 12-6 (*a*)



(*b*)

Calculations: Let us start with Eq. 12-9 ($\tau_{net,z} = 0$). Note that we are asked for the magnitude of force $\vec{T}_c$ and not of forces $\vec{F}_h$ and $\vec{F}_v$ acting at the hinge, at point O. To eliminate $\vec{F}_h$ and $\vec{F}_v$ from the torque calculation, we should calculate torques about an axis that is perpendicular to the figure at point O. Then $\vec{F}_h$ and $\vec{F}_v$ will have moment arms of zero. The lines of action for $\vec{T}_c$, $\vec{T}_r$, and $m\vec{g}$ are dashed in Fig. 12-6*b*. The corresponding moment arms are a, b, and $b/2$.

Writing torques in the form of $r_\perp F$ and using our rule about signs for torques, the balancing equation $\tau_{net,z} = 0$ becomes

$$(a)(T_c) - (b)(T_r) - (\tfrac{1}{2}b)(mg) = 0. \quad (12\text{-}19)$$

Substituting Mg for T_r and solving for T_c, we find that

$$T_c = \frac{gb(M + \frac{1}{2}m)}{a} = \frac{(9.8 \text{ m/s}^2)(2.5 \text{ m})(430 \text{ kg} + 85/2 \text{ kg})}{1.9 \text{ m}} = 6093 \text{ N} \approx 6100 \text{ N}. \quad \text{(Answer)}$$

Example, static equilibrium, cont.:

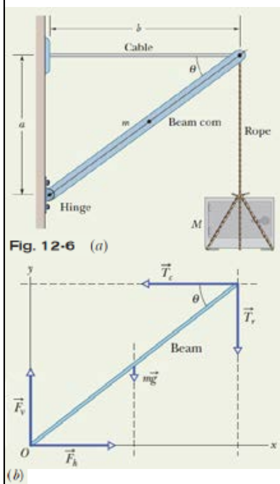


Fig. 12-6 (a) (b)

(b) Find the magnitude F of the net force on the beam from the hinge.

Calculations: For the horizontal balance, we write $F_{net,x} = 0$ as

$$F_h - T_c = 0, \qquad (12\text{-}20)$$

and so

$$F_h = T_c = 6093 \text{ N}.$$

For the vertical balance, we write $F_{net,y} = 0$ as

$$F_v - mg - T_r = 0.$$

Substituting Mg for T_r and solving for F_v, we find that

$$F_v = (m + M)g = (85 \text{ kg} + 430 \text{ kg})(9.8 \text{ m/s}^2)$$
$$= 5047 \text{ N}.$$

From the Pythagorean theorem, we now have

$$F = \sqrt{F_h^2 + F_v^2}$$
$$= \sqrt{(6093 \text{ N})^2 + (5047 \text{ N})^2} \approx 7900 \text{ N}. \qquad \text{(Answer)}$$

Note that F is substantially greater than either the combined weights of the safe and the beam, 5000 N, or the tension in the horizontal wire, 6100 N.

Definitions Associated With Deformation

- **Stress**
  - **Is proportional to the force causing the deformation**
  - **It is the external force acting on the object per unit area**

- **Strain**
  - **Is the result of a stress**
  - **Is a measure of the degree of deformation**

Elastic Modulus

- The elastic modulus is the constant of proportionality between the stress and the strain.
- The elastic modulus in general relates what is done to a solid object to how that object responds.

$$\text{elastic modulus} = \frac{\text{stress}}{\text{strain}}$$

- Various types of deformation have unique elastic modulus.

Three Types of Modulus

- **Young's Modulus**
  - Measures the resistance of a solid to a change in its length.
- **Shear Modulus**
  - Measures the resistance to motion of the planes within a solid parallel to each other.
- **Bulk Modulus**
  - Measures the resistance of solids or liquids to changes in their volume.

Young's Modulus

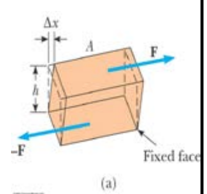
- **The tension strain is the ratio of the change in length to the original length**
- **Young's modulus, *Y*, is the ratio of those two ratios:**

$$Y \equiv \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L / L_i}$$

- **Units are N / m^2**

Shear Modulus

- **Another type of deformation occurs when a force acts parallel to one of its faces while the opposite face is held fixed by another force This is called a *shear stress.***



- **For small deformations, no change in volume occurs with this deformation.**
  - **A good first approximation**
- **The shear stress is F / A**
  - **F is the tangential force**
  - **A is the area of the face being sheared**
- **The shear strain is $\Delta x / h$**
  - **Δx is the horizontal distance the sheared face moves**
  - **h is the height of the object**

- **The shear modulus is the ratio of the shear stress to the shear strain**

$$S = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x/h}$$

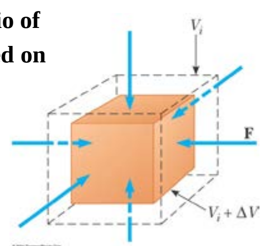
- **Units are N / m^2**

Bulk Modulus

- **Another type of deformation occurs when a force of uniform magnitude is applied perpendicularly over the entire surface of the object**
- **The object will undergo a change in volume, but not in shape**

- **The volume stress is defined as the ratio of the magnitude of the total force, *F*, exerted on the surface to the area, *A*, of the surface this is also called the *pressure*.**

- **The volume strain is the ratio of the change in volume to the original volume.**



- **The bulk modulus is the ratio of the volume stress to the volume strain.**

$$B = \frac{\text{volume stress}}{\text{volume strain}} = -\frac{\Delta F / A}{\Delta V / V_i} = -\frac{\Delta P}{\Delta V / V_i}$$

- **The negative indicates that an increase in pressure will result in a decrease in volume.**

Elasticity

A **stress** is defined as deforming force per unit area, which produces a **strain**, or unit deformation.

Stress and strain are, within a certain range, proportional to each other. The constant of proportionality is called a **modulus of elasticity.**

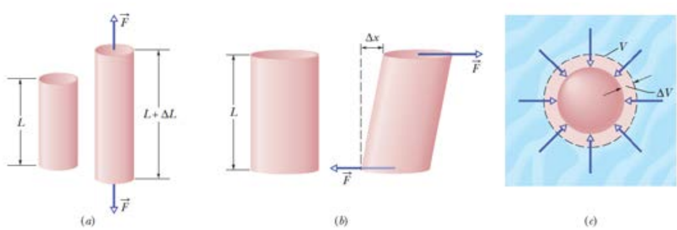


Fig. 12-10 (*a*) A cylinder subject to *tensile stress* stretches by an amount ΔL. (*b*) A cylinder subject to *shearing stress* deforms by an amount Δx, somewhat like a pack of playing cards would. (*c*) A solid sphere subject to uniform *hydraulic stress* from a fluid shrinks in volume by an amount ΔV. All the deformations shown are greatly exaggerated.

Elasticity: Tension and Compression

For simple tension or compression, the stress on the object is defined as F/A, where F is the magnitude of the force applied perpendicularly to an area A on the object.

The strain, or unit deformation, is then the dimensionless quantity $\Delta L/L$, the fractional change in a length of the specimen.

The modulus for tensile and compressive stresses is called the Young's modulus and is represented in engineering practice by the symbol E.

$$\frac{F}{A} = E\frac{\Delta L}{L}.$$

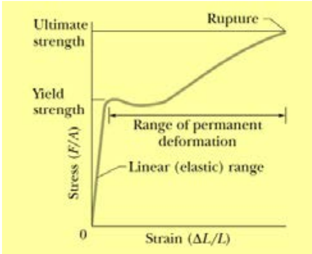
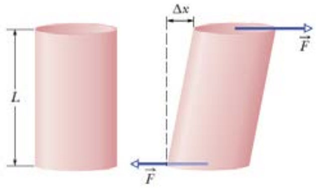


Fig. 12-12 A stress–strain curve for a steel test specimen. The specimen deforms permanently when the stress is equal to the **yield strength** of the specimen's material. It ruptures when the stress is equal to the **ultimate strength** of the material.

Elasticity: Shearing

In the case of shearing, the stress is also a force per unit area, but the force vector lies in the plane of the area rather than perpendicular to it. The strain is the dimensionless ratio $\Delta x/L$, with the quantities defined as shown in the figure. The corresponding modulus, which is given the symbol G in engineering practice, is called the shear modulus.

$$\frac{F}{A} = G\frac{\Delta x}{L}.$$



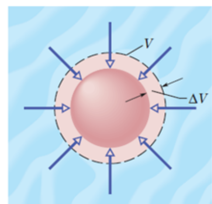
Elasticity: Hydraulic Stress

In the figure, the stress is the fluid pressure p on the object, where pressure is a force per unit area.

The strain is DV/V, where V is the original volume of the specimen and DV is the absolute value of the change in volume.

The corresponding modulus, with symbol *B*, is called the bulk modulus of the material. The object is said to be under hydraulic compression, and the pressure can be called the hydraulic stress.

$$p = B\,\frac{\Delta V}{V}$$



12.7: Elasticity

Table 12-1

Some Elastic Properties of Selected Materials of Engineering Interest

Material	Density ρ (kg/m^3)	Young's Modulus E ($10^9\ N/m^2$)	Ultimate Strength S_u ($10^6\ N/m^2$)	Yield Strength S_y ($10^6\ N/m^2$)
Steel[a]	7860	200	400	250
Aluminum	2710	70	110	95
Glass	2190	65	50[b]	—
Concrete[c]	2320	30	40[b]	—
Wood[d]	525	13	50[b]	—
Bone	1900	9[b]	170[b]	—
Polystyrene	1050	3	48	—

[a]Structural steel (ASTM-A36).
[b]In compression.
[c]High strength
[d]Douglas fir.

Example, elongated rod

One end of a steel rod of radius $R = 9.5$ mm and length $L = 81$ cm is held in a vise. A force of magnitude $F = 62$ kN is then applied perpendicularly to the end face (uniformly across the area) at the other end, pulling directly away from the vise. What are the stress on the rod and the elongation ΔL and strain of the rod?

KEY IDEAS

(1) Because the force is perpendicular to the end face and uniform, the stress is the ratio of the magnitude F of the force to the area A. The ratio is the left side of Eq. 12-23. (2) The elongation ΔL is related to the stress and Young's modulus E by Eq. 12-23 ($F/A = E\,\Delta L/L$). (3) Strain is the ratio of the elongation to the initial length L.

Calculations: To find the stress, we write

$$\text{stress} = \frac{F}{A} = \frac{F}{\pi R^2} = \frac{6.2 \times 10^4\ \text{N}}{(\pi)(9.5 \times 10^{-3}\ \text{m})^2}$$
$$= 2.2 \times 10^8\ \text{N/m}^2. \qquad \text{(Answer)}$$

The yield strength for structural steel is 2.5×10^8 N/m², so this rod is dangerously close to its yield strength.

We find the value of Young's modulus for steel in Table 12-1. Then from Eq. 12-23 we find the elongation:

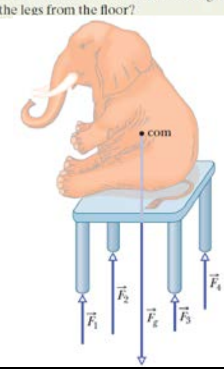
$$\Delta L = \frac{(F/A)L}{E} = \frac{(2.2 \times 10^8\ \text{N/m}^2)(0.81\ \text{m})}{2.0 \times 10^{11}\ \text{N/m}^2}$$
$$= 8.9 \times 10^{-4}\ \text{m} = 0.89\ \text{mm}. \qquad \text{(Answer)}$$

For the strain, we have

$$\frac{\Delta L}{L} = \frac{8.9 \times 10^{-4}\ \text{m}}{0.81\ \text{m}}$$
$$= 1.1 \times 10^{-3} = 0.11\%. \qquad \text{(Answer)}$$

Example, wobbly table

A table has three legs that are 1.00 m in length and a fourth leg that is longer by $d = 0.50$ mm, so that the table wobbles slightly. A steel cylinder with mass $M = 290$ kg is placed on the table (which has a mass much less than M) so that all four legs are compressed but unbuckled and the table is level but no longer wobbles. The legs are wooden cylinders with cross-sectional area $A = 1.0$ cm²; Young's modulus is $E = 1.3 \times 10^{10}$ N/m². What are the magnitudes of the forces on the legs from the floor?



We take the table plus steel cylinder as our system. The situation is like that in the figure. If the tabletop remains level, the legs must be compressed in the following ways: Each of the short legs must be compressed by the same amount (call it ΔL_3) and thus by the same force of magnitude F_3. The single long leg must be compressed by a larger amount ΔL_4 and thus by a force with a larger magnitude F_4.

$$\Delta L_4 = \Delta L_3 + d. \Rightarrow \frac{F_4 L}{AE} = \frac{F_3 L}{AE} + d.$$

$$F_{\text{net},y} = 0 \Rightarrow 3F_3 + F_4 - Mg = 0,$$

$$F_3 = \frac{Mg}{4} - \frac{dAE}{4L}$$
$$= \frac{(290\ \text{kg})(9.8\ \text{m/s}^2)}{4}$$
$$- \frac{(5.0 \times 10^{-4}\ \text{m})(10^{-4}\ \text{m}^2)(1.3 \times 10^{10}\ \text{N/m}^2)}{(4)(1.00\ \text{m})}$$
$$= 548\ \text{N} \approx 5.5 \times 10^2\ \text{N}. \qquad \text{(Answer)}$$

$$F_4 = Mg - 3F_3 = (290\ \text{kg})(9.8\ \text{m/s}^2) - 3(548\ \text{N})$$
$$\approx 1.2\ \text{kN}. \qquad \text{(Answer)}$$

9 In Fig. 12-22, a vertical rod is hinged at its lower end and attached to a cable at its upper end. A horizontal force $\vec{F}_a$ is to be applied to the rod as shown. If the point at which the force is applied is moved up the rod, does the tension in the cable increase, decrease, or remain the same?

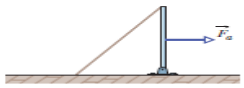


Figure 12-22 Question 9.

10 Figure 12-23 shows a horizontal block that is suspended by two wires, A and B, which are identical except for their original lengths. The center of mass of the block is closer to wire B than to wire A. (a) Measuring torques about the block's center of mass, state whether the magnitude of the torque due to wire A is greater than, less than, or equal to the magnitude of the torque due to wire B. (b) Which wire exerts more force on the block? (c) If the wires are now equal in length, which one was originally shorter (before the block was suspended)?

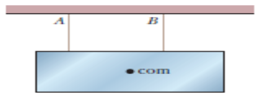
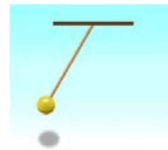


Figure 12-23 Question 10.

11 The table gives the initial lengths of three rods and the changes in their lengths when forces are applied to their ends to put them under strain. Rank the rods according to their strain, greatest first.

	Initial Length	Change in Length
Rod A	$2L_0$	ΔL_0
Rod B	$4L_0$	$2\Delta L_0$
Rod C	$10L_0$	$4\Delta L_0$



Oscillatory Motion
Motion (10)

Dr. Hind I. Abdulgafour

Periodic Motion

Period: time required for one cycle of periodic motion

Frequency: number of oscillations per unit time

$$f = \frac{1}{T}$$

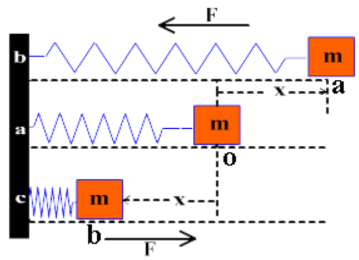
SI unit: cycle/second $= 1/\text{s} = \text{s}^{-1}$

$$1\ \text{Hz} = 1\ \text{cycle/second}$$

Simple Harmonic Motion (SHM)

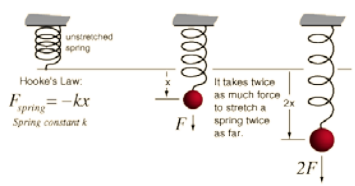
Back and forth motion that is caused by a force that is <u>directly proportional</u> to the displacement. The displacement centers around an equilibrium position.

$$F_s \alpha \ x$$



Springs – Hooke's Law

One of the simplest type of simple harmonic motion is called **Hooke's Law.** This is primarily in reference to SPRINGS.



$F_s \, \alpha \, x$

$k =$ Constant of Proportionality

$k =$ Spring Constant(Unit : N/m)

$F_s = kx \quad or \quad -kx$

Example

A load of 50 N attached to a spring hanging vertically stretches the spring 5.0 cm. The spring is now placed horizontally on a table and stretched 11.0 cm. What force is required to stretch the spring this amount?

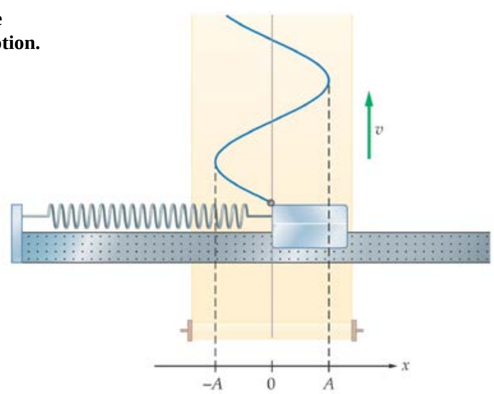
$$F_s = kx$$
$$50 = k(0.05)$$
$$k = 1000\ \text{N/m}$$

$$F_s = kx$$
$$F_s = (1000)(0.11)$$
$$F_s = 110\ \text{N}$$

Simple Harmonic Motion

A mass on a spring has a displacement as a function of time that is a sine or cosine curve:

Here, *A* is called the amplitude of the motion.



Simple Harmonic Motion

If we call the period of the motion T – this is the time to complete one full cycle – we can write the position as a function of time:

$$x = A\cos\left(\frac{2\pi}{T}t\right)$$

ightforward to show that the position at time t ıe as the position at time t, as we would

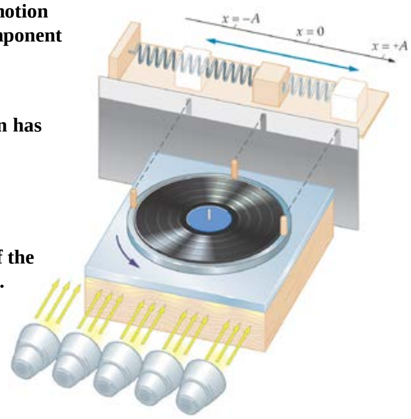
Connections between Uniform Circular Motion and Simple Harmonic Motion

An object in simple harmonic motion has the same motion as one component of an object in uniform circular motion:

Here, the object in circular motion has an angular speed of

$$\omega = 2\pi/T$$

the period of motion of the ımple harmonic motion.



Connections between Uniform Circular Motion and Simple Harmonic Motion

The position as a function of time:

$$x = A\cos\theta = A\cos(\omega t) = A\cos\left(\frac{2\pi}{T}t\right)$$

The angular frequency:

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\text{SI unit: rad/s} = \text{s}^{-1}$$

Connections between Uniform Circular Motion and Simple Harmonic Motion

The velocity as a function of time:

$$v = -A\omega\sin(\omega t)$$

eration:

$$a = -A\omega^2\cos(\omega t)$$

e found by taking components of the circular motion

The Period of a Mass on a Spring

Since the force on a mass on a spring is proportional to the displacement, and also to the acceleration, we find that $ma = -kx$.

Substituting the time dependencies of ***a*** **and** ***x*** **gives**

$$m[-A\omega^2 \cos(\omega t)] = -k[A \cos(\omega t)]$$

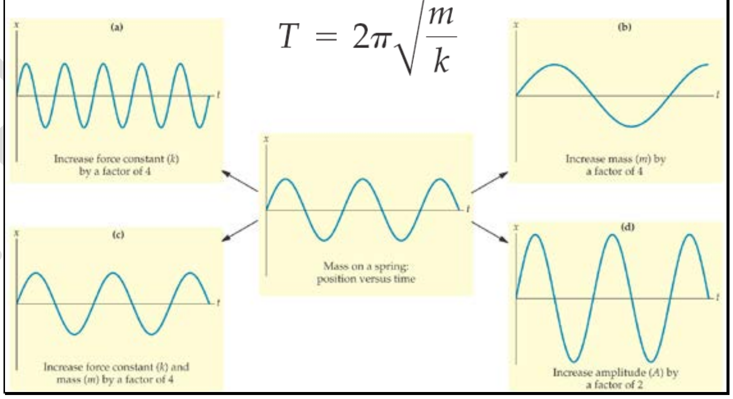
$$\omega^2 = k/m$$

$$\omega = \sqrt{\frac{k}{m}}$$

The Period of a Mass on a Spring

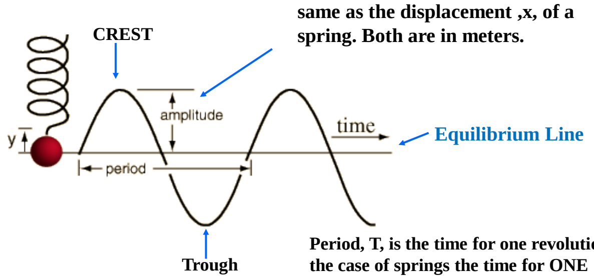
Therefore, the period is

$$T = 2\pi\sqrt{\frac{m}{k}}$$



Springs are like Waves and Circles

The amplitude, A, of a wave is the same as the displacement ,x, of a spring. Both are in meters.



T_s=sec/cycle. Let's assume that the wave crosses the equilibrium line in one second intervals. T =3.5 seconds/1.75 cycles. T = 2 sec.

Period, T, is the time for one revolution or in the case of springs the time for ONE COMPLETE oscillation (One crest and trough). Oscillations could also be called vibrations and cycles. In the wave above we have 1.75 cycles or waves or vibrations or oscillations.

Frequency

The FREQUENCY of a wave is the inverse of the PERIOD. That means that the frequency is the #cycles per sec. The commonly used unit is HERTZ(HZ).

$$Period = T = \frac{\text{seconds}}{\text{cycles}} = \frac{3.5s}{1.75cyc} = 2s$$

$$Frequency = f = \frac{\text{cycles}}{\text{seconds}} = \frac{1.75cyc}{3.5\text{sec}} = 0.5\,{}^{c}\!/_{s} = 0.5Hz$$

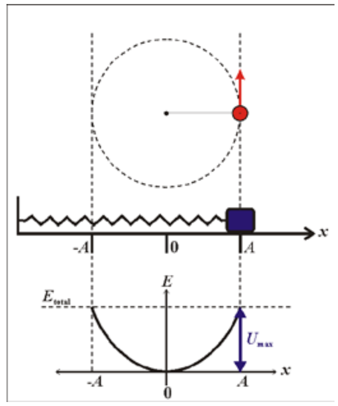
$$T = \frac{1}{f} \qquad f = \frac{1}{T}$$

SHM and Uniform Circular Motion

Springs and Waves behave very similar to objects that move in circles.

The radius of the circle is symbolic of the displacement, x, of a spring or the amplitude, A, of a wave.

$$x_{spring} = A_{wave} = r_{circle}$$



SHM and Uniform Circular Motion

$$v = \frac{2\pi r}{T}, \quad r = A$$

$$v = \frac{2\pi A}{T} \rightarrow \frac{T}{2\pi} = \frac{A}{v}$$

$$Energy_{before} = Energy_{after}$$

$$U_s = K$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \rightarrow kx^2 = mv^2, x = A$$

$$\frac{A^2}{v^2} = \frac{m}{k} \rightarrow \frac{A}{v} = \sqrt{\frac{m}{k}}$$

$$\frac{T}{2\pi} = \sqrt{\frac{m}{k}} \rightarrow T_{spring} = 2\pi\sqrt{\frac{m}{k}}$$

- **The radius of a circle is symbolic of the amplitude of a wave.**
- **Energy is conserved as the elastic potential energy in a spring can be converted into kinetic energy. Once again the displacement of a spring is symbolic of the amplitude of a wave**
- **Since BOTH algebraic expressions have the ratio of the Amplitude to the velocity we can set them equal to each other.**
- **This derives the PERIOD of a SPRING.**

Energy Conservation in Oscillatory Motion

In an ideal system with no nonconservative forces, the total mechanical energy is conserved. For a mass on a spring:

$$E = K + U = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$

now the position and velocity as functions of time, we can find um kinetic and potential energies:

$$U_{\text{max}} = \tfrac{1}{2}kA^2$$

$$K_{\text{max}} = \tfrac{1}{2}mA^2\omega^2 = \tfrac{1}{2}mA^2(k/m) = \tfrac{1}{2}kA^2$$

Energy Conservation in Oscillatory Motion

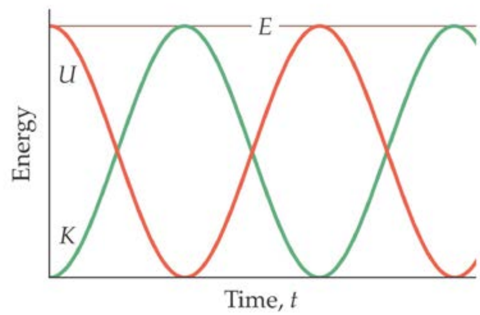
As a function of time,

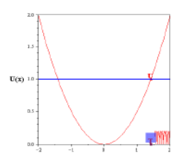
$$\begin{aligned} E &= U + K = \tfrac{1}{2}kA^2\cos^2(\omega t) + \tfrac{1}{2}kA^2\sin^2(\omega t) \\ &= \tfrac{1}{2}kA^2[\cos^2(\omega t) + \sin^2(\omega t)] = \tfrac{1}{2}kA^2 \end{aligned}$$

So the total energy is constant; as the kinetic energy increases, the potential energy decreases, and vice versa.

Energy Conservation in Oscillatory Motion

This diagram shows how the energy transforms from potential to kinetic and back, while the total energy remains the same.

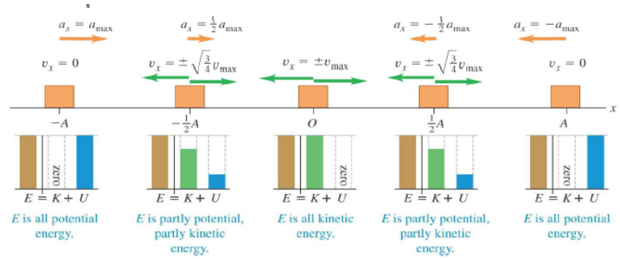




Energy in SHM

- Energy is conserved during SHM and the forms (potential and kinetic) interconvert as the position of the object in motion changes.

$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$$



Example

A slingshot consists of a light leather cup, containing a stone, that is pulled back against 2 rubber bands. It takes a force of 30 N to stretch the bands 1.0 cm (a) What is the potential energy stored in the bands when a 50.0 g stone is placed in the cup and pulled back 0.20 m from the equilibrium position? (b) With what speed does it leave the slingshot?

$a)\ F_s = kx \quad 30 = k(0.01) \quad k =$

$U_s = \tfrac{1}{2}kx^2 = 0.5(k)(.20)^2 =$

$b)\ E_B = E_A \quad U_s = K$

$U_s = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.050)v^2$

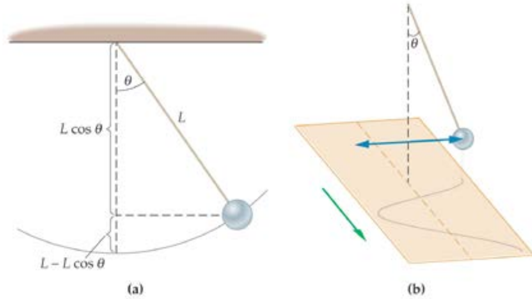
$v =$

The Pendulum

Pendulums, like springs, oscillate back and forth exhibiting simple harmonic behavior.

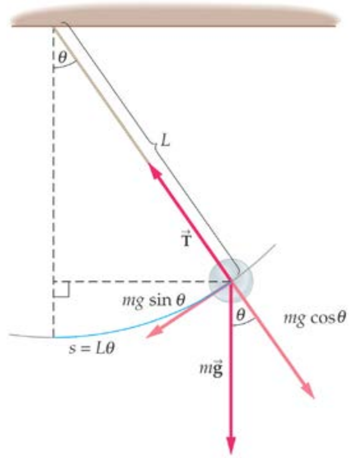
A simple pendulum consists of a mass m (of negligible size) suspended by a string or rod of length L (and negligible mass).

The angle it makes with the vertical varies with time as a sine or cosine.

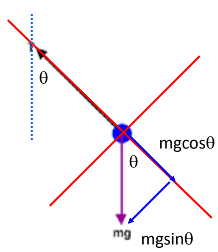


The Pendulum

Looking at the forces on the pendulum bob, we see that the restoring force is proportional to sin θ, whereas the restoring force for a spring is proportional to the displacement (which is θ in this case).



Pendulums



Consider the FBD for a pendulum. Here we have the weight and tension. Even though the weight isn't at an angle let's draw an axis along the tension.

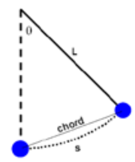
$$mg\sin\theta = \text{Restoring Force}$$

$$mg\sin\theta = kx$$

Pendulums

$$mg\sin\theta = \text{Restoring Force}$$
$$mg\sin\theta = kx$$

$$\theta = \frac{s}{R} = \frac{s}{L}$$
$$s = \theta L = Amplitude$$
$$mg\sin\theta = k\theta L$$
$$\sin\theta \cong \theta, if\ \ \theta = small$$
$$mg = kl$$
$$\frac{m}{k} = \frac{l}{g}$$
$$T_{spring} = 2\pi\sqrt{\frac{m}{k}}$$



$$\boxed{T_{pendulum} = 2\pi\sqrt{\frac{l}{g}}}$$

The Pendulum

Substituting θ for sin θ allows us to treat the pendulum in a mathematically identical way to the mass on a spring. Therefore, we find that the period of a pendulum depends only on the length of the string:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{(mg/L)}}$$
$$= 2\pi\sqrt{\frac{L}{g}}$$

Example

A visitor to a lighthouse wishes to determine the height of the tower. She ties a spool of thread to a small rock to make a simple pendulum, which she hangs down the center of a spiral staircase of the tower. The period of oscillation is 9.40 s. What is the height of the tower?

$$T_P = 2\pi\sqrt{\frac{l}{g}} \rightarrow l = height$$

$$T_P^2 = \frac{4\pi^2 l}{g} \rightarrow l = \frac{T_P^2 g}{4\pi^2} = \frac{9.4^2(9.8)}{4(3.141592)^2} =$$

Homework

Q1 If a pendulum has a period of 2.5 s on earth, what would be its period in a space station orbiting the earth? If a mass hung from a vertical spring has a period of 5.0 s on earth, what would its period be in the space station? Justify each of your answers.

Q2 A simple pendulum is mounted in an elevator. What happens to the period of the pendulum (does it increase, decrease, or remain the same) if the elevator (a) accelerates upward at 5.0 m/s^2; (b) moves upward at a steady 5.0 m/s; (c) accelerates downward at 5.0 m/s^2; (d) accelerates downward at 9.8 m/s^2? Justify your answers.

Q3 An object is moving with SHM of amplitude A on the end of a spring. If the amplitude is doubled, what happens to the total distance the object travels in one period? What happens to the period? What happens to the maximum speed of the object? Discuss how these answers are related.

Example 14.4 Velocity, acceleration, and energy in SHM

(a) Find the maximum and minimum velocities attained by the oscillating glider of Example 14.2. (b) Find the maximum and minimum accelerations. (c) Find the velocity v_x and acceleration a_x when the glider is halfway from its initial position to the equilibrium position $x = 0$. (d) Find the total energy, potential energy, and kinetic energy at this position.

SOLUTION

IDENTIFY and SET UP: The problem concerns properties of the motion at specified *positions*, not at specified *times*, so we can use the energy relationships of this section. Figure 14.13 shows our choice of x-axis. The maximum displacement from equilibrium is $A = 0.020$ m. We use Eqs. (14.22) and (14.4) to find v_x and a_x for a given x. We then use Eq. (14.21) for given x and v_x to find the total, potential, and kinetic energies E, U, and K.

EXECUTE: (a) From Eq. (14.22), the velocity v_x at any displacement x is

$$v_x = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}$$

The glider's maximum *speed* occurs when it is moving through $x = 0$:

$$v_{max} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{200\ \text{N/m}}{0.50\ \text{kg}}}(0.020\ \text{m}) = 0.40\ \text{m/s}$$

Its maximum and minimum (most negative) *velocities* are $+0.40$ m/s and -0.40 m/s, which occur when it is moving through $x = 0$ to the right and left, respectively.

(b) From Eq. (14.4), $a_x = -(k/m)x$. The glider's maximum (most positive) acceleration occurs at the most negative value of x, $x = -A$:

$$a_{max} = -\frac{k}{m}(-A) = -\frac{200\ \text{N/m}}{0.50\ \text{kg}}(-0.020\ \text{m}) = 8.0\ \text{m/s}^2$$

The minimum (most negative) acceleration is $a_{min} = -8.0\ \text{m/s}^2$, which occurs at $x = +A = +0.020$ m.

(c) The point halfway from $x = x_0 = A$ to $x = 0$ is $x = A/2 = 0.010$ m. From Eq. (14.22), at this point

$$v_x = -\sqrt{\frac{200\ \text{N/m}}{0.50\ \text{kg}}}\sqrt{(0.020\ \text{m})^2 - (0.010\ \text{m})^2} = -0.35\ \text{m/s}$$

We choose the negative square root because the glider is moving from $x = A$ toward $x = 0$. From Eq. (14.4),

$$a_x = -\frac{200\ \text{N/m}}{0.50\ \text{kg}}(0.010\ \text{m}) = -4.0\ \text{m/s}^2$$

Figure 14.14 shows the conditions at $x = 0$, $\pm A/2$, and $\pm A$.

(d) The energies are

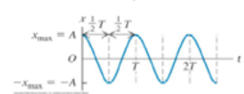
$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(200\ \text{N/m})(0.020\ \text{m})^2 = 0.040\ \text{J}$$
$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(200\ \text{N/m})(0.010\ \text{m})^2 = 0.010\ \text{J}$$
$$K = \tfrac{1}{2}mv_x^2 = \tfrac{1}{2}(0.50\ \text{kg})(-0.35\ \text{m/s})^2 = 0.030\ \text{J}$$

EVALUATE: At $x = A/2$, the total energy is one-fourth potential energy and three-fourths kinetic energy. You can confirm this by inspecting Fig. 14.15b.

SUMMARY

Periodic motion: motion that repeats itself in a defined cycle. $f = \frac{1}{T}$ $T = \frac{1}{f}$ $\omega = 2\pi f = \frac{2\pi}{T}$

Simple harmonic motion: if the restoring force is proportional to the distance from equilibrium, the motion will be of the SHM type. The angular frequency and period do not depend on the amplitude of oscillation.

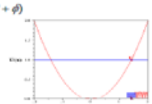


$$F_x = -kx \qquad a_x = \frac{F_x}{m} = -\frac{k}{m}x$$

$$\omega = \sqrt{\frac{k}{m}} \qquad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

$$x = A\cos(\omega t + \phi)$$

Energy in SHM: $E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$



Simple pendulum: $\omega = \sqrt{\frac{g}{L}}$ $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$ $T = 2\pi\sqrt{\frac{L}{g}}$

THE UNIVERSAL LAW OF GRAVITATION
(11)

Dr. Hind I. Abdulgafour

Sir Isaac Newton

What makes us to fall on the earth always?

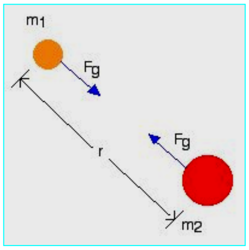
- **There is always some force acting on us that guides our direction of falling.**
- **No matter from where ever we jump, or we drop objects from anywhere they will always fall towards the earth.**

What makes the apple to always fall on the earth?

It was Isaac Newton who posed this question and answered it. Newton stated that all objects attract each other along the line joining their centers.

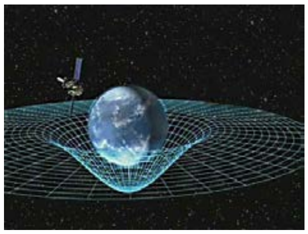
Every object in the universe attracts every other object towards itself

- This force with which the two objects attract each other is called the force of gravitation
- The force of gravitation acts even if there is nothing connecting the two objects.



Newton's Law of Gravitation

What causes YOU to be pulled down? THE EARTH….or more specifically…the EARTH'S MASS. Anything that has MASS has a gravitational pull towards it.



$$F_g \alpha\ Mm$$

What the proportionality above is saying is that for there to be a FORCE DUE TO GRAVITY on something there must be at least 2 masses involved, where one is larger than the other.

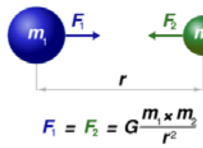
N.L.o.G.

As you move AWAY from the earth, your DISTANCE increases and your FORCE DUE TO GRAVITY decrease. This is a special INVERSE relationship called an Inverse-Square.

$$F_g\ \alpha \frac{1}{r^2}$$

" stands for SEPARATION DISTANC he distance between the CENTERS O of the 2 objects. We us the symbol "r" mbolizes the radius. Gravitation is related to circular motion as you will

N.L.o.G – Putting it all together



$$F_g \;\alpha\; \frac{m_1 m_2}{r^2}$$

$G =$ constant of proportionality

G = Universal Gravitational Constant

$$G = 6.67x10^{-27}\ Nm^2/kg^2$$

$$F_g = G\frac{m_1 m_2}{r^2}$$

$F_g = mg \rightarrow$ Use this when you are on the earth

$F_g = G\frac{m_1 m_2}{r^2} \rightarrow$ Use this when you are LEAVING the earth

Try this!

$F_g = mg \rightarrow$ Use this when you are on the earth

$F_g = G\frac{m_1 m_2}{r^2} \rightarrow$ Use this when you are LEAVING the earth

$$mg = G\frac{Mm}{r^2}$$

$$g = G\frac{M}{r^2}$$

$M =$ Mass of the Earth $= 5.97x10^{24} - kg$

$r =$ radius of the Earth $= 6.37x10^{6} - m$

$$g = \frac{(6.67x10^{-27})(5.97x10^{24})}{(6.37x10^{6})^2} = 9.81 m/s^2$$

The Universal Law of Gravitation states that

- **Any two point particles with masses m_1 and m_2 attract each other by a force whose magnitude is directly proportional to the product of the two masses, that is m_1m_2 and inversely proportional to the square of the distance R between them. The direction of the force is along the line joining the two masses.**

Universal Gravitation

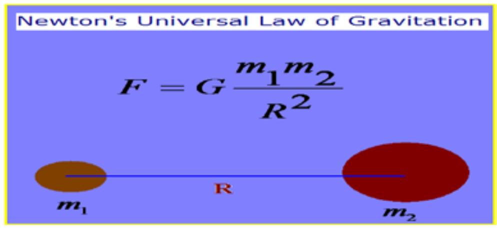
- **From this, Newton reasoned that the strength of the gravitational force is *not constant*, in fact, the magnitude of the force is *inversely proportional to the square of the distance* between the objects.**

- **Newton concluded that the gravitational force is:**
  - *Directly proportional* **to the** *masses* **of** *both* **objects.**
  - *Inversely proportional* **to the** *distance* **between the objects.**

Inverse Square Law

Newton's Law of Universal Gravitation is often called an *inverse square law*, since the force is inversely proportional to the square of the distance.

In the mathematical form the law can be represented as…



In the expression

$$F = G\frac{m_1 m_2}{R^2}$$

G is a gravitational constant. It does not depend on the value of masses or the distance between the masses. The constant G will remain the same for any two objects anywhere in the Universe.

Value of G is 6.6734×10^{-11} Nm^2 / kg^2

Uses of Gravitation

- It is the gravitational force that keep everything at its place. Otherwise we would have been floating in the air.
- It also keeps the earth, sun and other celestial bodies at their right places
- It is responsible for many natural phenomenon on the earths like tides and orbiting of moon around the earth.

It is due to gravitation that we are walking on the Earth.

➢The Gravitational Field

Objects with mass create an ***invisible disturbance in the space around them*** **that is felt by other massive objects - this is a** ***gravitational field*****.**

ان الأجسام ذات الكتلة تخلق اضطرابًا غير مرئي في الفضاء المحيط بها والتي تشعر بها كائنات ضخمة أخرى - وهذا مجال جاذبية.

➢ Gravitational Field Strength

- **To measure the strength of the gravitational field at any point, measure the gravitational force, F, exerted on any "test mass", m.**
- *Gravitational Field Strength, $g = F/m$*

Gravitational Field Strength

- Near the surface of the Earth, g = F/m = 9.8 N/kg = 9.8 m/s^2.
- In general, $g = GM/r^2$, where M is the mass of the object creating the field, r is the distance from the object's center, and G = 6.67 x10^{-11} Nm^2/kg^2.

Example 1

What is the gravitational force between the earth and a 100 kg man standing on the earth's surface?

$M = \text{Mass of the Earth} = 5.97x10^{24} - kg$

$r = \text{radius of the Earth} = 6.37x10^{6} - m$

$$F_g = G\frac{m_{man}M_{Earth}}{r^2} = 6.67x10^{-11}\frac{(100)(5.97x10^{24})}{(6.37x10^{6})^2} =$$

Because the force
can define this for

$F_g \ \alpha \ m_{man} \rightarrow F_g = m_{man}g$

$9.81x10^2 = 100g$

$g = 9.8 \ m/s/s$

Example 2: Law of Universal Gravitation

- **Jimmy is attracted to Betty. Jimmy's mass is 90.0 kg and Betty's mass is 57.0 kg. If Jim is standing 10.0 meters away from Betty, what is the gravitational force between them?**
  - $F_G = GM_1M_2 / r^2$
  - $F_G = (6.67\text{x}10^{-11}\ Nm^2/kg^2)(90.0\ kg)(57.0\ kg) / (10.0\ m)^2$
  - $F_G = (3.42\text{x}10^{-7}\ Nm^2) / (100.\ m^2)$
  - $F_G = 3.42\text{x}10^{-9}\ N = 3.42\ nN$
    - **In standard terms, that's 7.6 ten-billionths of a pound of force.**

Example 13.1 Calculating gravitational force

The mass m_1 of one of the small spheres of a Cavendish balance is 0.0100 kg, the mass m_2 of the nearest large sphere is 0.500 kg, and the center-to-center distance between them is 0.0500 m. Find the gravitational force F_g on each sphere due to the other.

SOLUTION

IDENTIFY, SET UP, and EXECUTE: Because the spheres are spherically symmetric, we can calculate F_g by treating them as *particles* separated by 0.0500 m, as in Fig. 13.2. Each sphere experiences the same magnitude of force from the other sphere. We use Newton's law of gravitation, Eq. (13.1), to determine F_g:

$$F_g = \frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(0.0100\ \text{kg})(0.500\ \text{kg})}{(0.0500\ \text{m})^2}$$
$$= 1.33 \times 10^{-10}\ \text{N}$$

EVALUATE: It's remarkable that such a small force could be measured—or even detected—more than 200 years ago. Only a very massive object such as the earth exerts a gravitational force we can feel.

Example 13.2 Acceleration due to gravitational attraction

Suppose the two spheres in Example 13.1 are placed with their centers 0.0500 m apart at a point in space far removed from all other bodies. What is the magnitude of the acceleration of each, relative to an inertial system?

SOLUTION

IDENTIFY, SET UP, and EXECUTE: Each sphere exerts on the other a gravitational force of the same magnitude F_g, which we found in Example 13.1. We can neglect any other forces. The *acceleration* magnitudes a_1 and a_2 are different because the masses are different. To determine these we'll use Newton's second law:

$$a_1 = \frac{F_g}{m_1} = \frac{1.33 \times 10^{-10}\ \text{N}}{0.0100\ \text{kg}} = 1.33 \times 10^{-8}\ \text{m/s}^2$$
$$a_2 = \frac{F_g}{m_2} = \frac{1.33 \times 10^{-10}\ \text{N}}{0.500\ \text{kg}} = 2.66 \times 10^{-10}\ \text{m/s}^2$$

EVALUATE: The larger sphere has 50 times the mass of the smaller one and hence has $\frac{1}{50}$ the acceleration. These accelerations are *not* constant; the gravitational forces increase as the spheres move toward each other.

Weight

The weight of a body is the total gravitational force exerted on the body by all other bodies in the universe.

When the body is near the surface of the earth, we can neglect all other gravitational forces and consider the weight as just the earth's gravitational attraction. At the surface of the *moon* we consider a body's weight to be the gravitational attraction of the moon, and so on.

If we again model the earth as a spherically symmetric body with radius R_E and mass m_E, the weight w of a small body of mass m at the earth's surface (a distance R_E from its center) is

$$w = F_g = \frac{Gm_E m}{R_E^2} \quad \text{(weight of a body of mass } m \text{ at the earth's surface)} \qquad (1)$$

But we also know from that the weight w of a body is the force that causes the acceleration g of free fall, so by Newton's second law, $w = mg$. Equating this with Eq. (1) and dividing by m, we find

$$g = \frac{Gm_E}{R_E^2} \quad \text{(acceleration due to gravity at the earth's surface)} \qquad (2)$$

The acceleration due to gravity g is independent of the mass m of the body because m doesn't appear in this equation. We already knew that, but we can now see how it follows from the law of gravitation.

We can *measure* all the quantities in Eq. (2) except for m_E, so this relationship allows us to compute the mass of the earth. Solving Eq. (2) for m_E and using $R_E = 6380 \text{ km} = 6.38 \times 10^6$ m and $g = 9.80 \text{ m/s}^2$, we find

$$m_E = \frac{gR_E^2}{G} = 5.98 \times 10^{24} \text{ kg}$$

At a point above the earth's surface a distance r from the center of the earth (a distance $r - R_E$ above the surface), the weight of a body is given by Eq. (1) with R_E replaced by r:

$$w = F_g = \frac{Gm_E m}{r^2} \qquad (3)$$

While the earth is an approximately spherically symmetric distribution of mass, it is *not* uniform throughout its volume. To demonstrate this, let's first calculate the average *density*, or mass per unit volume, of the earth. If we assume a spherical earth, the volume is

$$V_E = \tfrac{4}{3}\pi R_E^3 = \tfrac{4}{3}\pi(6.38 \times 10^6 \text{ m})^3 = 1.09 \times 10^{21} \text{ m}^3$$

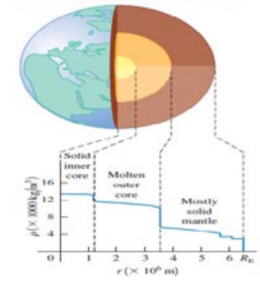


Fig. 4 The density of the earth decreases with increasing distance from its center.

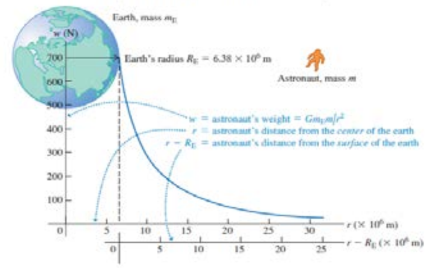


Fig. 3 An astronaut who weighs 700 N at the earth's surface experiences less gravitational attraction when above the surface. The relevant distance r is from the astronaut to the *center* of the earth (*not* from the astronaut to the earth's surface).

The average density ρ (the Greek letter rho) of the earth is the total mass divided by the total volume:

$$\rho = \frac{m_E}{V_E} = \frac{5.97 \times 10^{24}\ \text{kg}}{1.09 \times 10^{21}\ \text{m}^3}$$
$$= 5500\ \text{kg/m}^3 = 5.5\ \text{g/cm}^3$$

Example 13.4 Gravity on Mars

A robotic lander with an earth weight of 3430 N is sent to Mars, which has radius $R_M = 3.40 \times 10^6$ m and mass $m_M = 6.42 \times 10^{23}$ kg (see Appendix F). Find the weight F_g of the lander on the Martian surface and the acceleration there due to gravity, g_M.

SOLUTION

IDENTIFY and SET UP: To find F_g we use Eq. (1) replacing m_E and R_E with m_M and R_M. We determine the lander mass m from the lander's earth weight w and then find g_M from $F_g = mg_M$.

EXECUTE: The lander's earth weight is $w = mg$, so

$$m = \frac{w}{g} = \frac{3430\ \text{N}}{9.80\ \text{m/s}^2} = 350\ \text{kg}$$

The mass is the same no matter where the lander is. From Eq. (13.3), the lander's weight on Mars is

$$F_g = \frac{Gm_M m}{R_M^2}$$
$$= \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.42 \times 10^{23}\ \text{kg})(350\ \text{kg})}{(3.40 \times 10^6\ \text{m})^2}$$
$$= 1.30 \times 10^3\ \text{N}$$

The acceleration due to gravity on Mars is

$$g_M = \frac{F_g}{m} = \frac{1.30 \times 10^3\ \text{N}}{350\ \text{kg}} = 3.7\ \text{m/s}^2$$

Gravitational Potential Energy

$$F_g = -G\frac{m_1 m_2}{r^2}\vec{r}$$

$$W = \int_R^{\infty} F(r)\,dr$$

$$W = \int_R^{\infty} G\frac{mM}{r^2}\,dr$$

$$W = GmM\int_R^{\infty}\frac{1}{r^2}\,dr = GmM\int_R^{\infty} r^{-2}$$

$$W = U_g = -\frac{GmM}{r}$$

Work is the integral of a Force function with respect to displacement.

Putting in the basic expression for gravitational force

Pulling out the constants and bringing the denominator to the numerator.

The negative sign should not surprise you as we already knew that Work was equal to the negative change in "U" or mgh.

Example 13.5 "From the earth to the moon"

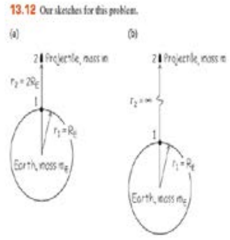
In Jules Verne's 1865 story with this title, three men went to the moon in a shell fired from a giant cannon sunk in the earth in Florida. (a) Find the minimum muzzle speed needed to shoot a shell straight up to a height above the earth equal to the earth's radius R_E. (b) Find the minimum muzzle speed that would allow a shell to escape from the earth completely (the *escape speed*). Neglect air resistance, the earth's rotation, and the gravitational pull of the moon. The earth's radius and mass are $R_E = 6.38 \times 10^6$ m and $m_E = 5.97 \times 10^{24}$ kg.

SOLUTION

IDENTIFY and SET UP: Once the shell leaves the cannon muzzle, only the (conservative) gravitational force does work. Hence we can use conservation of mechanical energy to find the speed at which the shell must leave the muzzle so as to come to a halt (a) at two earth radii from the earth's center and (b) at an infinite distance from earth. The energy-conservation equation is $K_1 + U_1 = K_2 + U_2$, with U given by Eq. (13.9).

Figure 13.12 shows our sketches. Point 1 is at $r_1 = R_E$, where the shell leaves the cannon with speed v_1 (the target variable). Point 2 is where the shell reaches its maximum height; in part

13.12 Our sketches for this problem.



(a) $r_2 = 2R_E$ (Fig. 13.12a), and in part (b) $r_2 = \infty$ (Fig 13.12b). In both cases $v_2 = 0$ and $K_2 = 0$. Let m be the mass of the shell (with passengers).

EXECUTE: (a) We solve the energy-conservation equation for v_1:

$$K_1 + U_1 = K_2 + U_2$$

$$\frac{1}{2}mv_1^2 + \left(-\frac{Gm_E m}{R_E}\right) = 0 + \left(-\frac{Gm_E m}{2R_E}\right)$$

$$v_1 = \sqrt{\frac{Gm_E}{R_E}} = \sqrt{\frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\ \text{kg})}{6.38 \times 10^6\ \text{m}}}$$

$$= 7900\ \text{m/s}\ (= 28{,}400\ \text{km/h} = 17{,}700\ \text{mi/h})$$

(b) Now $r_2 = \infty$ so $U_2 = 0$ (see Fig. 13.11). Since $K_2 = 0$, the total mechanical energy $K_2 + U_2$ is zero in this case. Again we solve the energy-conservation equation for v_1:

$$\frac{1}{2}mv_1^2 + \left(-\frac{Gm_E m}{R_E}\right) = 0 + 0$$

$$v_1 = \sqrt{\frac{2Gm_E}{R_E}}$$

$$= \sqrt{\frac{2(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\ \text{kg})}{6.38 \times 10^6\ \text{m}}}$$

$$= 1.12 \times 10^4\ \text{m/s}\ (= 40{,}200\ \text{km/h} = 25{,}000\ \text{mi/h})$$

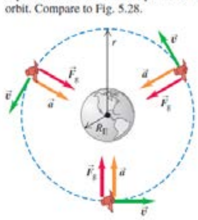
EVALUATE: Our result in part (b) doesn't depend on the mass of the shell or the direction of launch. A modern spacecraft launched from Florida must attain essentially the speed found in part (b) to escape the earth; however, before launch it's already moving at 410 m/s to the east because of the earth's rotation. Launching to the east takes advantage of this "free" contribution toward escape speed.

To generalize, the initial speed v_1 needed for a body to escape from the surface of a spherical body of mass M and radius R (ignoring air resistance) is $v_1 = \sqrt{2GM/R}$ (escape speed). This equation yields escape speeds of 5.02×10^3 m/s for Mars, 5.95×10^4 m/s for Jupiter, and 6.18×10^5 m/s for the sun.

Satellites: Circular Orbits

- **A *circular orbit like trajectory is the simplest case. It is also an* important case, since many artificial satellite have nearly circular orbits and the orbits of the planets around the sun are also fairly circular.**
- **The only force acting on a satellite in circular orbit around the earth is the earth's gravitational attraction, which is directed toward the center of the earth and hence toward the center of the orbit as shown in fig.5.**

Fig.5 The force $\vec{F}_g$ due to the earth's gravitational attraction provides the centripetal acceleration that keeps a satellite in orbit. Compare to Fig. 5.28.



The satellite is in a circular orbit: Its acceleration $\vec{a}$ is always perpendicular to its velocity $\vec{v}$, so its speed v is constant.

center of the circle. By the law of gravitation, the net force (gravitational force) on the satellite of mass m has magnitude $F_g = Gm_E m/r^2$ and is in the same direction as the acceleration. Newton's second law ($\sum \vec{F} = m\vec{a}$) then tells us that

$$\frac{Gm_E m}{r^2} = \frac{mv^2}{r}$$

Solving this for v, we find

$$v = \sqrt{\frac{Gm_E}{r}} \quad \text{(circular orbit)} \quad (13.10)$$

We can derive a relationship between the radius r of a circular orbit and the period T, the time for one revolution. The speed v is the distance $2\pi r$ traveled in one revolution, divided by the period:

$$v = \frac{2\pi r}{T} \tag{13.11}$$

To get an expression for T, we solve Eq. (13.11) for T and substitute v from Eq. (13.10):

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{Gm_{\mathrm{E}}}} = \frac{2\pi r^{3/2}}{\sqrt{Gm_{\mathrm{E}}}} \quad \text{(circular orbit)} \tag{13.12}$$

Since the speed v in a circular orbit is determined by Eq. (13.10) for a given orbit radius r, the total mechanical energy $E = K + U$ is determined as well.

$$E = K + U = \tfrac{1}{2}mv^2 + \left(-\frac{Gm_{\mathrm{E}}m}{r}\right) = \tfrac{1}{2}m\left(\frac{Gm_{\mathrm{E}}}{r}\right) - \frac{Gm_{\mathrm{E}}m}{r}$$
$$= -\frac{Gm_{\mathrm{E}}m}{2r} \quad \text{(circular orbit)} \tag{13.13}$$

The total mechanical energy in a circular orbit is negative and equal to one-half the potential energy. Increasing the orbit radius r means increasing the mechanical energy (that is, making E less negative).

Example 13.6 **A satellite orbit**

You wish to put a 1000-kg satellite into a circular orbit 300 km above the earth's surface. (a) What speed, period, and radial acceleration will it have? (b) How much work must be done to the satellite to put it in orbit? (c) How much additional work would have to be done to make the satellite escape the earth? The earth's radius and mass are given in Example 13.5 (Section 13.3).

SOLUTION

IDENTIFY and SET UP: The satellite is in a circular orbit, so we can use the equations derived in this section. In part (a), we first find the radius r of the satellite's orbit from its altitude. We then calculate the speed v and period T using Eqs. (13.10) and (13.12) and the acceleration from $a_{\mathrm{rad}} = v^2/r$. In parts (b) and (c), the work required is the difference between the initial and final mechanical energy, which for a circular orbit is given by Eq. (13.13).

EXECUTE: (a) The radius of the satellite's orbit is $r = 6380\ \mathrm{km} + 300\ \mathrm{km} = 6680\ \mathrm{km} = 6.68 \times 10^6\ \mathrm{m}$. From Eq. (13.10), the orbital speed is

$$v = \sqrt{\frac{Gm_{\mathrm{E}}}{r}} = \sqrt{\frac{(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(5.97 \times 10^{24}\ \mathrm{kg})}{6.68 \times 10^6\ \mathrm{m}}}$$
$$= 7720\ \mathrm{m/s}$$

We find the orbital period from Eq. (13.12):

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.68 \times 10^6\ \mathrm{m})}{7720\ \mathrm{m/s}} = 5440\ \mathrm{s} = 90.6\ \mathrm{min}$$

Finally, the radial acceleration is

$$a_{\mathrm{rad}} = \frac{v^2}{r} = \frac{(7720\ \mathrm{m/s})^2}{6.68 \times 10^6\ \mathrm{m}} = 8.92\ \mathrm{m/s^2}$$

This is the value of g at a height of 300 km above the earth's surface; it is about 10% less than the value of g at the surface.

(b) The work required is the difference between E_2, the total mechanical energy when the satellite is in orbit, and E_1, the total mechanical energy when the satellite was at rest on the launch pad. From Eq. (13.13), the energy in orbit is

$$E_2 = -\frac{Gm_{\mathrm{E}}m}{2r}$$
$$= -\frac{(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(5.97 \times 10^{24}\ \mathrm{kg})(1000\ \mathrm{kg})}{2(6.68 \times 10^6\ \mathrm{m})}$$
$$= -2.98 \times 10^{10}\ \mathrm{J}$$

The satellite's kinetic energy is zero on the launch pad ($r = R_{\mathrm{E}}$), so

$$E_1 = K_1 + U_1 = 0 + \left(-\frac{Gm_{\mathrm{E}}m}{R_{\mathrm{E}}}\right)$$
$$= -\frac{(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(5.97 \times 10^{24}\ \mathrm{kg})(1000\ \mathrm{kg})}{6.38 \times 10^6\ \mathrm{m}}$$
$$= -6.24 \times 10^{10}\ \mathrm{J}$$

Hence the work required is

$$W_{\mathrm{required}} = E_2 - E_1 = (-2.98 \times 10^{10}\ \mathrm{J}) - (-6.24 \times 10^{10}\ \mathrm{J})$$
$$= 3.26 \times 10^{10}\ \mathrm{J}$$

(c) We saw in part (b) of Example 13.5 that the minimum total mechanical energy for a satellite to escape to infinity is zero. Here, the total mechanical energy in the circular orbit is $E_2 = -2.98 \times 10^{10}\ \mathrm{J}$; to increase this to zero, an amount of work equal to $2.98 \times 10^{10}\ \mathrm{J}$ would have to be done on the satellite, presumably by rocket engines attached to it.

EVALUATE: In part (b) we ignored the satellite's initial kinetic energy (while it was still on the launch pad) due to the rotation of the earth. How much difference does this make? (See Example 13.5 for useful data.)

Kepler's Laws

There are three laws that Johannes Kepler formulated when he was studying the heavens

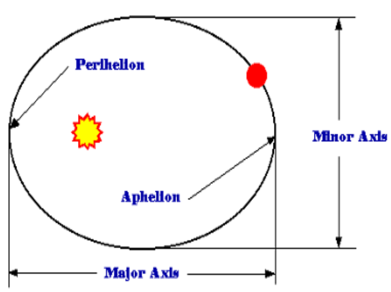
THE LAW OF ORBITS - "*All planets move in elliptical orbits, with the Sun at one focus."*

THE LAW OF AREAS - "*A line that connects a planet to the sun sweeps out equal areas in the plane of the planet's orbit in equal times, that is, the rate dA/dt at which it sweeps out area A is constant."*

THE LAW OF PERIODS - "*The square of the period of any planet is proportional to the cube of the semi major axis of its orbit."*

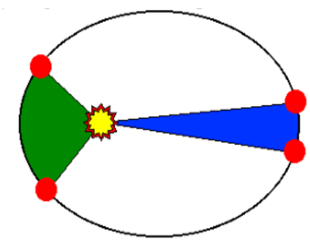
Kepler's 1st law – The Law of Orbits

"*All planets move in elliptical orbits, with the Sun at one focus."*



Kepler's 2nd Law – The Law of Areas

"A line that connects a planet to the sun sweeps out equal areas in the plane of the planet's orbit in equal times, that is, the rate dA/dt at which it sweeps out area A is constant."



Kepler's 3rd Law – The Law of Periods

"The square of the period of any planet is proportional to the cube of the semi major axis of its orbit."

$$T = \frac{2\pi a^{3/2}}{\sqrt{G m_S}} \quad \text{(elliptical orbit around the sun)} \qquad (13.17)$$

$$F_g = G\frac{mM}{r^2},\ F_c = \frac{mv^2}{r}$$

Gravitational forces are centripetal, thus we can set them equal to each other!

$$F_g = F_c$$

$$G\frac{mM}{r^2} = \frac{mv^2}{r}$$

Since we are moving in a circle we can substitute the appropriate velocity formula!

$$G\frac{M}{r} = v^2 \quad v = \frac{2\pi r}{T}$$

The expression in the RED circle derived by setting the centripetal force equal to the gravitational force is called ORBITAL SPEED.

$$G\frac{M}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

Using algebra, you can see that everything in the parenthesis is CONSTANT. Thus the proportionality holds true!

Example 13.8 **Kepler's third law**

The asteroid Pallas has an orbital period of 4.62 years and an orbital eccentricity of 0.233. Find the semi-major axis of its orbit.

SOLUTION

IDENTIFY and SET UP: This example uses Kepler's third law, which relates the period T and the semi-major axis a for an orbiting object (such as an asteroid). We use Eq. (13.17) to determine a; from Appendix F we have $m_S = 1.99 \times 10^{30}$ kg, and a conversion factor from Appendix E gives $T = (4.62\ \text{yr})(3.156 \times 10^7\ \text{s/yr}) = 1.46 \times 10^8$ s. Note that we don't need the value of the eccentricity.

EXECUTE: From Eq. (13.17), $a^{3/2} = [(Gm_S)^{1/2}T]/2\pi$. To solve for a, we raise both sides of this expression to the $\frac{2}{3}$ power and then substitute the values of G, m_S, and T:

$$a = \left(\frac{Gm_S T^2}{4\pi^2}\right)^{1/3} = 4.15 \times 10^{11}\ \text{m}$$

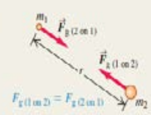
(Plug in the numbers yourself to check.)

EVALUATE: Our result is intermediate between the semi-major axes of Mars and Jupiter (see Appendix F). Most known asteroids orbit in an "asteroid belt" between the orbits of these two planets.

Summary

Newton's law of gravitation: Any two bodies with masses m_1 and m_2, a distance r apart, attract each other with forces inversely proportional to r^2. These forces form an action–reaction pair and obey Newton's third law. When two or more bodies exert gravitational forces on a particular body, the total gravitational force on that individual body is the vector sum of the forces exerted by the other bodies. The gravitational interaction between spherical mass distributions, such as planets or stars, is the same as if all the mass of each distribution were concentrated at the center. (See Examples 13.1–13.3 and 13.10.)

$$F_g = \frac{Gm_1m_2}{r^2} \qquad (13.1)$$



Gravitational force, weight, and gravitational potential energy: The weight w of a body is the total gravitational force exerted on it by all other bodies in the universe. Near the surface of the earth (mass m_E and radius R_E), the weight is essentially equal to the gravitational force of the earth alone. The gravitational potential energy U of two masses m and m_E separated by a distance r is inversely proportional to r. The potential energy is never positive; it is zero only when the two bodies are infinitely far apart. (See Examples 13.4 and 13.5.)

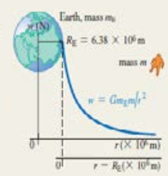
$$w = F_g = \frac{Gm_Em}{R_E^2} \qquad (13.3)$$

(weight at earth's surface)

$$g = \frac{Gm_E}{R_E^2} \qquad (13.4)$$

(acceleration due to gravity at earth's surface)

$$U = -\frac{Gm_Em}{r} \qquad (13.9)$$



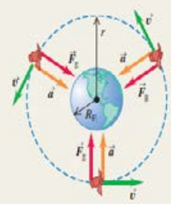
Orbits: When a satellite moves in a circular orbit, the centripetal acceleration is provided by the gravitational attraction of the earth. Kepler's three laws describe the more general case: an elliptical orbit of a planet around the sun or a satellite around a planet. (See Examples 13.6–13.9.)

$$v = \sqrt{\frac{Gm_E}{r}}$$

(speed in circular orbit) (13.10)

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{Gm_E}} = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$$

(period in circular orbit) (13.12)



Q13.1 A student wrote: "The only reason an apple falls downward to meet the earth instead of the earth rising upward to meet the apple is that the earth is much more massive and so exerts a much greater pull." Please comment.

Q13.2 A planet makes a circular orbit with period T around a star. If it were to orbit, at the same distance, a star with three times the mass of the original star, would the new period (in terms of T) be (a) $3T$, (b) $T\sqrt{3}$, (c) T, (d) $T/\sqrt{3}$, or (e) $T/3$?

Q13.3 If all planets had the same average density, how would the acceleration due to gravity at the surface of a planet depend on its radius?

Q13.4 Is a pound of butter on the earth the same amount as a pound of butter on Mars? What about a kilogram of butter? Explain.

THE UNIVERSAL LAW OF GRAVITATION

(11)

Dr. Hind I. Abdulgafour

Sir Isaac Newton

What makes us to fall on the earth always?

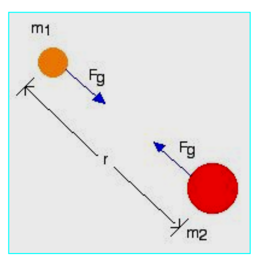
- **There is always some force acting on us that guides our direction of falling.**
- **No matter from where ever we jump, or we drop objects from anywhere they will always fall towards the earth.**

What makes the apple to always fall on the earth?

It was Isaac Newton who posed this question and answered it. Newton stated that all objects attract each other along the line joining their centers.

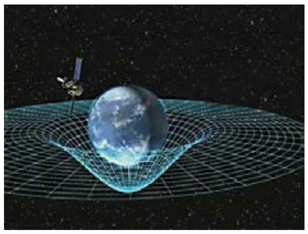
Every object in the universe attracts every other object towards itself

- This force with which the two objects attract each other is called the force of gravitation
- The force of gravitation acts even if there is nothing connecting the two objects.



Newton's Law of Gravitation

What causes YOU to be pulled down? THE EARTH….or more specifically…the EARTH'S MASS. Anything that has MASS has a gravitational pull towards it.



$$F_g \alpha \; Mm$$

What the proportionality above is saying is that for there to be a FORCE DUE TO GRAVITY on something there must be at least 2 masses involved, where one is larger than the other.

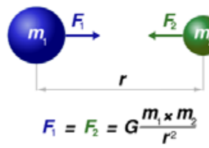
N.L.o.G.

As you move AWAY from the earth, your DISTANCE increases and your FORCE DUE TO GRAVITY decrease. This is a special INVERSE relationship called an Inverse-Square.

$$F_g \; \alpha \; \frac{1}{r^2}$$

" stands for SEPARATION DISTANC he distance between the CENTERS O of the 2 objects. We us the symbol "r" mbolizes the radius. Gravitation is related to circular motion as you will

N.L.o.G – Putting it all together



$$F_g \;\alpha\; \frac{m_1 m_2}{r^2}$$

G = constant of proportionality

G = Universal Gravitational Constant

$$G = 6.67x10^{-27}\ Nm^2/kg^2$$

$$F_g = G\frac{m_1 m_2}{r^2}$$

$F_g = mg \rightarrow$ Use this when you are on the earth

$F_g = G\dfrac{m_1 m_2}{r^2} \rightarrow$ Use this when you are LEAVING the earth

Try this!

$F_g = mg \rightarrow$ Use this when you are on the earth

$F_g = G\dfrac{m_1 m_2}{r^2} \rightarrow$ Use this when you are LEAVING the earth

$$mg = G\frac{Mm}{r^2}$$

$$g = G\frac{M}{r^2}$$

M = Mass of the Earth $= 5.97x10^{24} - kg$

r = radius of the Earth $= 6.37x10^{6} - m$

$$g = \frac{(6.67x10^{-27})(5.97x10^{24})}{(6.37x10^{6})^2} = 9.81 m/s^2$$

The Universal Law of Gravitation states that

- Any two point particles with masses m_1 and m_2 attract each other by a force whose magnitude is directly proportional to the product of the two masses, that is m_1m_2 and inversely proportional to the square of the distance R between them. The direction of the force is along the line joining the two masses.

Universal Gravitation

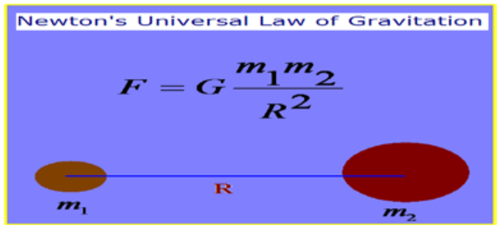
- From this, Newton reasoned that the strength of the gravitational force is *not constant*, in fact, the magnitude of the force is *inversely proportional to the square of the distance* between the objects.

- Newton concluded that the gravitational force is:
  - *Directly proportional* to the *masses* of *both* objects.
  - *Inversely proportional* to the *distance* between the objects.

Inverse Square Law

Newton's Law of Universal Gravitation is often called an *inverse square law*, since the force is inversely proportional to the square of the distance.

In the mathematical form the law can be represented as…



In the expression $$F = G\frac{m_1 m_2}{R^2}$$

G is a gravitational constant. It does not depend on the value of masses or the distance between the masses. The constant G will remain the same for any two objects anywhere in the Universe.

Value of G is 6.6734×10^{-11} Nm^2 / kg^2

Uses of Gravitation

- **It is the gravitational force that keep everything at its place. Otherwise we would have been floating in the air.**
- **It also keeps the earth, sun and other celestial bodies at their right places**
- **It is responsible for many natural phenomenon on the earths like tides and orbiting of moon around the earth.**

It is due to gravitation that we are walking on the Earth.

➢The Gravitational Field

Objects with mass create an *invisible disturbance in the space around them* **that is felt by other massive objects - this is a** *gravitational field*.

ان الأجسام ذات الكتلة تخلق اضطرابًا غير مرئي في الفضاء المحيط بها والتي تشعر بها كائنات ضخمة أخرى - وهذا مجال جاذبية.

➢ Gravitational Field Strength

- **To measure the strength of the gravitational field at any point, measure the gravitational force, F, exerted on any "test mass", m.**
- *Gravitational Field Strength, $g = F/m$*

Gravitational Field Strength

- Near the surface of the Earth, $g = F/m = 9.8$ N/kg = 9.8 m/s^2.
- In general, *$g = GM/r^2$*, where M is the mass of the object creating the field, r is the distance from the object's center, and G = 6.67×10^{-11} Nm^2/kg^2.

Example 1

What is the gravitational force between the earth and a 100 kg man standing on the earth's surface?

M = Mass of the Earth $= 5.97x10^{24} - kg$

r = radius of the Earth $= 6.37x10^{6} - m$

$$F_g = G\frac{m_{man}M_{Earth}}{r^2} = 6.67x10^{-11}\frac{(100)(5.97x10^{24})}{(6.37x10^{6})^2} =$$

Because the force
can define this for

$$F_g \;\alpha\; m_{man} \rightarrow F_g = m_{man}g$$

$$9.81x10^2 = 100g$$

$$g = 9.8\, m/s/s$$

Example 2: Law of Universal Gravitation

- **Jimmy is attracted to Betty. Jimmy's mass is 90.0 kg and Betty's mass is 57.0 kg. If Jim is standing 10.0 meters away from Betty, what is the gravitational force between them?**
  - $F_G = GM_1M_2 / r^2$
  - $F_G = (6.67 \times 10^{-11}\ Nm^2/kg^2)(90.0\ kg)(57.0\ kg) / (10.0\ m)^2$
  - $F_G = (3.42 \times 10^{-7}\ Nm^2) / (100.\ m^2)$
  - $F_G = 3.42 \times 10^{-9}\ N = 3.42\ nN$
    - **In standard terms, that's 7.6 ten-billionths of a pound of force.**

Example 13.1 Calculating gravitational force

The mass m_1 of one of the small spheres of a Cavendish balance is 0.0100 kg, the mass m_2 of the nearest large sphere is 0.500 kg, and the center-to-center distance between them is 0.0500 m. Find the gravitational force F_g on each sphere due to the other.

SOLUTION

IDENTIFY, SET UP, and EXECUTE: Because the spheres are spherically symmetric, we can calculate F_g by treating them as *particles* separated by 0.0500 m, as in Fig. 13.2. Each sphere experiences the same magnitude of force from the other sphere. We use Newton's law of gravitation, Eq. (13.1), to determine F_g:

$$F_g = \frac{(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(0.0100\ \mathrm{kg})(0.500\ \mathrm{kg})}{(0.0500\ \mathrm{m})^2} = 1.33 \times 10^{-10}\ \mathrm{N}$$

EVALUATE: It's remarkable that such a small force could be measured—or even detected—more than 200 years ago. Only a very massive object such as the earth exerts a gravitational force we can feel.

Example 13.2 Acceleration due to gravitational attraction

Suppose the two spheres in Example 13.1 are placed with their centers 0.0500 m apart at a point in space far removed from all other bodies. What is the magnitude of the acceleration of each, relative to an inertial system?

SOLUTION

IDENTIFY, SET UP, and EXECUTE: Each sphere exerts on the other a gravitational force of the same magnitude F_g, which we found in Example 13.1. We can neglect any other forces. The *acceleration* magnitudes a_1 and a_2 are different because the masses are different. To determine these we'll use Newton's second law:

$$a_1 = \frac{F_g}{m_1} = \frac{1.33 \times 10^{-10}\ \mathrm{N}}{0.0100\ \mathrm{kg}} = 1.33 \times 10^{-8}\ \mathrm{m/s^2}$$

$$a_2 = \frac{F_g}{m_2} = \frac{1.33 \times 10^{-10}\ \mathrm{N}}{0.500\ \mathrm{kg}} = 2.66 \times 10^{-10}\ \mathrm{m/s^2}$$

EVALUATE: The larger sphere has 50 times the mass of the smaller one and hence has $\frac{1}{50}$ the acceleration. These accelerations are *not* constant; the gravitational forces increase as the spheres move toward each other.

Weight

The weight of a body is the total gravitational force exerted on the body by all other bodies in the universe.

When the body is near the surface of the earth, we can neglect all other gravitational forces and consider the weight as just the earth's gravitational attraction. At the surface of the *moon* we consider a body's weight to be the gravitational attraction of the moon, and so on.

If we again model the earth as a spherically symmetric body with radius R_E and mass m_E, the weight w of a small body of mass m at the earth's surface (a distance R_E from its center) is

$$w = F_g = \frac{Gm_E m}{R_E^2} \quad \text{(weight of a body of mass } m \text{ at the earth's surface)} \qquad \textbf{(1)}$$

But we also know from that the weight w of a body is the force that causes the acceleration g of free fall, so by Newton's second law, $w = mg$. Equating this with Eq. **(1)** and dividing by m, we find

$$g = \frac{Gm_E}{R_E^2} \quad \text{(acceleration due to gravity at the earth's surface)} \qquad \textbf{(2)}$$

The acceleration due to gravity g is independent of the mass m of the body because m doesn't appear in this equation. We already knew that, but we can now see how it follows from the law of gravitation.

We can *measure* all the quantities in Eq. **(2)** except for m_E, so this relationship allows us to compute the mass of the earth. Solving Eq. **(2)** for m_E and using $R_E = 6380\ \text{km} = 6.38 \times 10^6$ m and $g = 9.80\ \text{m/s}^2$, we find

$$m_E = \frac{gR_E^2}{G} = 5.98 \times 10^{24}\ \text{kg}$$

At a point above the earth's surface a distance r from the center of the earth (a distance $r - R_E$ above the surface), the weight of a body is given by Eq. **(1)** with R_E replaced by r:

$$w = F_g = \frac{Gm_E m}{r^2} \qquad \textbf{(3)}$$

While the earth is an approximately spherically symmetric distribution of mass, it is *not* uniform throughout its volume. To demonstrate this, let's first calculate the average *density*, or mass per unit volume, of the earth. If we assume a spherical earth, the volume is

$$V_E = \tfrac{4}{3}\pi R_E^3 = \tfrac{4}{3}\pi(6.38 \times 10^6\ \text{m})^3 = 1.09 \times 10^{21}\ \text{m}^3$$

Fig. 4 The density of the earth decreases with increasing distance from its center.

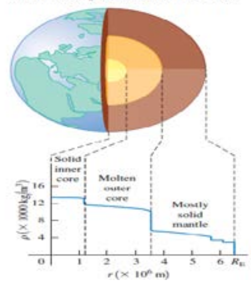
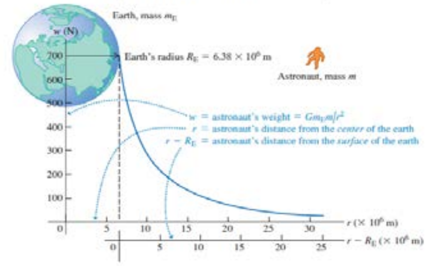


Fig. 3 An astronaut who weighs 700 N at the earth's surface experiences less gravitational attraction when above the surface. The relevant distance r is from the astronaut to the *center* of the earth (*not* from the astronaut to the earth's surface).



The average density ρ (the Greek letter rho) of the earth is the total mass divided by the total volume:

$$\rho = \frac{m_E}{V_E} = \frac{5.97 \times 10^{24}\ \text{kg}}{1.09 \times 10^{21}\ \text{m}^3}$$
$$= 5500\ \text{kg/m}^3 = 5.5\ \text{g/cm}^3$$

Example 13.4 Gravity on Mars

A robotic lander with an earth weight of 3430 N is sent to Mars, which has radius $R_M = 3.40 \times 10^6$ m and mass $m_M = 6.42 \times 10^{23}$ kg (see Appendix F). Find the weight F_g of the lander on the Martian surface and the acceleration there due to gravity, g_M.

SOLUTION

IDENTIFY and SET UP: To find F_g we use Eq. **(1)** replacing m_E and R_E with m_M and R_M. We determine the lander mass m from the lander's earth weight w and then find g_M from $F_g = mg_M$.

EXECUTE: The lander's earth weight is $w = mg$, so

$$m = \frac{w}{g} = \frac{3430\ \text{N}}{9.80\ \text{m/s}^2} = 350\ \text{kg}$$

The mass is the same no matter where the lander is. From Eq. (13.3), the lander's weight on Mars is

$$F_g = \frac{Gm_Mm}{R_M^2}$$
$$= \frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(6.42 \times 10^{23}\ \text{kg})(350\ \text{kg})}{(3.40 \times 10^6\ \text{m})^2}$$
$$= 1.30 \times 10^3\ \text{N}$$

The acceleration due to gravity on Mars is

$$g_M = \frac{F_g}{m} = \frac{1.30 \times 10^3\ \text{N}}{350\ \text{kg}} = 3.7\ \text{m/s}^2$$

Gravitational Potential Energy

$$F_g = -G\frac{m_1 m_2}{r^2}\vec{r}$$

$$W = \int_R^\infty F(r)dr$$

$$W = \int_R^\infty G\frac{mM}{r^2}dr$$

$$W = GmM\int_R^\infty \frac{1}{r^2}dr = GmM\int_R^\infty r^{-2}$$

$$W = U_g = -\frac{GmM}{r}$$

Work is the integral of a Force function with respect to displacement.

Putting in the basic expression for gravitational force

Pulling out the constants and bringing the denominator to the numerator.

The negative sign should not surprise you as we already knew that Work was equal to the negative change in "U" or mgh.

Example 13.5 "From the earth to the moon"

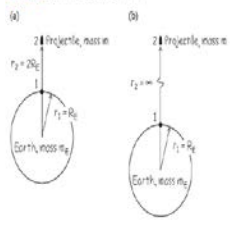
In Jules Verne's 1865 story with this title, three men went to the moon in a shell fired from a giant cannon sunk in the earth in Florida. (a) Find the minimum muzzle speed needed to shoot a shell straight up to a height above the earth equal to the earth's radius R_E. (b) Find the minimum muzzle speed that would allow a shell to escape from the earth completely (the *escape speed*). Neglect air resistance, the earth's rotation, and the gravitational pull of the moon. The earth's radius and mass are $R_E = 6.38 \times 10^6$ m and $m_E = 5.97 \times 10^{24}$ kg.

SOLUTION

IDENTIFY and SET UP: Once the shell leaves the cannon muzzle, only the (conservative) gravitational force does work. Hence we can use conservation of mechanical energy to find the speed at which the shell must leave the muzzle so as to come to a halt (a) at two earth radii from the earth's center and (b) at an infinite distance from earth. The energy-conservation equation is $K_1 + U_1 = K_2 + U_2$, with U given by Eq. (13.9).

Figure 13.12 shows our sketches. Point 1 is at $r_1 = R_E$, where the shell leaves the cannon with speed v_1 (the target variable). Point 2 is where the shell reaches its maximum height; in part

13.12 Our sketches for this problem.



(a) $r_2 = 2R_E$ (Fig. 13.12a), and in part (b) $r_2 = \infty$ (Fig 13.12b). In both cases $v_2 = 0$ and $K_2 = 0$. Let m be the mass of the shell (with passengers).

EXECUTE: (a) We solve the energy-conservation equation for v_1:

$$K_1 + U_1 = K_2 + U_2$$

$$\frac{1}{2}mv_1^2 + \left(-\frac{Gm_E m}{R_E}\right) = 0 + \left(-\frac{Gm_E m}{2R_E}\right)$$

$$v_1 = \sqrt{\frac{Gm_E}{R_E}} = \sqrt{\frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\ \text{kg})}{6.38 \times 10^6\ \text{m}}}$$

$$= 7900\ \text{m/s}\ (= 28{,}400\ \text{km/h} = 17{,}700\ \text{mi/h})$$

(b) Now $r_2 = \infty$ so $U_2 = 0$ (see Fig. 13.11). Since $K_2 = 0$, the total mechanical energy $K_2 + U_2$ is zero in this case. Again we solve the energy-conservation equation for v_1:

$$\frac{1}{2}mv_1^2 + \left(-\frac{Gm_E m}{R_E}\right) = 0 + 0$$

$$v_1 = \sqrt{\frac{2Gm_E}{R_E}}$$

$$= \sqrt{\frac{2(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\ \text{kg})}{6.38 \times 10^6\ \text{m}}}$$

$$= 1.12 \times 10^4\ \text{m/s}\ (= 40{,}200\ \text{km/h} = 25{,}000\ \text{mi/h})$$

EVALUATE: Our result in part (b) doesn't depend on the mass of the shell or the direction of launch. A modern spacecraft launched from Florida must attain essentially the speed found in part (b) to escape the earth; however, before launch it's already moving at 410 m/s to the east because of the earth's rotation. Launching to the east takes advantage of this "free" contribution toward escape speed.

To generalize, the initial speed v_1 needed for a body to escape from the surface of a spherical body of mass M and radius R (ignoring air resistance) is $v_1 = \sqrt{2GM/R}$ (escape speed). This equation yields escape speeds of 5.02×10^3 m/s for Mars, 5.95×10^4 m/s for Jupiter, and 6.18×10^5 m/s for the sun.

Satellites: Circular Orbits

- **A *circular orbit like trajectory is the simplest case. It is also an* important case, since many artificial satellite have nearly circular orbits and the orbits of the planets around the sun are also fairly circular.**
- **The only force acting on a satellite in circular orbit around the earth is the earth's gravitational attraction, which is directed toward the center of the earth and hence toward the center of the orbit as shown in fig.5.**

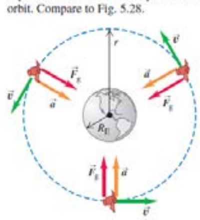
center of the circle. By the law of gravitation, the net force (gravitational force) on the satellite of mass m has magnitude $F_g = Gm_E m/r^2$ and is in the same direction as the acceleration. Newton's second law ($\sum \vec{F} = m\vec{a}$) then tells us that

$$\frac{Gm_E m}{r^2} = \frac{mv^2}{r}$$

Solving this for v, we find

$$v = \sqrt{\frac{Gm_E}{r}} \quad \text{(circular orbit)} \qquad (13.10)$$

Fig.5 The force $\vec{F}_g$ due to the earth's gravitational attraction provides the centripetal acceleration that keeps a satellite in orbit. Compare to Fig. 5.28.



The satellite is in a circular orbit: Its acceleration $\vec{a}$ is always perpendicular to its velocity $\vec{v}$, so its speed v is constant.

We can derive a relationship between the radius r of a circular orbit and the period T, the time for one revolution. The speed v is the distance $2\pi r$ traveled in one revolution, divided by the period:

$$v = \frac{2\pi r}{T} \tag{13.11}$$

To get an expression for T, we solve Eq. (13.11) for T and substitute v from Eq. (13.10):

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{Gm_E}} = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}} \quad \text{(circular orbit)} \tag{13.12}$$

Since the speed v in a circular orbit is determined by Eq. (13.10) for a given orbit radius r, the total mechanical energy $E = K + U$ is determined as well.

$$E = K + U = \tfrac{1}{2}mv^2 + \left(-\frac{Gm_E m}{r}\right) = \tfrac{1}{2}m\left(\frac{Gm_E}{r}\right) - \frac{Gm_E m}{r}$$
$$= -\frac{Gm_E m}{2r} \quad \text{(circular orbit)} \tag{13.13}$$

The total mechanical energy in a circular orbit is negative and equal to one-half the potential energy. Increasing the orbit radius r means increasing the mechanical energy (that is, making E less negative).

Example 13.6 A satellite orbit

You wish to put a 1000-kg satellite into a circular orbit 300 km above the earth's surface. (a) What speed, period, and radial acceleration will it have? (b) How much work must be done to the satellite to put it in orbit? (c) How much additional work would have to be done to make the satellite escape the earth? The earth's radius and mass are given in Example 13.5 (Section 13.3).

SOLUTION

IDENTIFY and SET UP: The satellite is in a circular orbit, so we can use the equations derived in this section. In part (a), we first find the radius r of the satellite's orbit from its altitude. We then calculate the speed v and period T using Eqs. (13.10) and (13.12) and the acceleration from $a_{rad} = v^2/r$. In parts (b) and (c), the work required is the difference between the initial and final mechanical energy, which for a circular orbit is given by Eq. (13.13).

EXECUTE: (a) The radius of the satellite's orbit is $r = 6380\ \text{km} + 300\ \text{km} = 6680\ \text{km} = 6.68 \times 10^6\ \text{m}$. From Eq. (13.10), the orbital speed is

$$v = \sqrt{\frac{Gm_E}{r}} = \sqrt{\frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\ \text{kg})}{6.68 \times 10^6\ \text{m}}}$$
$$= 7720\ \text{m/s}$$

We find the orbital period from Eq. (13.12):

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.68 \times 10^6\ \text{m})}{7720\ \text{m/s}} = 5440\ \text{s} = 90.6\ \text{min}$$

Finally, the radial acceleration is

$$a_{rad} = \frac{v^2}{r} = \frac{(7720\ \text{m/s})^2}{6.68 \times 10^6\ \text{m}} = 8.92\ \text{m/s}^2$$

This is the value of g at a height of 300 km above the earth's surface; it is about 10% less than the value of g at the surface.

(b) The work required is the difference between E_2, the total mechanical energy when the satellite is in orbit, and E_1, the total mechanical energy when the satellite was at rest on the launch pad. From Eq. (13.13), the energy in orbit is

$$E_2 = -\frac{Gm_E m}{2r}$$
$$= -\frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\ \text{kg})(1000\ \text{kg})}{2(6.68 \times 10^6\ \text{m})}$$
$$= -2.98 \times 10^{10}\ \text{J}$$

The satellite's kinetic energy is zero on the launch pad ($r = R_E$), so

$$E_1 = K_1 + U_1 = 0 + \left(-\frac{Gm_E m}{R_E}\right)$$
$$= -\frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\ \text{kg})(1000\ \text{kg})}{6.38 \times 10^6\ \text{m}}$$
$$= -6.24 \times 10^{10}\ \text{J}$$

Hence the work required is

$$W_{required} = E_2 - E_1 = (-2.98 \times 10^{10}\ \text{J}) - (-6.24 \times 10^{10}\ \text{J})$$
$$= 3.26 \times 10^{10}\ \text{J}$$

(c) We saw in part (b) of Example 13.5 that the minimum total mechanical energy for a satellite to escape to infinity is zero. Here, the total mechanical energy in the circular orbit is $E_2 = -2.98 \times 10^{10}\ \text{J}$; to increase this to zero, an amount of work equal to $2.98 \times 10^{10}\ \text{J}$ would have to be done on the satellite, presumably by rocket engines attached to it.

EVALUATE: In part (b) we ignored the satellite's initial kinetic energy (while it was still on the launch pad) due to the rotation of the earth. How much difference does this make? (See Example 13.5 for useful data.)

Kepler's Laws

There are three laws that Johannes Kepler formulated when he was studying the heavens

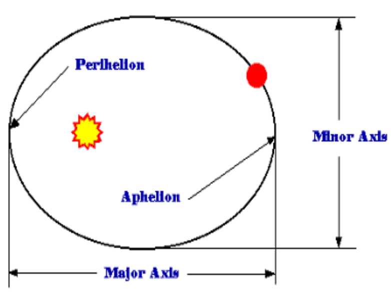
THE LAW OF ORBITS - "*All planets move in elliptical orbits, with the Sun at one focus."*

THE LAW OF AREAS - "*A line that connects a planet to the sun sweeps out equal areas in the plane of the planet's orbit in equal times, that is, the rate dA/dt at which it sweeps out area A is constant."*

THE LAW OF PERIODS - "*The square of the period of any planet is proportional to the cube of the semi major axis of its orbit."*

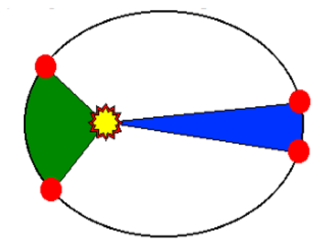
Kepler's 1st law – The Law of Orbits

"*All planets move in elliptical orbits, with the Sun at one focus."*



Kepler's 2nd Law – The Law of Areas

"A line that connects a planet to the sun sweeps out equal areas in the plane of the planet's orbit in equal times, that is, the rate dA/dt at which it sweeps out area A is constant."



Kepler's 3rd Law – The Law of Periods

"The square of the period of any planet is proportional to the cube of the semi major axis of its orbit."

$$T = \frac{2\pi a^{3/2}}{\sqrt{Gm_S}} \quad \text{(elliptical orbit around the sun)} \qquad (13.17)$$

$$F_g = G\frac{mM}{r^2}, F_c = \frac{mv^2}{r}$$

Gravitational forces are centripetal, thus we can set them equal to each other!

$$F_g = F_c$$

$$G\frac{mM}{r^2} = \frac{mv^2}{r}$$

Since we are moving in a circle we can substitute the appropriate velocity formula!

$$G\frac{M}{r} = v^2 \quad v = \frac{2\pi r}{T}$$

The expression in the RED circle derived by setting the centripetal force equal to the gravitational force is called ORBITAL SPEED.

$$G\frac{M}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

Using algebra, you can see that everything in the parenthesis is CONSTANT. Thus the proportionality holds true!

Example 13.8 Kepler's third law

The asteroid Pallas has an orbital period of 4.62 years and an orbital eccentricity of 0.233. Find the semi-major axis of its orbit.

SOLUTION

IDENTIFY and SET UP: This example uses Kepler's third law, which relates the period T and the semi-major axis a for an orbiting object (such as an asteroid). We use Eq. (13.17) to determine a; from Appendix F we have $m_S = 1.99 \times 10^{30}$ kg, and a conversion factor from Appendix E gives $T = (4.62\ \text{yr})(3.156 \times 10^7\ \text{s/yr}) = 1.46 \times 10^8$ s. Note that we don't need the value of the eccentricity.

EXECUTE: From Eq. (13.17), $a^{3/2} = [(Gm_S)^{1/2}T]/2\pi$. To solve for a, we raise both sides of this expression to the $\frac{2}{3}$ power and then substitute the values of G, m_S, and T:

$$a = \left(\frac{Gm_S T^2}{4\pi^2}\right)^{1/3} = 4.15 \times 10^{11}\ \text{m}$$

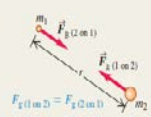
(Plug in the numbers yourself to check.)

EVALUATE: Our result is intermediate between the semi-major axes of Mars and Jupiter (see Appendix F). Most known asteroids orbit in an "asteroid belt" between the orbits of these two planets.

Summary

Newton's law of gravitation: Any two bodies with masses m_1 and m_2, a distance r apart, attract each other with forces inversely proportional to r^2. These forces form an action–reaction pair and obey Newton's third law. When two or more bodies exert gravitational forces on a particular body, the total gravitational force on that individual body is the vector sum of the forces exerted by the other bodies. The gravitational interaction between spherical mass distributions, such as planets or stars, is the same as if all the mass of each distribution were concentrated at the center. (See Examples 13.1–13.3 and 13.10.)

$$F_g = \frac{Gm_1m_2}{r^2} \qquad (13.1)$$



Gravitational force, weight, and gravitational potential energy: The weight w of a body is the total gravitational force exerted on it by all other bodies in the universe. Near the surface of the earth (mass m_E and radius R_E), the weight is essentially equal to the gravitational force of the earth alone. The gravitational potential energy U of two masses m and m_E separated by a distance r is inversely proportional to r. The potential energy is never positive; it is zero only when the two bodies are infinitely far apart. (See Examples 13.4 and 13.5.)

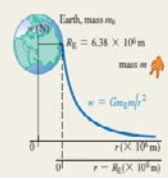
$$w = F_g = \frac{Gm_Em}{R_E^2} \qquad (13.3)$$

(weight at earth's surface)

$$g = \frac{Gm_E}{R_E^2} \qquad (13.4)$$

(acceleration due to gravity at earth's surface)

$$U = -\frac{Gm_Em}{r} \qquad (13.9)$$



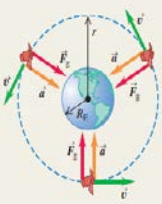
Orbits: When a satellite moves in a circular orbit, the centripetal acceleration is provided by the gravitational attraction of the earth. Kepler's three laws describe the more general case: an elliptical orbit of a planet around the sun or a satellite around a planet. (See Examples 13.6–13.9.)

$$v = \sqrt{\frac{Gm_E}{r}} \tag{13.10}$$

(speed in circular orbit)

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{Gm_E}} = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}} \tag{13.12}$$

(period in circular orbit)



Q13.1 A student wrote: "The only reason an apple falls downward to meet the earth instead of the earth rising upward to meet the apple is that the earth is much more massive and so exerts a much greater pull." Please comment.

Q13.2 A planet makes a circular orbit with period T around a star. If it were to orbit, at the same distance, a star with three times the mass of the original star, would the new period (in terms of T) be (a) $3T$, (b) $T\sqrt{3}$, (c) T, (d) $T/\sqrt{3}$, or (e) $T/3$?

Q13.3 If all planets had the same average density, how would the acceleration due to gravity at the surface of a planet depend on its radius?

Q13.4 Is a pound of butter on the earth the same amount as a pound of butter on Mars? What about a kilogram of butter? Explain.

Fluid Mechanics
(12)

Dr. Hind I. Abdulgafour

INTRODUCTION

Mechanics: **The oldest physical science that deals with both stationary and moving bodies under the influence of forces.**

Statics: **The branch of mechanics that deals with bodies at rest.**

Dynamics: **The branch that deals with bodies in motion.**

Fluid mechanics: **The science that deals with the behavior of fluids at rest (*fluid statics*) or in motion (*fluid dynamics*), and the interaction of fluids with solids or other fluids at the boundaries.**

Fluid dynamics: **Fluid mechanics is also referred to as fluid dynamics by considering fluids at rest as a special case of motion with zero velocity.**

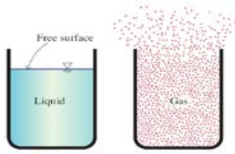
Fluid mechanics deals with liquids and gases in motion or at rest.

2

Fluids

- **At ordinary temperature, matter exists in one of three states**
  - **Solid - has a shape and forms a surface**
  - **Liquid - has no shape but forms a surface**
  - **Gas - has no shape and forms no surface**

- **What do we mean by "fluids"?**
  - **Fluids are "substances that flow"…. "substances that take the shape of the container"**
  - **Atoms and molecules are free to move.**
  - **No long range correlation between positions.**



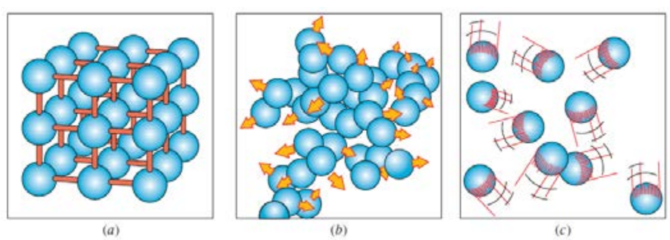
Unlike a liquid, a gas does not form a free surface, and it expands to fill the entire available space.

Intermolecular bonds are strongest in solids and weakest in gases.

Solid: The molecules in a solid are arranged in a pattern that is repeated throughout.

Liquid: In liquids molecules can rotate and translate freely.

Gas: In the gas phase, the molecules are far apart from each other, and molecular ordering is nonexistent.



The arrangement of atoms in different phases: (*a*) molecules are at relatively fixed positions in a solid, (*b*) groups of molecules move about each other in the liquid phase, and (*c*) individual molecules move about at random in the gas phase.

4

Fluids

- **What parameters do we use to describe fluids?**
- **Density: An important property of any material is its density, defined as its mass per unit volume.**

$$\rho = \frac{\Delta m}{\Delta V}$$

units :
$kg/m^3 = 10^{-3}\ g/cm^3$

Densities of Some Common Substances

$\rho(water) = 1.000 \times 10^3\ kg/m^3 = 1.000\ g/cm^3$

$\rho(ice) = 0.917 \times 10^3\ kg/m^3 = 0.917\ g/cm^3$

$\rho(air) = 1.29\ kg/m^3 = 1.29 \times 10^{-3}\ g/cm^3$

$\rho(Hg) = 13.6 \times 10^3\ kg/m^3 = 13.6\ g/cm^3$

Example 1:

Find the mass and weight of the air at 20°C in a living room with a 4.0 m × 5.0 m floor and a ceiling 3.0 m high, and the mass and weight of an equal volume of water.

EXECUTE: We have $V = (4.0\ m)(5.0\ m)(3.0\ m) = 60\ m^3$

$$m_{air} = \rho_{air}V = (1.20\ kg/m^3)(60\ m^3) = 72\ kg$$
$$w_{air} = m_{air}g = (72\ kg)(9.8\ m/s^2) = 700\ N = 160\ lb$$

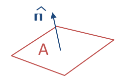
The mass and weight of an equal volume of water are

$$m_{water} = \rho_{water}V = (1000\ kg/m^3)(60\ m^3) = 6.0 \times 10^4\ kg$$
$$w_{water} = m_{water}g = (6.0 \times 10^4\ kg)(9.8\ m/s^2)$$
$$= 5.9 \times 10^5\ N = 1.3 \times 10^5\ lb = 66\ tons$$

Fluids

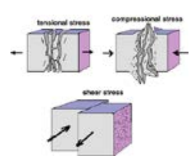
$$p = \frac{\Delta F}{\Delta A}$$

$$\boldsymbol{F} = pA\hat{\boldsymbol{n}}$$



1atm = 1.013 x10^5 Pa
= 1013 mbar
= 760 Torr
= 14.7 lb/m^2 (=PSI)

- **Fluids**
  - **do not sustain shearing or tensile stresses like solids**
  - **they only hold compression stresses from all sides**



- **Forces applied by static fluids**
  - **always perpendicular to objects surface (area)**
  - **depending on the area (surface) of the object, the force is applied in different ways**

- **Pressure**
  - **force applied to a surface area**
  - **Pressure is a scalar, not a vector**

$$P = \frac{F}{A}\left[Pa = \frac{N}{m^2}\right]$$

Example 2:

In the room described in Example 12.1, what is the total downward force on the floor due to an air pressure of 1.00 atm?

EXECUTE: We have $A = (4.0\ \text{m})(5.0\ \text{m}) = 20\ \text{m}^2$, so from Eq. (12.3),

$$F_{\perp} = pA = (1.013 \times 10^5\ \text{N/m}^2)(20\ \text{m}^2)$$
$$= 2.0 \times 10^6\ \text{N} = 4.6 \times 10^5\ \text{lb} = 230\ \text{tons}$$

Pascal's Principle

- **So far we have discovered (using Newton's Laws):**
  - **Pressure depends on depth: $\Delta p = \rho g \Delta y$**
- **Pascal's Principle addresses how a change in pressure is transmitted through a fluid.**

Any change in the pressure applied to an enclosed fluid is transmitted to every portion of the fluid and to the walls of the containing vessel.

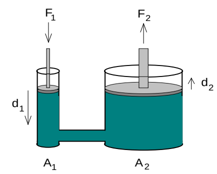
تم نقل أي تغيير في الضغط المطبق على سائل مغلق إلى كل جزء من السائل وإلى جدران الوعاء المحتوي.

- **Pascal's Principle explains the working of hydraulic lifts**

  ← i.e. the application of a small force at one place can result in the creation of a large force in another.

Pascal's Principle

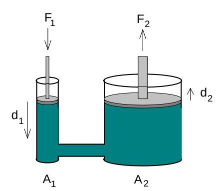
- **Consider the system shown:**
  - **A downward force F_1 is applied to the piston of area A_1.**
  - **This force is transmitted through the liquid to create an upward force F_2.**
  - **Pascal's Principle says that increased pressure from F_1 (F_1/A_1) is transmitted throughout the liquid.**



$$\Rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow \boxed{F_2 = F_1 \frac{A_2}{A_1}}$$

Pascal's Principle

- **Consider F_1 moving through a distance d_1.**
  - **How large is the volume of the liquid displaced?**



$$\Delta V_1 = d_1 A_1$$

$$\Delta V_2 = \Delta V_1 \Rightarrow \boxed{d_2 = d_1 \frac{A_1}{A_2}}$$

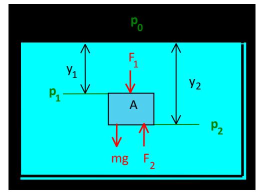
$$\Rightarrow W_2 = F_2 d_2 = F_1 \frac{A_2}{A_1} d_1 \frac{A_1}{A_2} = W_1$$

F_1 equals the work done by F_2
ł "something for nothing".

Pressure vs. Depth

Incompressible Fluids (liquids) السوائل غير قابلة للضغط

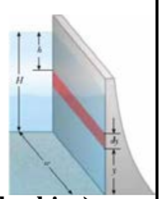
- When the pressure is much less than the bulk modulus of the fluid, we treat the density as constant independent of pressure: incompressible fluid
- For an incompressible fluid, the density is the same everywhere, but the pressure is NOT!



- Consider an imaginary fluid volume (a cube, face area A)
  - ← The sum of all the forces on this volume must be ZERO as it is in equilibrium: $F_2 - F_1 - mg = 0$

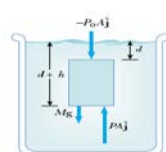
$$F_2 - F_1 = p_2A - p_1A$$
$$mg = \rho(y_2 - y_1)Ag \Rightarrow \boxed{p_2 = p_1 + \rho g(y_2 - y_1)}$$

Pressure varies with Depth



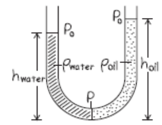
- Divers know that…
  - pressure increases with depth
- In Planes…
  - pressure decreases with height (pressurized cabins)
- Why?
  - Density: mass by unit of volume (volume/mass)
  - Incompressible fluid: the density is the uniform throughout the liquid

$$PA - P_0A - \rho Ahg = 0$$
$$PA - P_0A = \rho Ahg$$
$$P = P_0 + \rho gh$$



Example 3:

A manometer tube is partially filled with water. Oil (which does not mix with water) is poured into the left arm of the tube until the oil–water interface is at the midpoint of the tube as shown. Both arms of the tube are open to the air. Find a relationship between the heights h_{oil} and h_{water}.



$P = P_0 + \rho g h$

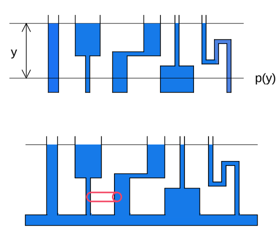
$$p = p_0 + \rho_{water} g h_{water}$$
$$p = p_0 + \rho_{oil} g h_{oil}$$

Since the pressure p at the bottom of the tube is the same for both fluids, we set these two expressions equal to each other and solve for h_{oil} in terms of h_{water}. You can show that the result is

$$h_{oil} = \frac{\rho_{water}}{\rho_{oil}} h_{water}$$

Pressure vs. Depth

- **For a fluid in an open container pressure same at a given depth <u>independent</u> of the container**

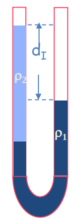


ıect two regions at the same depth.

- **If the pressures were different, fluid would flow in the tube!**
- **However, if fluid did flow, then the system was NOT in equilibrium since no equilibrium system will spontaneously leave equilibrium.**

Example

- *What happens with two fluids??*
  Consider a U tube containing liquids of density r_1 and r_2 as shown:
  - Compare the densities of the liquids:



A) $\rho_1 < \rho_2$ B) $\rho_1 = \rho_2$ C) $\boxed{\rho_1 > \rho_2}$

Archimedes' Principle

Buoyancy is a familiar phenomenon: A body immersed in water seems to weigh less than when it is in air. When the body is less dense than the fluid, it floats. The human body usually floats in water, and a helium-filled balloon floats in air.

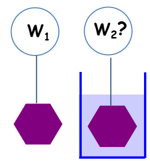
Archimedes's principle: When a body is completely or partially immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.

Archimedes' Principle

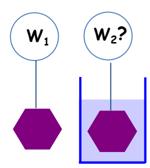
- **Suppose we weigh an object in air (1) and in water (2).**
  - **How do these weights compare?**

Why?

» **Since the pressure at the bottom of the object is greater than that at the top of the object, the water exerts a net upward force, <u>the buoyant force</u>, on the object.**



- **The buoyant force is equal to the difference in the pressures times the area.**



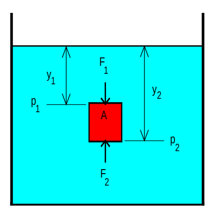
$$F_B = (p_2 - p_1) \cdot A = \rho g(y_2 - y_1)A$$

$$\boxed{F_B = \rho_{liquid} g V_{liquid} = M_{liquid} \cdot g = W_{liquid}}$$

imedes:

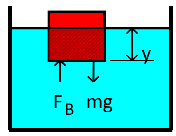
e buoyant force is equal to the weight the liquid displaced.

yant force determines whether an ill sink or float. How does this work?



Sink or Float?

- **The buoyant force is equal to the weight of the liquid that is displaced.**
- **If the buoyant force is larger than the weight of the object, it will float; otherwise it will sink.**



- **We can calculate how much of a floating object will be submerged in the liquid:**

←Object is in equilibrium ➡ $F_B = mg$

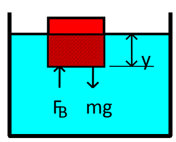
➡ $\rho_{liquid} \cdot g \cdot V_{liquid} = \rho_{object} \cdot g \cdot V_{object}$

➡ $\frac{V_{liquid}}{V_{object}} = \frac{\rho_{object}}{\rho_{liquid}}$

The Tip of the Iceberg

- **What fraction of an iceberg is submerged?**

$$\frac{V_{liquid}}{V_{object}} = \frac{\rho_{object}}{\rho_{liquid}}$$



$$\frac{V_{water}}{V_{ice}} = \frac{\rho_{ice}}{\rho_{water}} = \frac{917\ kg/m^3}{1024\ kg/m^3} = 90\%$$

Example 12.5 Buoyancy

A 15.0-kg solid gold statue is raised from the sea bottom (Fig. 12.13a). What is the tension in the hoisting cable (assumed massless) when the statue is (a) at rest and completely underwater and (b) at rest and completely out of the water?

SOLUTION

IDENTIFY and SET UP: In both cases the statue is in equilibrium and experiences three forces: its weight, the cable tension, and a buoyant force equal in magnitude to the weight of the fluid displaced by the statue (seawater in part (a), air in part (b)). Figure 12.13b shows the free-body diagram for the statue. Our target variables are the values of the tension in seawater (T_{sw}) and in air (T_{air}). We are given the mass m_{statue}, and we can calculate the buoyant force in seawater (B_{sw}) and in air (B_{air}) using Archimedes's principle.

EXECUTE: (a) To find B_{sw}, we first find the statue's volume V using the density of gold from Table 12.1:

$$V = \frac{m_{statue}}{\rho_{gold}} = \frac{15.0\ \text{kg}}{19.3 \times 10^3\ \text{kg/m}^3} = 7.77 \times 10^{-4}\ \text{m}^3$$

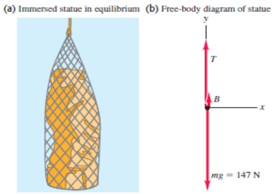
The buoyant force B_{sw} equals the weight of this same volume of seawater. Using Table 12.1 again:

$$\begin{aligned} B_{sw} &= w_{sw} = m_{sw}g = \rho_{sw}Vg \\ &= (1.03 \times 10^3\ \text{kg/m}^3)(7.77 \times 10^{-4}\ \text{m}^3)(9.80\ \text{m/s}^2) \\ &= 7.84\ \text{N} \end{aligned}$$

The statue is at rest, so the net external force acting on it is zero. From Fig. 12.13b,

$$\begin{aligned} \sum F_y &= B_{sw} + T_{sw} + (-m_{statue}g) = 0 \\ T_{sw} &= m_{statue}g - B_{sw} = (15.0\ \text{kg})(9.80\ \text{m/s}^2) - 7.84\ \text{N} \\ &= 147\ \text{N} - 7.84\ \text{N} = 139\ \text{N} \end{aligned}$$

12.13 What is the tension in the cable hoisting the statue?



A spring scale attached to the upper end of the cable will indicate a tension 7.84 N less than the statue's actual weight $m_{statue}g =$ 147 N.

(b) The density of air is about 1.2 kg/m^3, so the buoyant force of air on the statue is

$$\begin{aligned} B_{air} &= \rho_{air}Vg = (1.2\ \text{kg/m}^3)(7.77 \times 10^{-4}\ \text{m}^3)(9.80\ \text{m/s}^2) \\ &= 9.1 \times 10^{-3}\ \text{N} \end{aligned}$$

This is negligible compared to the statue's actual weight $m_{statue}g =$ 147 N. So within the precision of our data, the tension in the cable with the statue in air is $T_{air} = m_{statue}g = 147$ N.

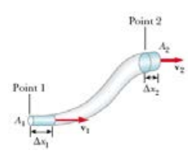
EVALUATE: Note that the buoyant force is proportional to the density of the *fluid* in which the statue is immersed, *not* the density of

Continued

Equation of continuity of fluids

- **The product of the area and the fluid speed at all points along a pipe is constant for an incompressible fluid**

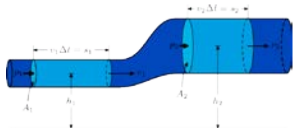
$$A_1 v_1 = A_2 v_2$$



Bernoulli principle

$$W = F\Delta x = PA\Delta x = PV$$

$$W = \Delta K + \Delta U$$



$W = (P_1 - P_2)V$

$$\Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \qquad \Delta U = mgh_2 - mgh_1$$

$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant} \qquad P_1 - P_2 = \rho g(y_2 - y_1) = \rho g h$$

FLUID FLOW
VISCOSITY
POISEUILLE'S LAW

Why do cars need different oils in hot and cold countries?

Why does the engine runs more freely as it heats up?

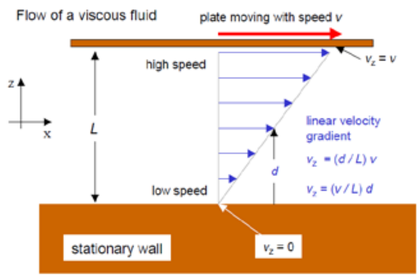
Have you noticed that skin lotions are easier to pour in summer than winter?

Why is honey sticky?

- Viscous fluids tend to cling to a solid surface.
- Syrup and honey are more viscous than water.
- Grease is more viscous than engine oils.
- Liquids are more viscous than gases.
- Lava is an example of a very viscous material.

Coefficient of viscosity

When a fluid (e.g. air) flows past a stationary wall (e.g. table top), the fluid right close to the wall does not move. However, away from the wall the flow speed is not zero. So a velocity gradient exists. This is due to adhesive, cohesive and frictional forces. We find that the magnitude of this gradient (how fast the speed changes with distance) is characteristic of the fluid. This is used to define the coefficient of viscosity η (Greek letter eta).



$F \propto A$ $\quad$ A = area of either plate

$F \propto (v / L)$ $\quad$ (v / L) = velocity gradient

The constant of proportionality for the fluid is called the coefficient of viscosity η

$$F = \eta\, A\, v / L$$

The greater the coefficient of viscosity η, the greater the force required to move the plate at a velocity v.

Viscosity

$\eta = (F / A)(L / v)$ $\quad$ $(N.m^{-2})(m).(m^{-1}.s) = Pa.s$

SI unit for viscosity is Pa.s

A common unit is the poise P where 1 Pa.s = 10 P

1 mPa.s = 10^{-2} P

Fluid	η (mPa.s)
water (0 °C)	1.8
water (20 °C)	1.0
water (100 °C)	0.3
white blood (37 °C)	~4
blood plasma (37 °)	~1.5
engine oil (AE10)	~ 200
air	0.018

Thermodynamic (13)

Dr. Hind I. Abdulgafour

Thermodynamics

- Thermodynamics is the study of the effects of work, heat, and energy on a system
- Thermodynamics is only concerned with macroscopic (large-scale) changes and observations

Thermodynamics

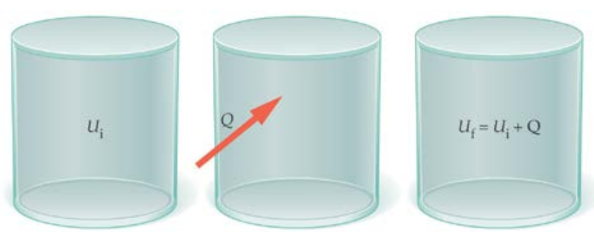
- All of thermodynamics can be expressed in terms of four quantities
  - Temperature (T)
  - Internal Energy (U)
  - Entropy (S)
  - Heat (Q)
- These quantities will be defined as we progress through the lesson

The First Law of Thermodynamics

The first law of thermodynamics is a statement of the conservation of energy.

If a system's volume is constant, and heat is added, its internal energy increases.

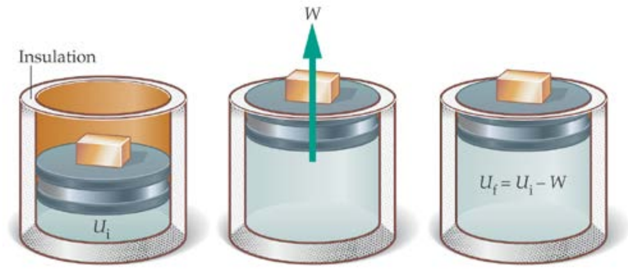
$$\Delta U = U_f - U_i = Q$$



The First Law of Thermodynamics

If a system does work on the external world, and no heat is added, its internal energy decreases.

$$\Delta U = U_f - U_i = -W$$



The First Law of Thermodynamics

Combining these gives the first law of thermodynamics. The change in a system's internal energy is related to the heat Q and the work W as follows:

$$\Delta U = Q - W$$

e signs of Q and W.

TABLE 18–1 Signs of Q and W	
Q positive	System *gains* heat
Q negative	System *loses* heat
W positive	Work done *by* system
W negative	Work done *on* system

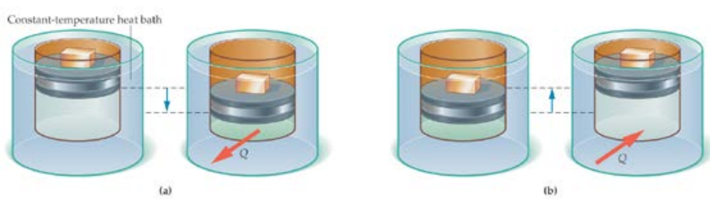
First law of thermodynamic

- The first law of thermodynamics is an extension of the law of conservation of energy
- The change in internal energy of a system is equal to the heat added to the system minus the work done by the system

$$\Delta U = Q - W$$

Thermal Processes

This is an idealized reversible process. The gas is compressed; the temperature is constant, so heat leaves the gas. As the gas expands, it draws heat from the reservoir, returning the gas and the reservoir to their initial states. The piston is assumed frictionless.



Process Terminology

- **Adiabatic – no heat transferred**
- **Isothermal – constant temperature**
- **Isobaric – constant pressure**
- **Isochoric – constant volume**

1- Adiabatic Process

- **An adiabatic process transfers no heat**
  - **therefore $Q = 0$**
- **$\Delta U = Q - W$**
- **When a system expands adiabatically, W is positive (the system does work) so ΔU is negative.**
- **When a system compresses adiabatically, W is negative (work is done on the system) so ΔU is positive.**

2- Isothermal Process

- An isothermal process is a constant temperature process. Any heat flow into or out of the system must be slow enough to maintain thermal equilibrium
- For ideal gases, if ΔT is zero, $\Delta U = 0$
- Therefore, $Q = W$
  - Any energy entering the system (Q) must leave as work (W)

3- Isobaric Process

- An isobaric process is a constant pressure process. ΔU, W, and Q are generally non-zero, but calculating the work done by an ideal gas is straightforward

$$W = P \cdot \Delta V$$

- Water boiling in a saucepan is an example of an isobar process

4-Isochoric Process

- An isochoric process is a constant volume process. When the volume of a system doesn't change, it will do no work on its surroundings. $W = 0$

$$\Delta U = Q$$

- Heating gas in a closed container is an isochoric process

Heat Capacity

- The amount of heat required to raise a certain mass of a material by a certain temperature is called heat capacity

$$Q = mc_x\Delta T$$

- The constant c_x is called the specific heat of substance x, (SI units of J/kg·K)

Heat Capacity of Ideal Gas

- C_V = heat capacity at constant volume

$$C_V = 3/2\ R$$

- C_P = heat capacity at constant pressure

$$C_P = 5/2\ R$$

- For constant volume

$$Q = nC_V\Delta T = \Delta U$$

- The universal gas constant R = 8.314 J/mol·K

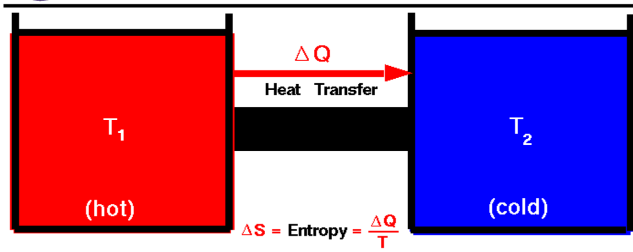
The second Law of thermodynamic

- We observe that heat always flows spontaneously from a warmer object to a cooler one, although the opposite would not violate the conservation of energy. This direction of heat flow is one of the ways of expressing the second law of thermodynamics:

- When objects of different temperatures are brought into thermal contact, the spontaneous flow of heat that results is always from the high temperature object to the low temperature object. Spontaneous heat flow never proceeds in the reverse direction.

Second Law of Thermodynamics

Glenn Research Center



$$\Delta S = \text{Entropy} = \frac{\Delta Q}{T}$$

There exists a useful thermodynamic variable called entropy (S). A natural process that starts in one equilibrium state and ends in another will go in the direction that causes the entropy of the system plus the environment to increase for an irreversible process and to remain constant for a reversible process.

$S_f = S_i$ (reversible) $S_f > S_i$ (irreversible)

Slide courtesy of NASA

Concerning the 2nd Law

- The second law of thermodynamics introduces the notion of entropy (S), a measure of system disorder (messiness).
- U is the quantity of a system's energy, S is the quality of a system's energy.

Direction of a Process

- The 2^{nd} Law helps determine the preferred direction of a process
- A reversible process is one which can change state and then return to the original state
- This is an idealized condition – all real processes are irreversible

Mechanical Waves (14)

Dr. Hind I. Abdulgafour

What are Waves?

Rhythmic disturbances that carry energy without carrying matter

Mechanical Waves

- A mechanical wave is a disturbance in matter that carries energy from one place to another
- Mechanical waves require matter to travel through
- A mechanical wave is created when a source of energy causes a vibration to travel through a medium

Types of Waves

- Mechanical Waves – need matter (or medium) to transfer energy
  - A medium is the substance through which a wave can travel. Ex. Air; water; particles; strings; solids; liquids; gases
- Electromagnetic Waves – DO NOT NEED matter (or medium) to transfer energy
  - They do not need a medium, but they can go through matter (medium), such as air, water, and glass

Mechanical Waves

Waves that need matter (medium) to transfer energy:

Examples: Sound waves, ocean waves, ripples in water, earthquakes, wave of people at a sporting event

Examples of Mediums

- **A medium is the matter through which a wave travels**
- **Solids, liquids and gases can all act as a medium**
- **In a wave pool, the water is the medium**
- **When you shake a rope, the rope is the medium**

Types of mechanical waves

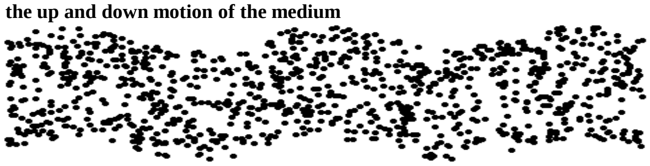
- The three main types of mechanical waves are:
- Transverse waves
- Longitudinal (compressional) waves
- Surface waves

Transverse (Mechanical) Waves

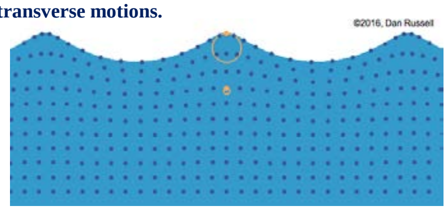
- Energy causes the matter in the medium to move up and down or back and forth at right angles to the direction the wave travels.
- Examples: waves in water

Transverse (Mechanical) Waves

•In a transverse wave the wave causes the medium to vibrate at right angles to the direction in which the wave travels
•The wave carries energy from left to right, in a direction perpendicular to the up and down motion of the medium

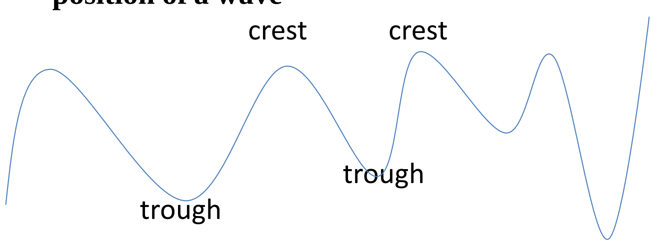


Water waves are an example of waves that involve a combination of both longitudinal and transverse motions.



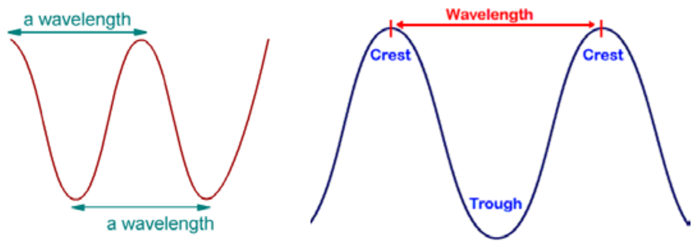
Transverse Waves

- **A crest is the highest point above the rest position of a wave**
- **A trough is the lowest point below the rest position of a wave**



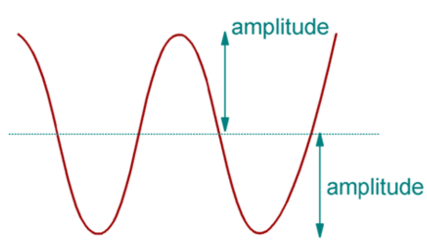
Parts of a Transverse Wave

The wavelength is the horizontal distance, either between the crests or troughs of two consecutive waves.

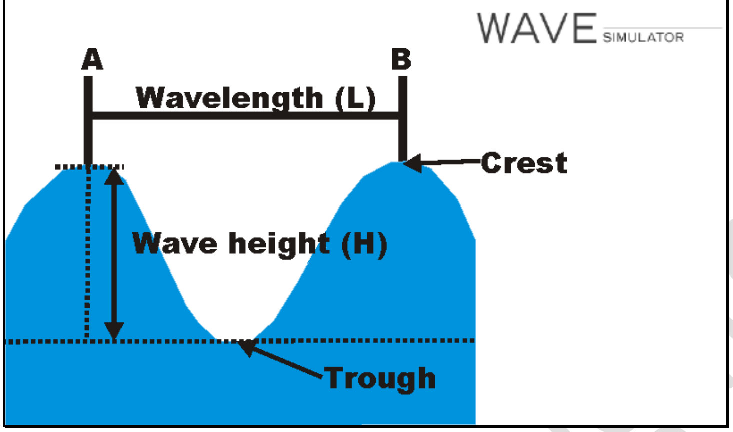


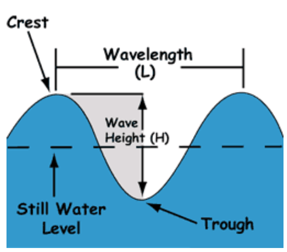
Parts of a Transverse Wave

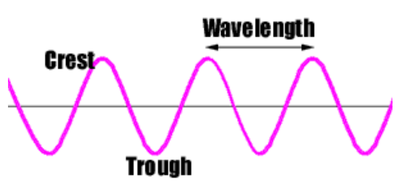
The **amplitude** is the peak (greatest) value (either positive or negative) of a wave. The distance from the undisturbed level to the trough or crest.

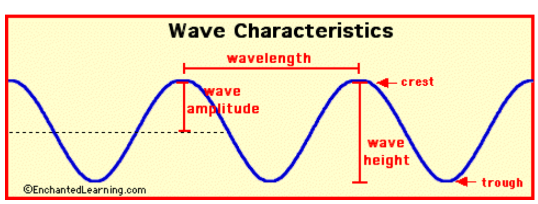


An ocean wave is an example of a mechanical transverse wave





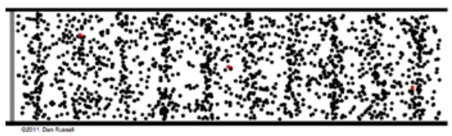


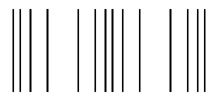


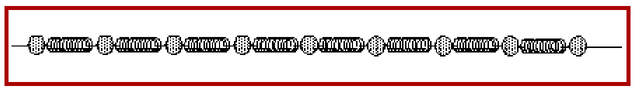
Compressional Wave (longitudinal)

- A mechanical wave in which matter in the medium moves forward and backward along the same direction that the wave travels.
- Ex. Sound waves

A slinky is a good illustration of how a compressional wave moves





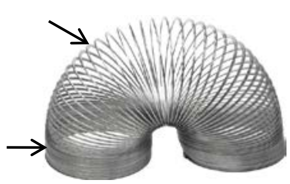


Longitudinal Waves

- A compression is an area where particles in a medium are spaced close together
- A rarefaction is an area where particles in a medium are spread out
- This can be seen in spring toys like slinkies

rarefaction

compression



Longitudinal Waves

- **A longitudinal wave is a wave in which the vibration of the medium is parallel to the direction the wave travels.**
- **The vibration is a back and forth motion that is parallel to or in the same direction that the wave moves.**

Surface Waves

- **A surface wave is a wave that travels along a surface separating two mediums.**
- **An ocean wave travels at the surface between the water and the air.**
- **Waves behave differently near shore, when they topple over themselves they bring the medium and anything floating in it with them.**

Electromagnetic Waves

• Waves that DO NOT NEED matter (medium) to transfer energy.

Examples: radiation, TV & radio waves, X-rays, microwaves, lasers, energy from the sun, visible light.

•Electromagnetic waves are considered transverse waves because they have similar characteristics; therefore, they have the same parts.

Longitudinal waves are waves in which the displacement of the medium is in the same direction as, or the opposite direction to, the direction of propagation of the wave.
Transverse wave is a moving wave that consists of oscillations occurring perpendicular (right angled) to the direction of energy transfer (or the propagation of the wave).

- **Most waves are either longitudinal or transverse.**
- **Sound waves are longitudinal.**
- **But all electromagnetic waves are transverse…**

Do all waves require a medium to travel?

- Almost all waves require a medium to travel.
- Exception: Electromagnetic waves (radio wave, microwave, visible light, ultraviolet, X-ray, gamma rays, etc) can travel through empty space (vacuum)

Wave Relationships

- **Notice from the definitions we can relate the properties of a wave to one another**

$$frequency = {}^{1}\!/_{period}$$

$$velocity = \frac{wavelength}{period} = wavelength \times frequency$$

Wave Relationships

- **Frequency is usually expressed in the unit of Hertz**
  - **This unit is named after a German scientist who studied radio waves**

$$1Hz = \frac{1}{s}$$

  - **For example, if a wave has a period of 10 seconds, the frequency of the wave would be 1/10 Hz, or 0.1 Hz**

- **Note that light is always traveling at the same speed ($c \sim 3 \times 10^8$ m/s)**
  - **Remember:** *velocity* = *wavelength* x *frequency*
    - **If frequency increases, wavelength decreases**
    - **If frequency decreases, wavelength increases**

Example

Sound waves are longitudinal waves in air. The speed of sound depends on temperature; at 20°C it is 344 m/s (1130 ft/s). What is the wavelength of a sound wave in air at 20°C if the frequency is 262 Hz (the approximate frequency of middle C on a piano)?

SOLUTION

IDENTIFY: This problem involves the relationship among wave speed, wavelength, and frequency for a periodic wave. The target variable is the wavelength λ.

SET UP: The wave speed $v = 344$ m/s and the frequency $f = 262$ Hz are given, so we can use the relationship in Eq. (15.1) among v, λ, and f.

EXECUTE: We solve Eq. (15.1) for the target variable λ:

$$\lambda = \frac{v}{f} = \frac{344 \text{ m/s}}{262 \text{ Hz}} = \frac{344 \text{ m/s}}{262 \text{ s}^{-1}} = 1.31 \text{ m}$$

The Linear Wave Equation

- The linear wave equation applies in general to various types of waves.
- For waves on strings, y represents the vertical displacement of the string.
- For sound waves, y corresponds to displacement of air molecules from equilibrium or variations in either the pressure or the density of the gas through which the sound waves are propagating.
- In the case of electromagnetic waves, y corresponds to electric or magnetic field components.

Mathematical Representation of Waves

Let $y = y(x,t)$ be a mathematical representation of the wave (a disturbance); it is a function of both position x and time t. The evolution of the wave (disturbance) in time and space is governed by an equation of the following form:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \qquad \text{(Wave Eqn.)}$$

Often, waves solving the Wave Eqn. above are expressed in terms of sines and/or cosines. Below are important results and relations pertaining to *sinusoidal* waves:

- **Traveling-Wave solutions** to Wave Equation:

$$y(x,t) = A\sin(kx - \omega t) \quad \text{(wave travels along } +x \text{ direction)}$$
$$y(x,t) = A\sin(kx + \omega t) \quad \text{(wave travels along } -x \text{ direction),}$$

provided that $\omega/k = v$. (This is seen by plugging the above solutions directly into the Wave Equation and differentiating; you should do this!) The parameter v is called the phase velocity while A is the amplitude of the wave; the parameters ω and k are defined further below.

- **Standing-Wave solution** to Wave Equation:

$$y(x,t) = A(\sin kx)(\sin \omega t)$$

again provided that $\omega/k = v$.

- Wavelength (λ) and Wavenumber (k):

$$k \equiv \frac{2\pi}{\lambda}$$

- Period (T), Frequency (f), and Angular Frequency (ω):

$$f \equiv \frac{1}{T} \equiv \frac{\omega}{2\pi}$$

- Relations among frequency, wavelength, and phase velocity:

$$v = \frac{\Delta x}{\Delta t} = \frac{\lambda}{T} = \lambda f = \frac{\omega}{k}$$

Wave speed

- Wave speed depends on the properties of the medium

$$v=\sqrt{\frac{\text{Magnitude of restoring force}}{\text{Inertia of the medium}}}$$

e.g. wave speed of a rope under tension:

$$v=\sqrt{\frac{F}{\mu}}; \quad F = tension, \quad \mu = \frac{mass}{length} = \frac{kg}{m}$$

check unit: $F=kg\frac{m}{s^2}$

$$\sqrt{\frac{F}{\mu}} = \sqrt{\frac{kgm}{s^2}\bullet\frac{m}{kg}} = \frac{m}{s}$$

Other examples:
(1) Sound velocity depends on the medium: air vs. solid (both magnitude of restoring force and inertia are different in these two media).
(2) Longitudinal earthquake waves (P-wave) is faster than the transverse earthquake wave (S-wave) - the magnitude of compressional restoring force (P-wave) is greater than the shear restoring force (S-wave).

- Traveling wave pulse - generated by a pulsed driving force
- Traveling periodic wave (harmonic wave) - generated by a continuous driving force

Mathematically (the amplitude of) a travelling is described by:

$\vec{A}(x,t) = f(x - vt)\hat{n}$

(a)The function f describes the shape of the wave

(e.g. for periodic wave, it may be cosine: $\vec{A}(x,t) = A_{max}\hat{n}\cos(x - vt)$)

(b)The dependence on **x** and t in this form (**x**-vt) signifies that it is a travelling wave along the **x**-direction;

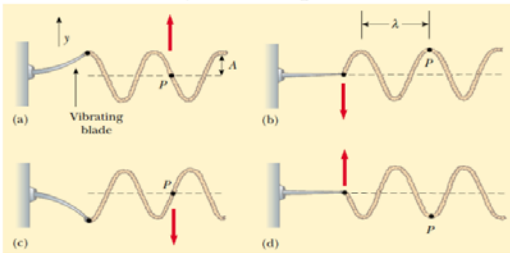
v = positive => travelling in positive **x**-direction

v=negative => travelling in negative **x**-direction

(c) The direction of the amplitude is denoted by the unit vector n̂.

Sinusoidal Wave on Strings

- **Each particle of the string, such as that at *P*, oscillates vertically with simple harmonic motion.**



Sinusoidal Wave on Strings

- **Note that although each segment oscillates in the y direction, the wave travels in the x direction with a speed v. Of course, this is the definition of a transverse wave.**

$$y = A\sin(kx - \omega t)$$

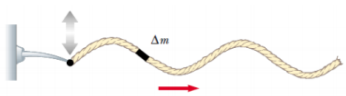
$$v_{y,\max} = \omega A$$

$$a_{y,\max} = \omega^2 A$$

$$v_y = \left.\frac{dy}{dt}\right]_{x\,=\,\text{constant}} = \frac{\partial y}{\partial t} = -\omega A\cos(kx - \omega t)$$

$$a_y = \left.\frac{dv_y}{dt}\right]_{x\,=\,\text{constant}} = \frac{\partial v_y}{\partial t} = -\omega^2 A\sin(kx - \omega t)$$

Rate of Energy Transfer



$$\Delta U = \tfrac{1}{2}(\Delta m)\,\omega^2 y^2$$
$$= \tfrac{1}{2}(\mu\Delta x)\,\omega^2 y^2$$

$$dU = \tfrac{1}{2}\mu\omega^2[A\sin(kx - \omega t)]^2\,dx = \tfrac{1}{2}\mu\omega^2 A^2\sin^2(kx - \omega t)\,dx$$

We have learned that for simple harmonic oscillation, the total energy E = K + U is a constant, i.e.,

$$dE = \frac{1}{2}\mu\omega^2 A^2 dx$$

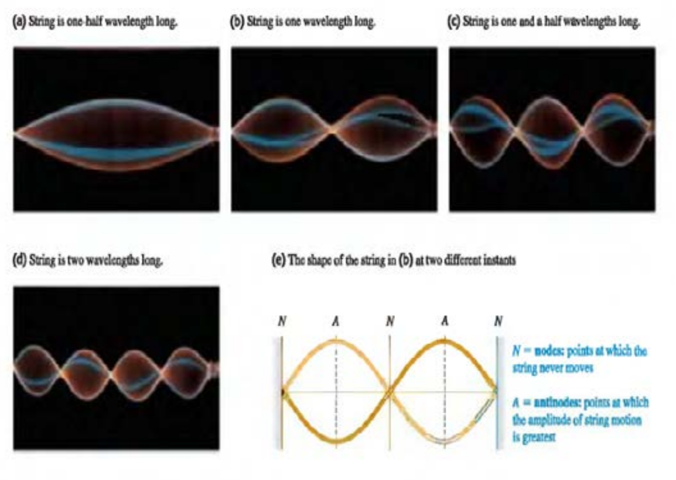
The rate of energy transfer: $P = \dfrac{dE}{dt} = \dfrac{1}{2}\mu\omega^2 A^2 v$

The Principle of Superposition

- The superposition principle states that when two or more waves move in the same linear medium, the net displacement of the medium (that is, the resultant wave) at any point equals the algebraic sum of all the displacements caused by the individual waves.
  - Interference: Same frequency, wavelength, amplitude, direction. Different phase.
  - Standing waves: Same frequency, wavelength, amplitude. Different direction.
  - Beats: Different frequency.

Standing Waves

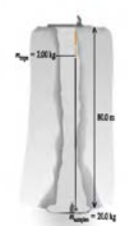
- A standing wave is an oscillation pattern with a stationary outline that results from the superposition of two identical waves traveling in opposite directions.
  - No sense of motion in the direction of propagation of either of the original waves.
  - Every particle of the medium oscillates in simple harmonic motion with the same frequency.
  - Need to distinguish between the amplitude of the individual waves and the amplitude of the simple harmonic motion of the particles of the medium.



Example

One end of a nylon rope is tied to a stationary support at the top of a vertical mine shaft 80.0 m deep (Fig. 15.14). The rope is stretched taut by a box of mineral samples with mass 20.0 kg attached at the lower end. The mass of the rope is 2.00 kg. The geologist at the bottom of the mine signals to his colleague at the top by jerking the rope sideways. (a) What is the speed of a trans-

15.14 Sending signals along a vertical rope using transverse waves.



EXECUTE: (a) The tension in the rope (due to the sample box) is

$$F = m_{sample}g = (20.0\text{ kg})(9.80\text{ m/s}^2) = 196\text{ N}$$

and the mass per unit length of the rope is

$$\mu = \frac{m_{rope}}{L} = \frac{2.00\text{ kg}}{80.0\text{ m}} = 0.0250\text{ kg/m}$$

Hence, from Eq. (15.13), the wave speed is

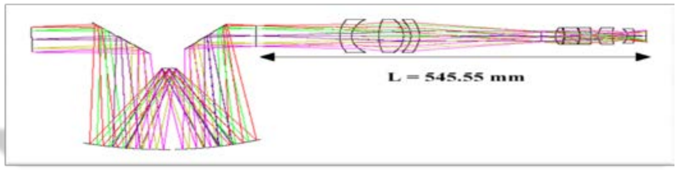
$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{196\text{ N}}{0.0250\text{ kg/m}}} = 88.5\text{ m/s}$$

(b) From Eq. (15.1),

$$\lambda = \frac{v}{f} = \frac{88.5\text{ m/s}}{2.00\text{ s}^{-1}} = 44.3\text{ m}$$

The length of the rope is 80.0 m, so the number of wave cycles in the rope is

$$\frac{80.0\text{ m/s}}{44.3\text{ m/cycle}} = 1.81\text{ cycles}$$

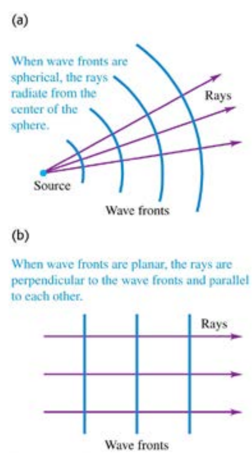


Optics
Light, Mirrors and Lenses
(15)

Dr. Hind I. Abdulgafour

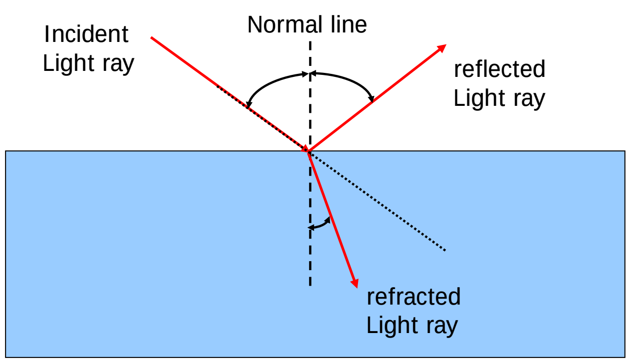
The nature of light

- **Light has properties of *both* waves and particles. The wave model is easier for explaining propagation, but some other behavior requires the particle model.**
- **The *rays* are perpendicular to the *wave fronts*. See Figure 1 at the right.**



Reflection and refraction at a surface

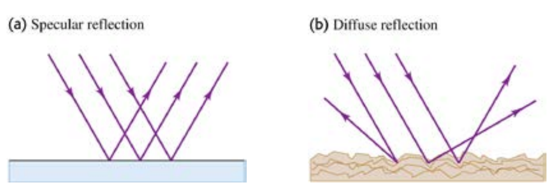
Index of refraction n = c/v > 1



3

Specular and diffuse reflection

- *Specular reflection* **occurs at a very smooth surface (left figure).**
- *Diffuse reflection* **occurs at a rough surface (right figure).**
- **Our primary concern is with specular reflection.**



Laws of reflection and refraction

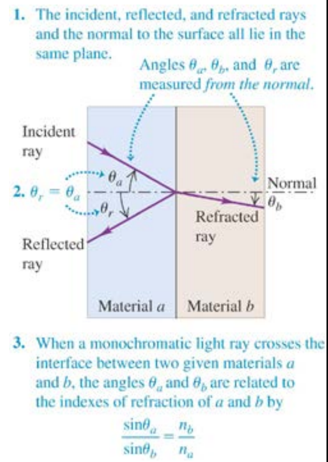
- **The frequency does not change on passing through a surface, but velocity does, and so wavelength.**
- $f = f_0 \Rightarrow v/\lambda = v_0/\lambda_0$
- **The *index of refraction* is**

  $n = c/v > 1$
- **Angles are measured with respect to the *normal*.**
- ***Reflection*: The angle of reflection is equal to the angle of incidence.**
- ***Refraction*: Snell's law applies.**

  In a material

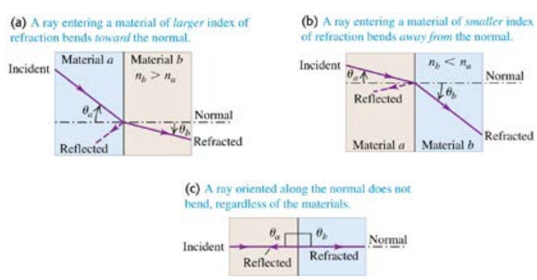
  $\lambda = \lambda_0 / n$

Figure 3 **(right) illustrates the laws of reflection and refraction.**



Reflection and refraction in three cases

- **Figure 4 below shows three important cases:**
  - ✓**If $n_b > n_a$ the refracted ray is bent *toward* the normal.**
  - ✓**If $n_b < n_a$ the refracted ray is bent *away from* the normal.**
  - ✓**A ray oriented along the normal never bends.**



33.8: Reflection and Refraction:

Table 33-1

Some Indexes of Refraction[a]

Medium	Index	Medium	Index
Vacuum	Exactly 1	Typical crown glass	1.52
Air (STP)[b]	1.00029	Sodium chloride	1.54
Water (20°C)	1.33	Polystyrene	1.55
Acetone	1.36	Carbon disulfide	1.63
Ethyl alcohol	1.36	Heavy flint glass	1.65
Sugar solution (30%)	1.38	Sapphire	1.77
Fused quartz	1.46	Heaviest flint glass	1.89
Sugar solution (80%)	1.49	Diamond	2.42

[a]For a wavelength of 589 nm (yellow sodium light).
[b]STP means "standard temperature (0°C) and pressure (1 atm)."

An example of reflection and refraction

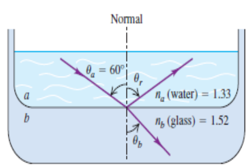
Example 33.1 Reflection and refraction

In Fig. 33.11, material a is water and material b is glass with index of refraction 1.52. The incident ray makes an angle of 60.0° with the normal; find the directions of the reflected and refracted rays.

SOLUTION

IDENTIFY and SET UP: This is a problem in geometric optics. We are given the angle of incidence $\theta_a = 60.0°$ and the indexes of refraction $n_a = 1.33$ and $n_b = 1.52$. We must find the angles of reflection and refraction θ_r and θ_b; to do this we use Eqs. (33.2) and (33.4), respectively. Figure 33.11 shows the rays and angles; n_b is slightly greater than n_a, so by Snell's law [Eq. (33.4)] θ_b is slightly smaller than θ_a, as the figure shows.

33.11 Reflection and refraction of light passing from water to glass.



EXECUTE: According to Eq. (33.2), the angle the reflected ray makes with the normal is the same as that of the incident ray, so $\theta_r = \theta_a = 60.0°$.

To find the direction of the refracted ray we use Snell's law, Eq. (33.4):

$$n_a \sin\theta_a = n_b \sin\theta_b$$

$$\sin\theta_b = \frac{n_a}{n_b}\sin\theta_a = \frac{1.33}{1.52}\sin 60.0° = 0.758$$

$$\theta_b = \arcsin(0.758) = 49.3°$$

EVALUATE: The second material has a larger refractive index than the first, as in Fig. 33.8a. Hence the refracted ray is bent toward the normal and $\theta_b < \theta_a$.

Example 33.2 Index of refraction in the eye

The wavelength of the red light from a helium-neon laser is 633 nm in air but 474 nm in the aqueous humor inside your eyeball. Calculate the index of refraction of the aqueous humor and the speed and frequency of the light in it.

SOLUTION

IDENTIFY and SET UP: The key ideas here are (i) the definition of index of refraction n in terms of the wave speed v in a medium and the speed c in vacuum, and (ii) the relationship between wavelength λ_0 in vacuum and wavelength λ in a medium of index n. We use Eq. (33.1), $n = c/v$; Eq. (33.5), $\lambda = \lambda_0/n$; and $v = \lambda f$.

EXECUTE: The index of refraction of air is very close to unity, so we assume that the wavelength λ_0 in vacuum is the same as that in air, 633 nm. Then from Eq. (33.5),

$$\lambda = \frac{\lambda_0}{n} \qquad n = \frac{\lambda_0}{\lambda} = \frac{633\ \text{nm}}{474\ \text{nm}} = 1.34$$

This is about the same index of refraction as for water. Then, using $n = c/v$ and $v = \lambda f$, we find

$$v = \frac{c}{n} = \frac{3.00 \times 10^8\ \text{m/s}}{1.34} = 2.25 \times 10^8\ \text{m/s}$$

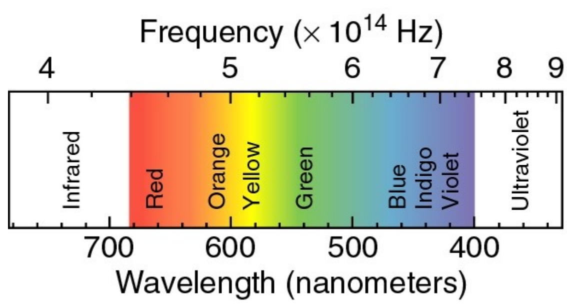
$$f = \frac{v}{\lambda} = \frac{2.25 \times 10^8\ \text{m/s}}{474 \times 10^{-9}\ \text{m}} = 4.74 \times 10^{14}\ \text{Hz}$$

EVALUATE: Note that while the speed and wavelength have different values in air and in the aqueous humor, the *frequency* in air, f_0, is the same as the frequency f in the aqueous humor:

$$f_0 = \frac{c}{\lambda_0} = \frac{3.00 \times 10^8\ \text{m/s}}{633 \times 10^{-9}\ \text{m}} = 4.74 \times 10^{14}\ \text{Hz}$$

When a light wave passes from one material into another, the wave speed and wavelength both change but the wave frequency is unchanged.

VISIBLE LIGHT



Color ➔ WAVELENGTH OR FREQUENCY

Wavelength × Frequency = c (speed of light)
$= 3 \times 10^8$ m/s

10

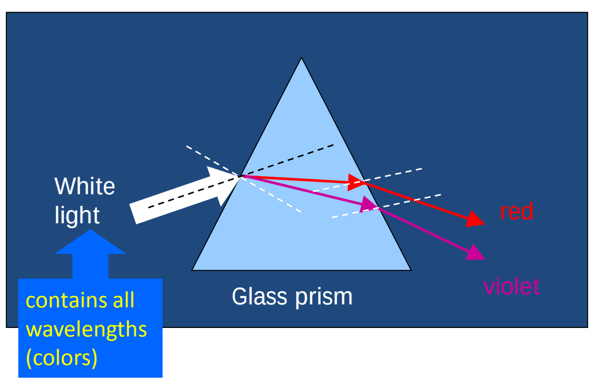
The index of refraction (n) depends of the color (wavelength) of the light

color	Wavelength (nm)	n
Red	660	1.520
orange	610	1.522
yellow	580	1.523
green	550	1.526
blue	470	1.531
violet	410	1.538

1 nanometer (nm) = 1 × 10^{-9} m

11

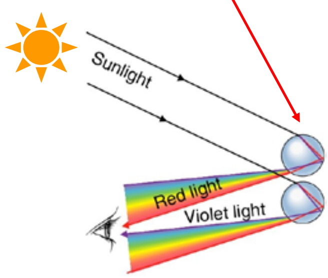
Different colors are refracted (bent) by different amounts this effect is called DISPERSION



12

The rainbow

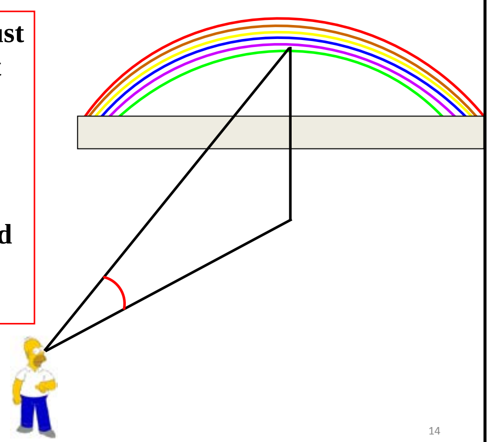
- **Rainbows are caused by dispersion of sunlight from water droplets which act as tiny prisms**



13

Why is it a rain BOW ?

The rain drops must be at just the right angle (42 degrees) between your eyes and the sun to see the rainbow. This angle is maintained along the arc of a circle.



14

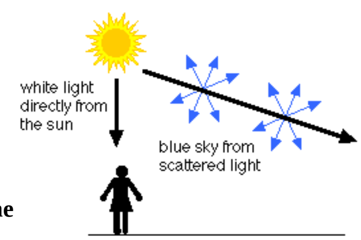
Atmospheric scattering

- Why is the sky blue and sunsets red?
- It is due to the way that sunlight is scattered by the atmosphere (N_2 and O_2)
- *Scattering→ atoms absorb light energy and re-emit it, but not at the same wavelength*
- Sunlight contains a full range of wavelengths in the visible region.

15

Atmospheric scattering: blue sky

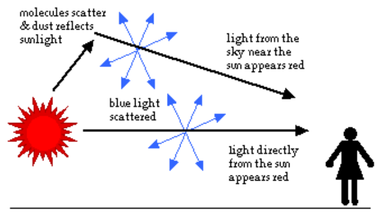
- Short wavelengths are scattered more than long wavelengths.
- Blue light (short) is scattered 10 times more than red light.
- The light that we see in the sky when not looking directly at the sun is scattered blue light.



16

Atmospheric scattering: red sunset

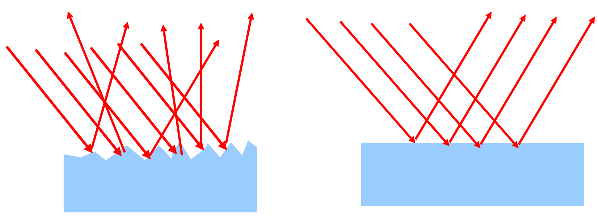
- At sunset, the sun is low on the horizon.
- When looking at the sun it appears red because much of the blue light is scattered out leaving only the red.



17

Mirrors ➔ reflection

- Light does not pass thru metals – it is reflected at the surface
- Two types of reflection: diffuse and specular

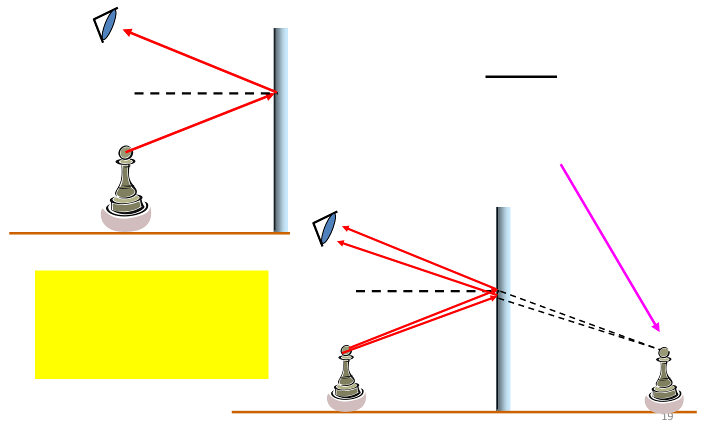


Diffuse reflection: Fuzzy or no image

Specular reflection: Sharp image

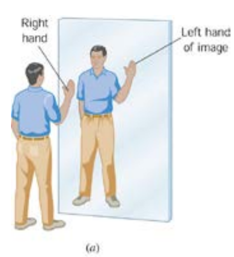
18

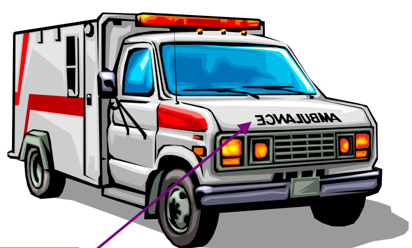
image formation by plane mirrors



19

The image of your right hand is your left hand

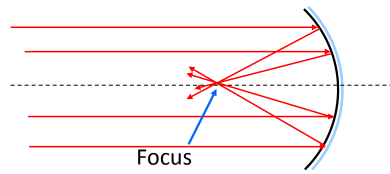




20

Spherical or curved mirrors

Concave mirror



parallel light rays are focused to one point

21

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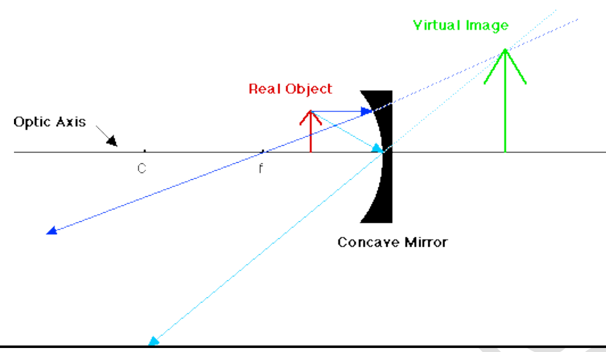
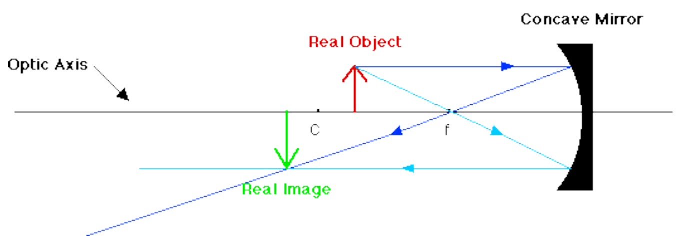
Concave Mirrors

- Curves inward
- May be real or virtual image

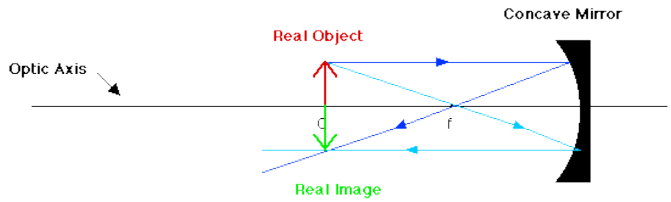
For a real object between f and the mirror, a virtual image is formed behind the mirror. The image is upright and larger than the object.



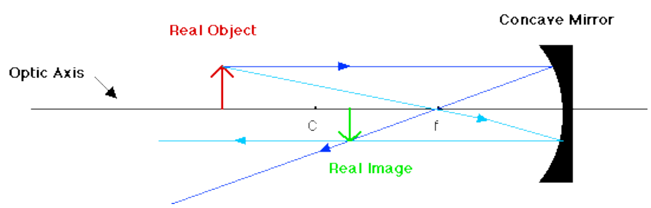
For a real object between C and f, a real image is formed outside of C. The image is inverted and larger than the object.



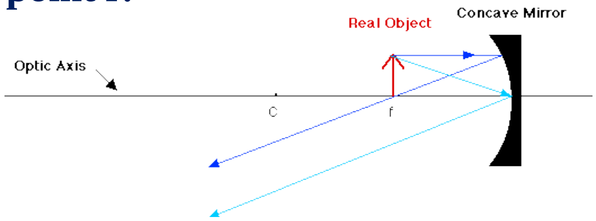
For a real object at C, the real image is formed at C. The image is inverted and the same size as the object.



For a real object close to the mirror but outside of the center of curvature, the real image is formed between C and f. The image is inverted and smaller than the object.



What size image is formed if the real object is placed at the focal point f?



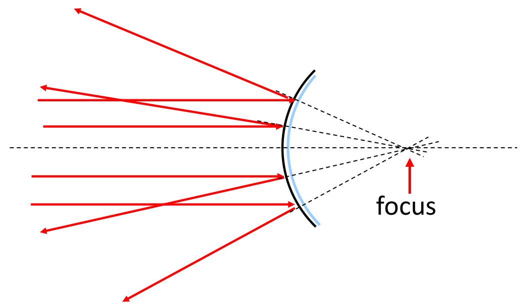
For a real object at f, no image is formed. The reflected rays are parallel and never converge.

Convex Mirrors

- **Curves outward**
- **Reduces images**
- **Virtual images**

–Use: Rear view mirrors, store security…

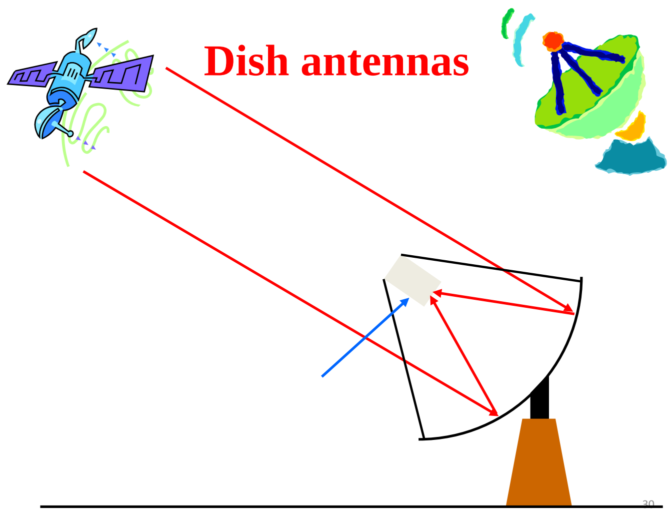
convex mirror



parallel rays diverge from a focus behind the mirror

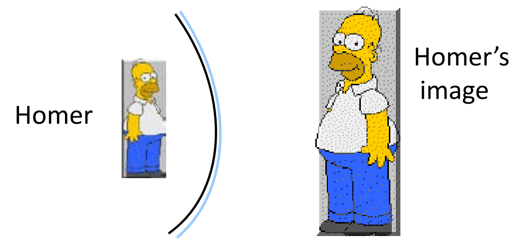
29

Dish antennas



30

Magnifying mirrors



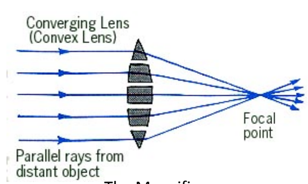
when something placed within the focus of a concave mirror, an enlarged, upright image is formed.
this principle is used in a shaving or makeup mirror

31

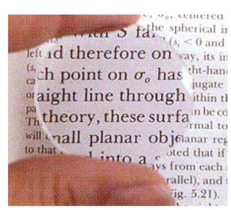
Convex Lenses

Thicker in the center than edges.

- **Lens that converges (brings together) light rays.**
- **Forms real images and virtual images depending on position of the object**



The Magnifier



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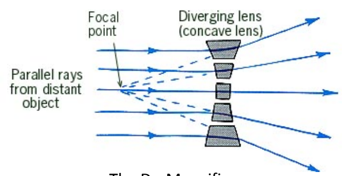
Concave Lenses

- **Lenses that are thicker at the edges and thinner in the center.**
  - **Diverges light rays**
  - **All images are erect and reduced.**



The De-Magnifier