AL-Karkh University of Science
College of Geophysics and Remote Sensing
Department of Geophysics

PHYSICS I

For Geophysics / First Semester

2016-2017

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Lecture (1)
Physics and Physical Measurements

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References

- **FUNDAMENTALS OF PHYSICS. Tenth edition, Halliday & Resnick.**

- **University Physics, with modern physics. Hugh D. Young and Roger A. Freedmen, 13th edition.**

Out lines of studies:

- **Review of physical in measurement and terminology**
- **Vectors**
- **Motion in two and three dimension**
- **Force and work**
- **Energy and motion**
- **Center of mass and linear momentum**
- **Rotation**
- **Rolling torque and angular momentum**
- **Oscillation**

Physics &Measurement

- **What is physics?**
- **Measurements in physics**
- **SI Standards**
- **Accuracy and Precision**

What is physics?

- A way of describing the physical world.
- Physics comes from the Greek “physis” meaning “nature” and the Latin “physica” meaning natural things.
- Physics is understanding the behavior and structure of matter.

Physical in measurement

- **Science and engineering are based on measurements and comparisons.**

- **We discover physics by learning how to measure the quantities involved in physics.**
- **we have two types of physical quantities.**

➢ **Fundamental quantities**
➢ **Derived quantities**

Fundamental

- These are independent
- Eg: Mass, Length, Time ,….
- They have fundamental units like kg → mass
  m → Length
  S → Time

Derived

- These are dependent on Fundamental
- Eg: area, volume, density, force,….
- They have derived units like
  m^2 → area
  m^3 → Volume
  kg/m^3 → Density

The International System of Units (SI)

In this course we shall use the SI (system international) system of units as follows:

- **Many SI derived units are defined in terms of these base units. Table 1-1 shows the units for the three base quantities - length, mass, and time .**

Parameter	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s

SI Unit Prefixes - Part I

Name	Symbol	Factor
deci-	d	10^{-1}
centi-	c	10^{-2}
milli-	m	10^{-3}
micro-	μ	10^{-6}
nano-	n	10^{-9}
pico-	p	10^{-12}
femto-	f	10^{-15}

SI Unit Prefixes - Part II

Name	Symbol	Factor
tera-	T	10^{12}
giga-	G	10^{9}
mega-	M	10^{6}
kilo-	k	10^{3}
hecto-	h	10^{2}
deka-	da	10^{1}

Fundamental SI units

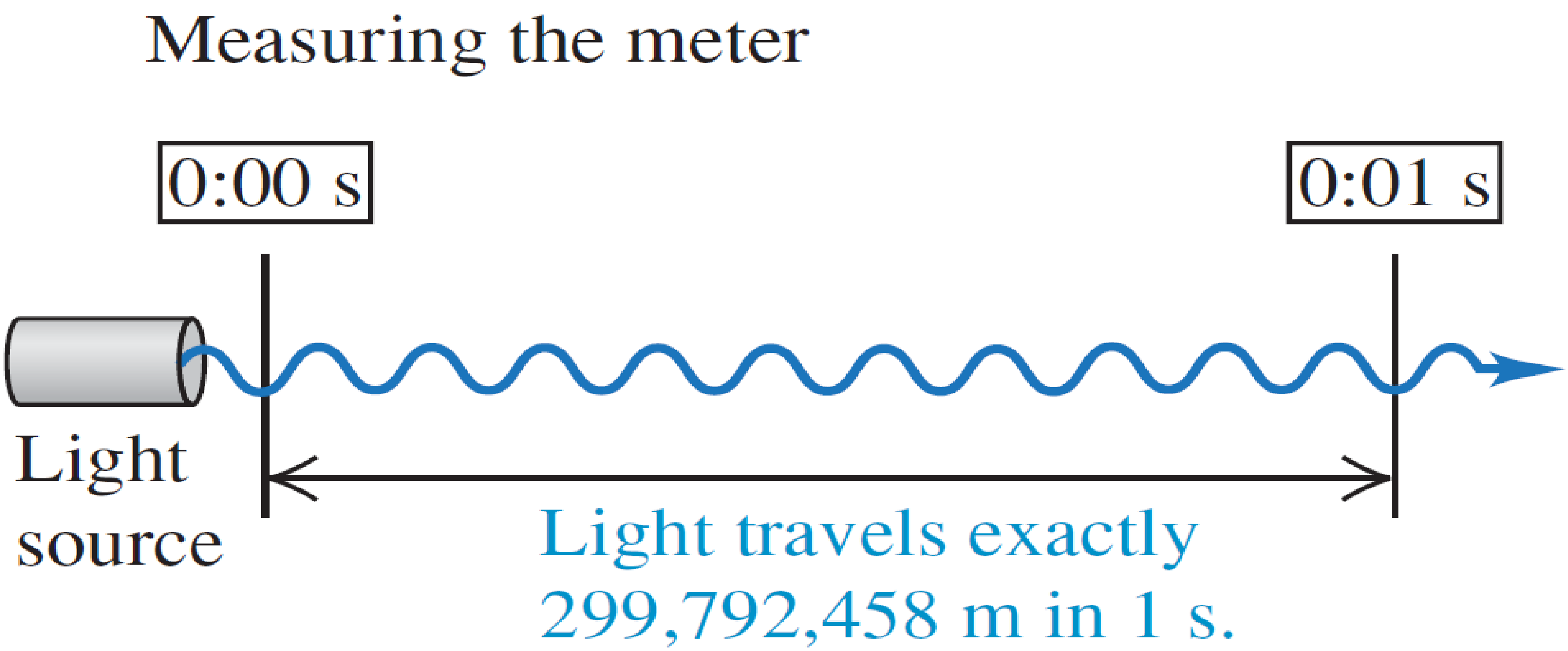
Quantity	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Temperature	kelvin	K
Time	second	s
Amount of Substance	mole	mol
Luminous Intensity	candela	cd
Electric Current	ampere	A

Derived SI Units (examples)

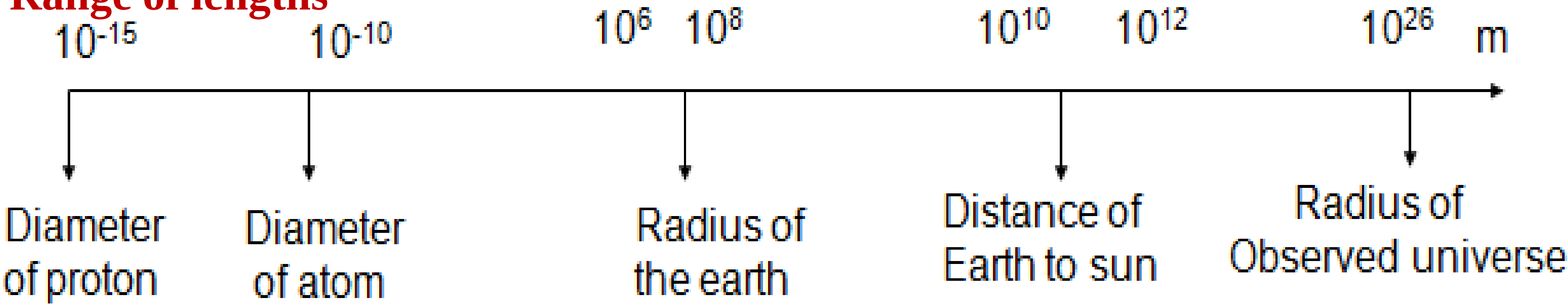
Quantity	unit	Symbol
Volume	cubic meter	m^3
Density	kilograms per cubic meter	kg/m^3
Speed	meter per second	m/s
Newton	$kg\ m/\ s^2$	N
Energy	Joule ($kg\ m^2/s^2$)	J
Pressure	Pascal ($kg/(ms^2)$)	Pa

The Length

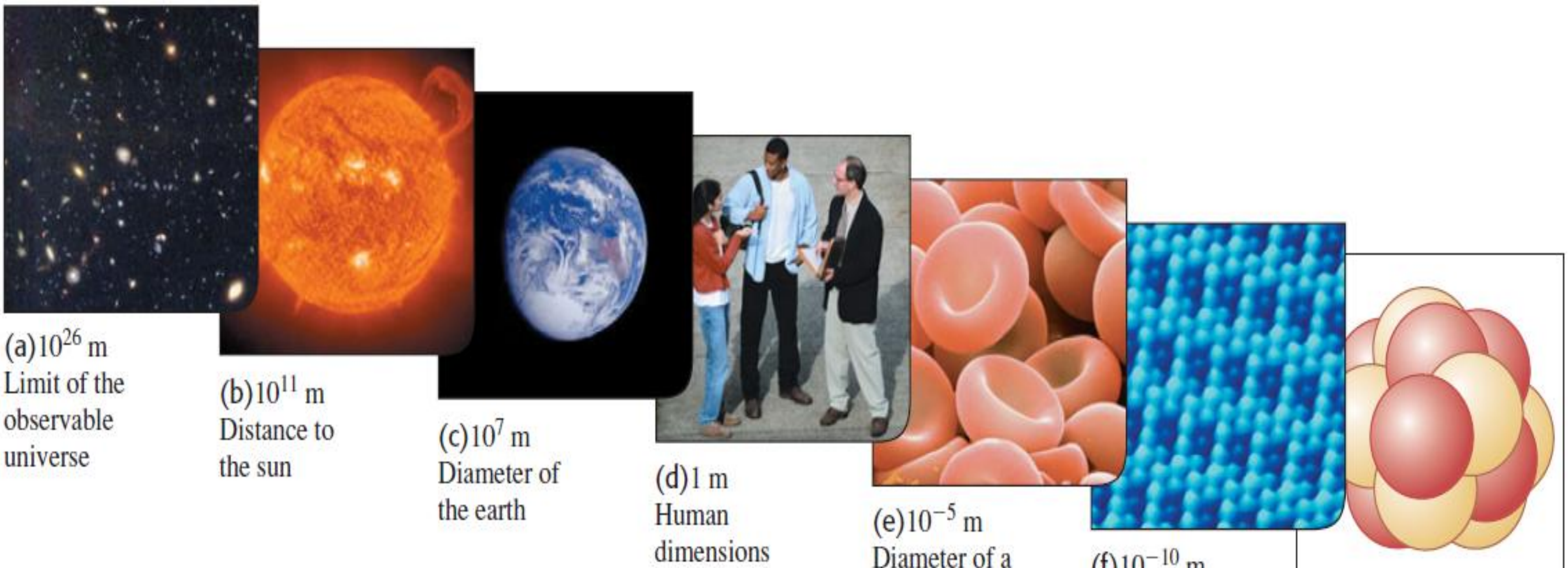
- **The definition of the meter (abbreviated m) is the distance that light travels in vacuum in 1/ 299,792,458 second.**



Range of lengths



Some typical lengths in the universe. (f) is a scanning tunneling microscope image of atoms on a crystal surface; (g) is an artist's impression.



(a) 10^{26} m Limit of the observable universe

(b) 10^{11} m Distance to the sun

(c) 10^{7} m Diameter of the earth

(d) 1 m Human dimensions

(e) 10^{-5} m Diameter of a red blood cell

(f) 10^{-10} m Radius of an atom

(g) 10^{-14} m Radius of an atomic nucleus

The time

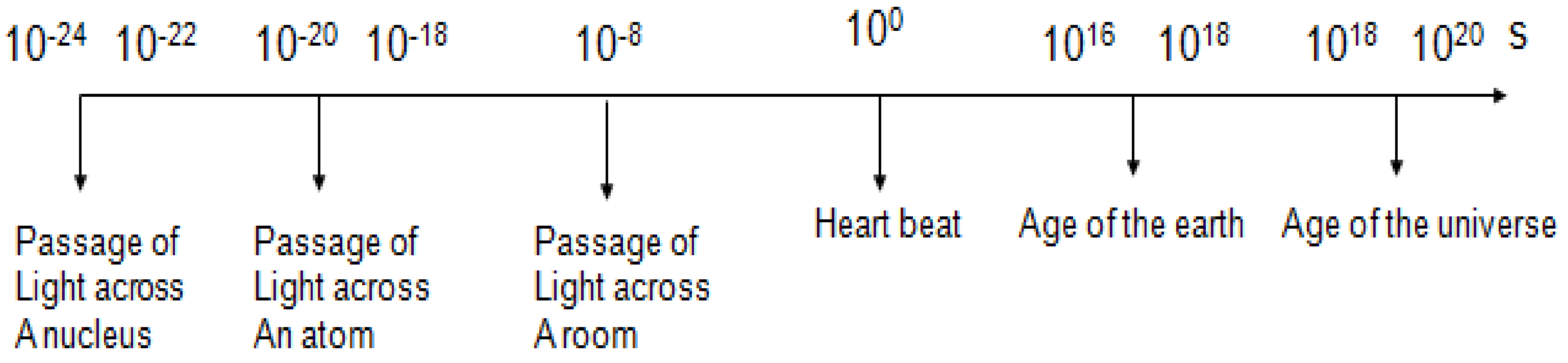
-It is based on an atomic clock, which uses the energy difference between the two lowest energy states of the cesium atom.

الزمن يعتمد على الساعة الذرية وهي تستخدم الفرق في الطاقة بين اوطئ - مستويين طاقيين لذرة السيزيوم

- The SI unit of time is *Second* (abbreviated s) is defined as the time.

-One second is the time taken by 9 192 631 770 oscillations of the light (of a specified wavelength) emitted by a cesium-133 atom.

Range of Time



1day=24hours

1hour=60minutes

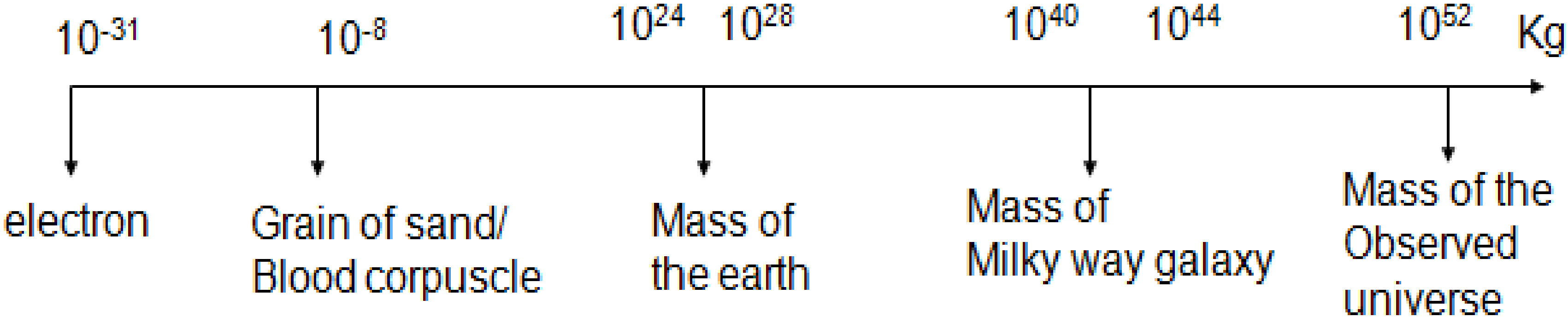
1 minute = 60 seconds

Thus: 1 day 24× 60 ×60 = 86400 s

The mass

- **The SI unit of mass is the *Kilogram*, which is defined as the mass of a specific platinum– iridium alloy cylinder.**

Range of masses



Some Units of Length, Mass, and Time

Length	Mass	Time
1 nanometer = 1 nm = 10^{-9} m (a few times the size of the largest atom)	1 microgram = 1 μg = 10^{-6} g = 10^{-9} kg (mass of a very small dust particle)	1 nanosecond = 1 ns = 10^{-9} s (time for light to travel 0.3 m)
1 micrometer = 1 μm = 10^{-6} m (size of some bacteria and living cells)	1 milligram = 1 mg = 10^{-3} g = 10^{-6} kg (mass of a grain of salt)	1 microsecond = 1 μs = 10^{-6} s (time for space station to move 8 mm)
1 millimeter = 1 mm = 10^{-3} m (diameter of the point of a ballpoint pen)	1 gram = 1 g = 10^{-3} kg (mass of a paper clip)	1 millisecond = 1 ms = 10^{-3} s (time for sound to travel 0.35 m)
1 centimeter = 1 cm = 10^{-2} m (diameter of your little finger)		
1 kilometer = 1 km = 10^{3} m (a 10-minute walk)		

Some Approximate Lengths

Measurement	Length in Meters
Distance to the first galaxies formed	2×10^{26}
Distance to the Andromeda galaxy	2×10^{22}
Distance to the nearby star Proxima Centauri	4×10^{16}
Distance to Pluto	6×10^{12}
Radius of Earth	6×10^{6}
Height of Mt. Everest	9×10^{3}
Thickness of this page	1×10^{-4}
Length of a typical virus	1×10^{-8}
Radius of a hydrogen atom	5×10^{-11}
Radius of a proton	1×10^{-15}

- **Example: Convert 5km to m:**
- **Multiply the original measurement by a conversion factor.**

Conversion between units

- $Kg.m.s^{-2} = 1000*100 = 10^5\ g.cm.s^{-2}$
- $Kg.m^2.s^{-2} = 1000*10000 = 10^7\ g.cm^2.s^{-2}$
- $1\ kwh = 3.6\times10^6\ J$
- $1\ eV = 1.602\times10^{-19}\ J$

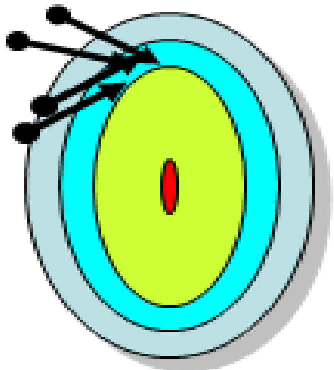
Uncertainty in Measurement
Accuracy & Precision

- **There is no such thing as a perfectly accurate measurement. Each and every measurement has an uncertainty due to:**
- **The observer المراقب**
- **The instrument الاجهزة**
- **The procedure used الاجراءات المتبعة**
- **Accuracy tells how close the measured value is to the true value of the quantity.**
- **Precision tells us to what resolution or limit the quantity is measured.**

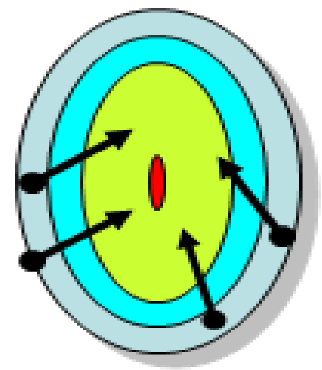
E.g. **suppose the true value of a certain length is 4.859 cm. in one exp, using instrument of precision 0.1 cm, the measured value found to be 4.7 cm, while in another exp using instrument of precision 0.01 cm, the length found to be 4.56 cm.**

- ***Here first measurement has more accurate but less precision.***
- ***Second measurement has more precision but less accurate.***

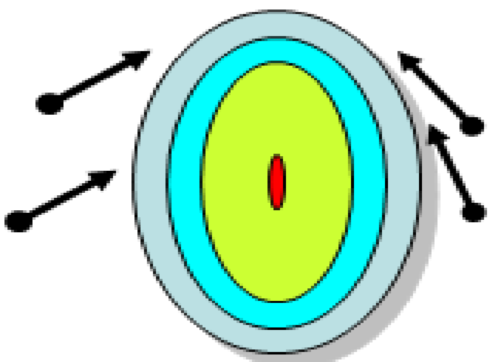
Bull's eye game



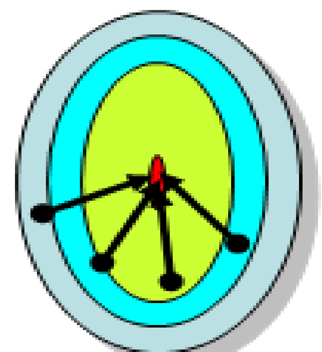
Precise, not accurate



Accurate, not precise



Neither precise nor accurate



Both accurate and precise

Uncertainty or error in measurement

- **The difference in the true value and measured value is called error.**
- **Types of error**
  - **Random error**
  - **Systematic error**

Random error

- **usually random errors are caused by the person doing the experiment.**
- **Causes –**
  - **changes in the experimental conditions like temp, pressure or humidity etc..**
  - **A different person reading the instrument**
- **Systematic error**
- **This error is due to the system or apparatus being used.**
- **Causes –**
  - **An observer consistently making the same mistake**
  - **An instrument with zero error**

Measuring tools

- **Metre scale**
- **Beam balance**
- **Spring balance**
- **Stop watch**

Mathematical representation of uncertainties

- ***Absolute error (Absolute uncertainty)***

It is the magnitude of difference between true value of quantity and the measurement value.

If p is the measured quantity then absolute error expressed as $\pm\Delta p$

- ***Relative error (Fractional uncertainty)***

The ratio of absolute error to the true value of the physical quantity is called relative error.

Here $\frac{\pm\Delta p}{P}$ is the relative error.

- ***Percentage error (Percentage Uncertainty)***

relative error X 100% = $\frac{\pm\Delta p}{P}$ X 100%

- **Example:**

- **mass of a body is (20 ± 0.2) Kg. Here**

- **absolute uncertainty = ± 0.2 kg**
  relative uncertainty = $\frac{0.2}{20}$ = ± 0.01

- **Percentage uncertainty = 0.01 X 100% = 1%**

So mass of a body = 20Kg ± 0.01 or 20 kg ± 1%

Questions:

1- How do you find the units of acceleration?

$h = gt^2/2$ Solve this equation for g

2- Two bodies have masses (20 ± 0.2) Kg and (30 ± 0.4) Kg. what is the total mass of these bodies?

3- Area of a rectangle field is A = l X b , l = (200 ± 5) m and b = (50 ± 2) m. find the percentage error in A?

Lecture (2)

Vector and Scalar

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Vector & Scalar

- **Physics deals with a great many quantities that have both size and direction, and it needs a special mathematical to describe those quantities.**

الكميات الفيزيائية يمكن تقسيمها الى قسمين كميات قياسية ويمكن تحديدها بمقدارها فقط مثل كتلة الجسم 50 كغم

.

اما الكمية المتجهة تحتاج الى تحديد اتجاهها بالاضافة الى مقدارها مثل سرعة الرياح 10km/h غربا.

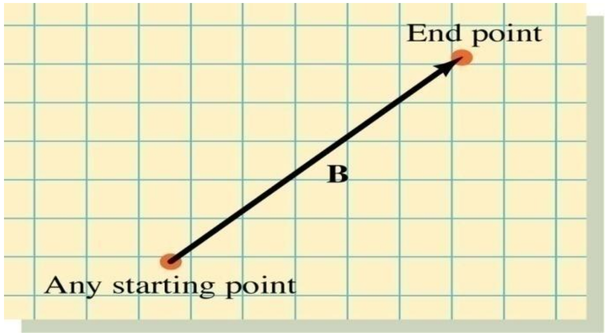
Physical Quantities → Scalars
Physical Quantities → Vectors

➢A scalar is completely described by a number. Some physical quantities, such as time, temperature, mass, and density, can be described completely by a single number with a unit.E.g. mass (m), temperature (T), etc

➢A vector quantity is a quantity that has both a magnitude and a direction and thus can be represented with a vector.

A vector is completely described by :

- Its magnitude
- Its direction



Example: The displacement vector متجه الازاحة

magnitude = 30 paces　　　　direction = northeast

A displacement vector is the simplest vector quantity which represent the change of position. (Similarly, we have velocity vectors and acceleration vectors).

Vector notation

- **A vector is denoted either by an arrow on top or by bold print. Example: The vector of acceleration a is**

written either as: $\vec{a}$ **or as: a Both methods are used**

- **The magnitude of a vector is denoted either by the symbol: | | or by the symbol of the vector written with regular type. Example: the magnitude of the acceleration vector can be written either as:** $|\vec{a}|$ **or as: a**

- **A vector is represented by an arrow whose length is proportional to the vector's magnitude. The arrow has the same direction as the vector**

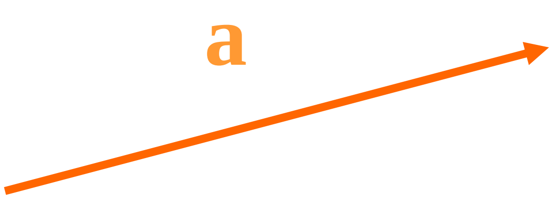


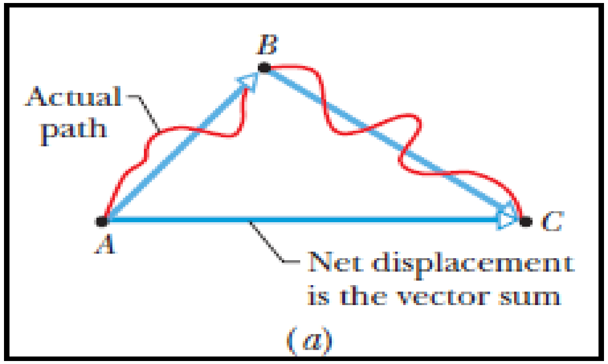
Table 1. explained the scalar and vectors quantities

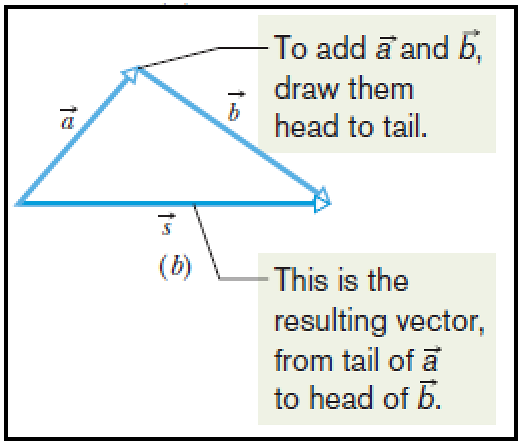
Vector Quantity	Scalar Quantity
Displacement	Length
Velocity	Mass
Force	Speed
Acceleration	Power
Field	Energy
Momentum	Work

Properties of Vectors
1- Adding Vector (geometric method)

- Suppose that, as in the vector diagram of Fig. a, a particle moves from A to B and then later from B to C. We can represent its overall displacement (no matter what its actual path) with two successive displacement vectors, AB and BC.
- The net displacement of these two displacements is a single displacement from A to C.
- We call AC the vector sum (or resultant) of the vectors AB and BC. This sum is not the usual algebraic sum.

Fig.(a) AC is the vector sum of the vectors AB and BC.
(b) The same vectors relabeled.





➢ To add vector $\vec{A}$ to vector $\vec{B}$

- As shown in Fig. c the resultant
- The sum vector extends from the tail of the first vector to the head of

  the last vector

  vector is:

$$R = \vec{A} + \vec{B}$$

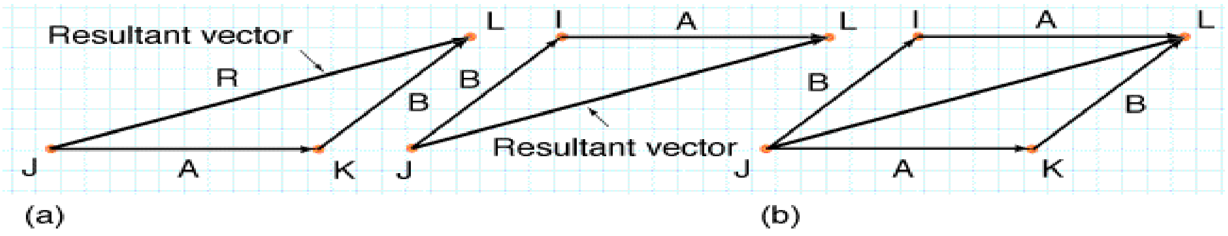


Fig. c shows the resultant

Vector R.

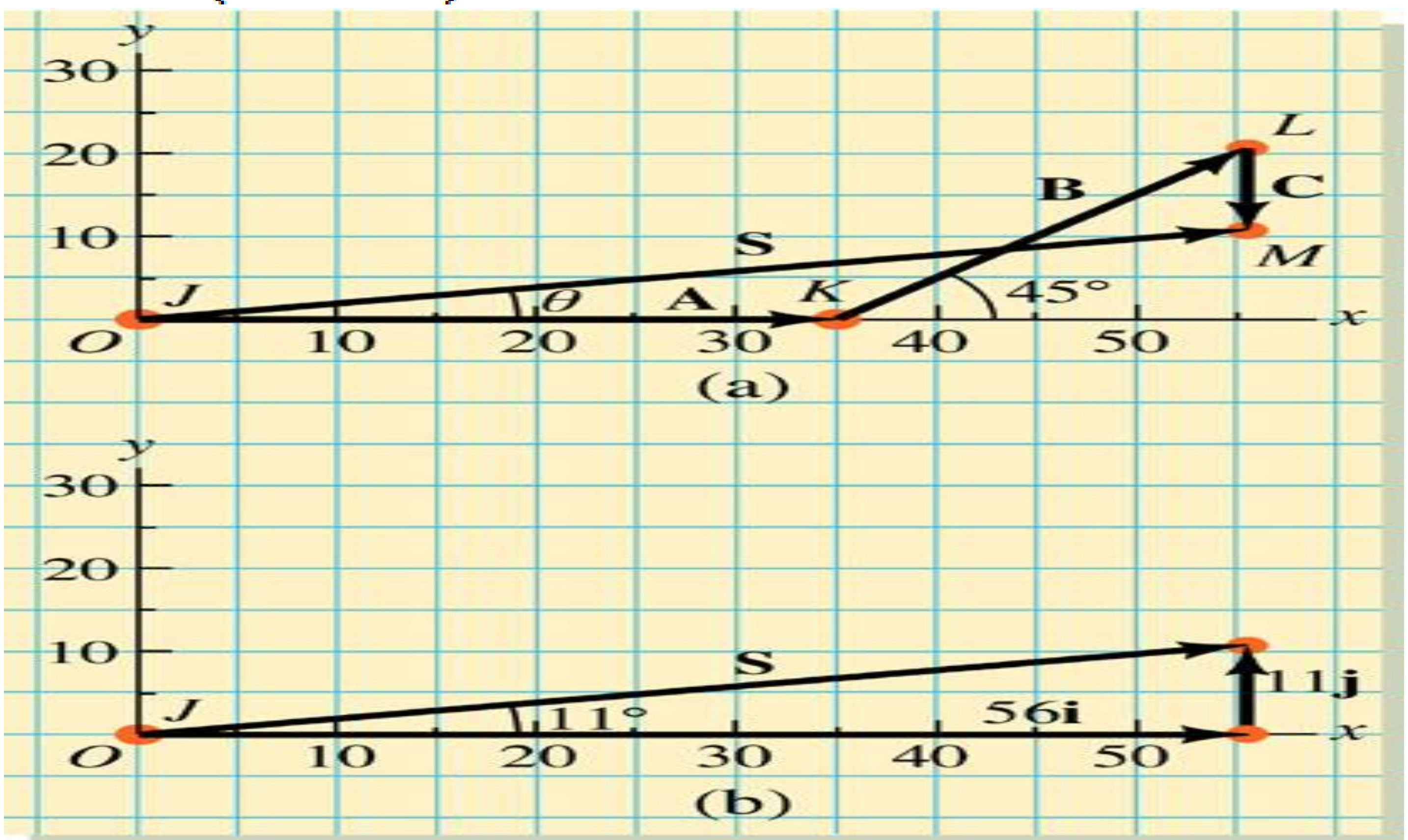
➢ Notice that the vector addition obeys the commutative law i.e. (خاصية التبادل)

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

➢ **Notice that the vector addition obeys the associative law. i.e. (خاصية الاشتراك او الترابط)**

$$\vec{A}+\left(\vec{B}+\vec{C}\right)=\left(\vec{A}+\vec{B}\right)+\vec{C} \qquad ....2$$

$$\vec{R}=\vec{A}+\left(\vec{B}+\vec{C}\right) \qquad ....3$$



2- Vector subtraction

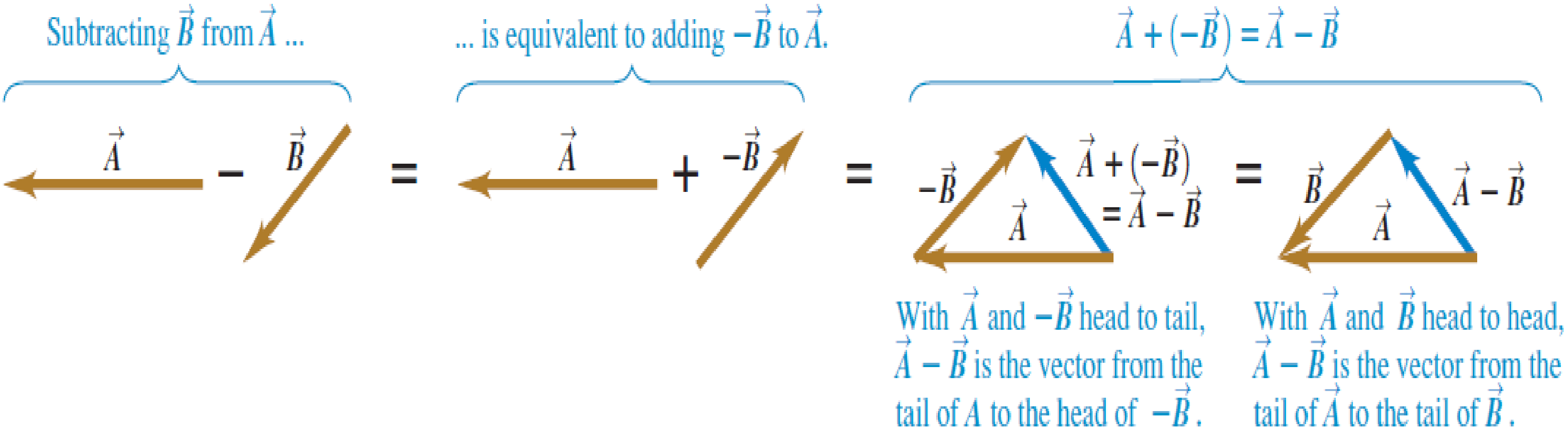
The vector subtraction $\vec{A} - \vec{B}$ is evaluated as the vector subtraction

$$\vec{A} - \vec{B} = \vec{A} + \left(-\vec{B}\right)$$

Where the vector $-\vec{B}$ is the negative vector of $\vec{B}$

$$\vec{B} + \left(-\vec{B}\right) = 0$$

To construct the vector difference $\vec{A} - \vec{B}$, you can either place the tail of $-\vec{B}$ at the head of $\vec{A}$ or place the two vectors $\vec{A}$ and $\vec{B}$ head to head.

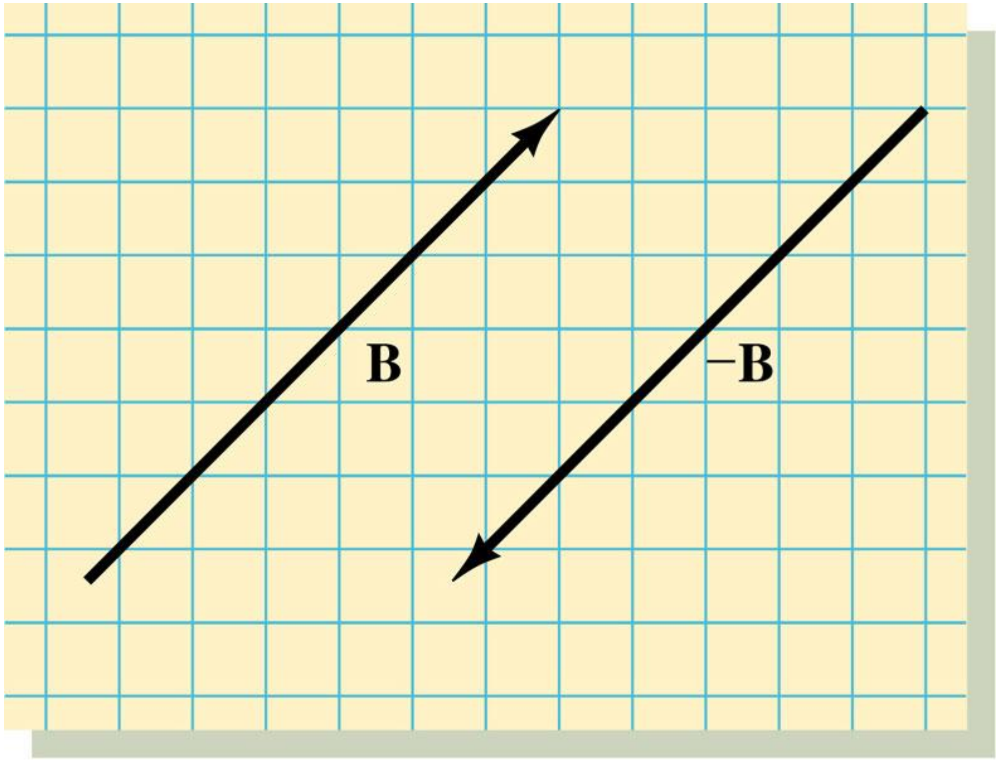


Negative of a vector

We are given vector B and are asked to determine –B

1. Vector –B has the same magnitude as B

2. Vector –B has the opposite direction

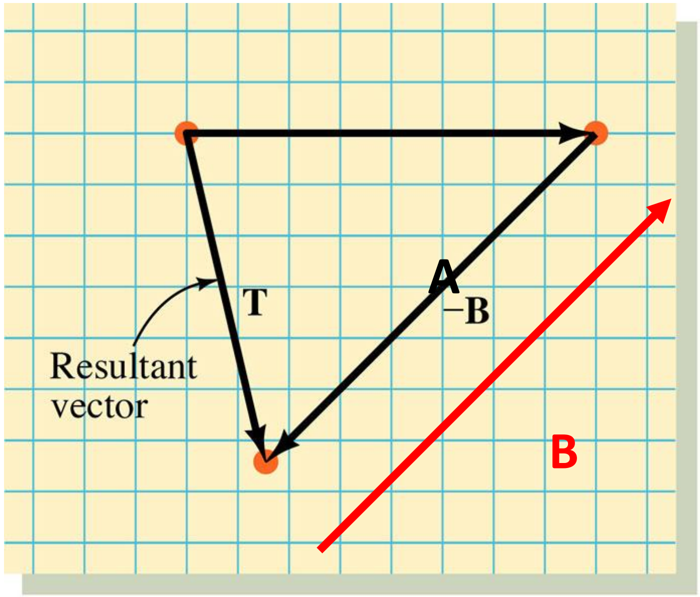


$$\vec{B} + (-\vec{B}) = 0$$

3- Vector Subtraction (geometric method)

We are given vectors A and B are asked to determine T = A – B

1. **Determine –B from B**
2. **Add vector (-B) to vector A**



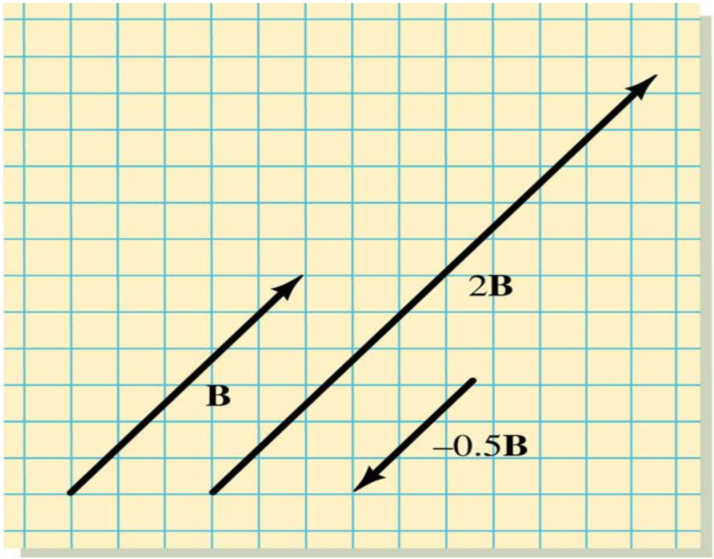
4- Multiplication of a vector

Multiplication of a vector B by a scalar b; determine bB

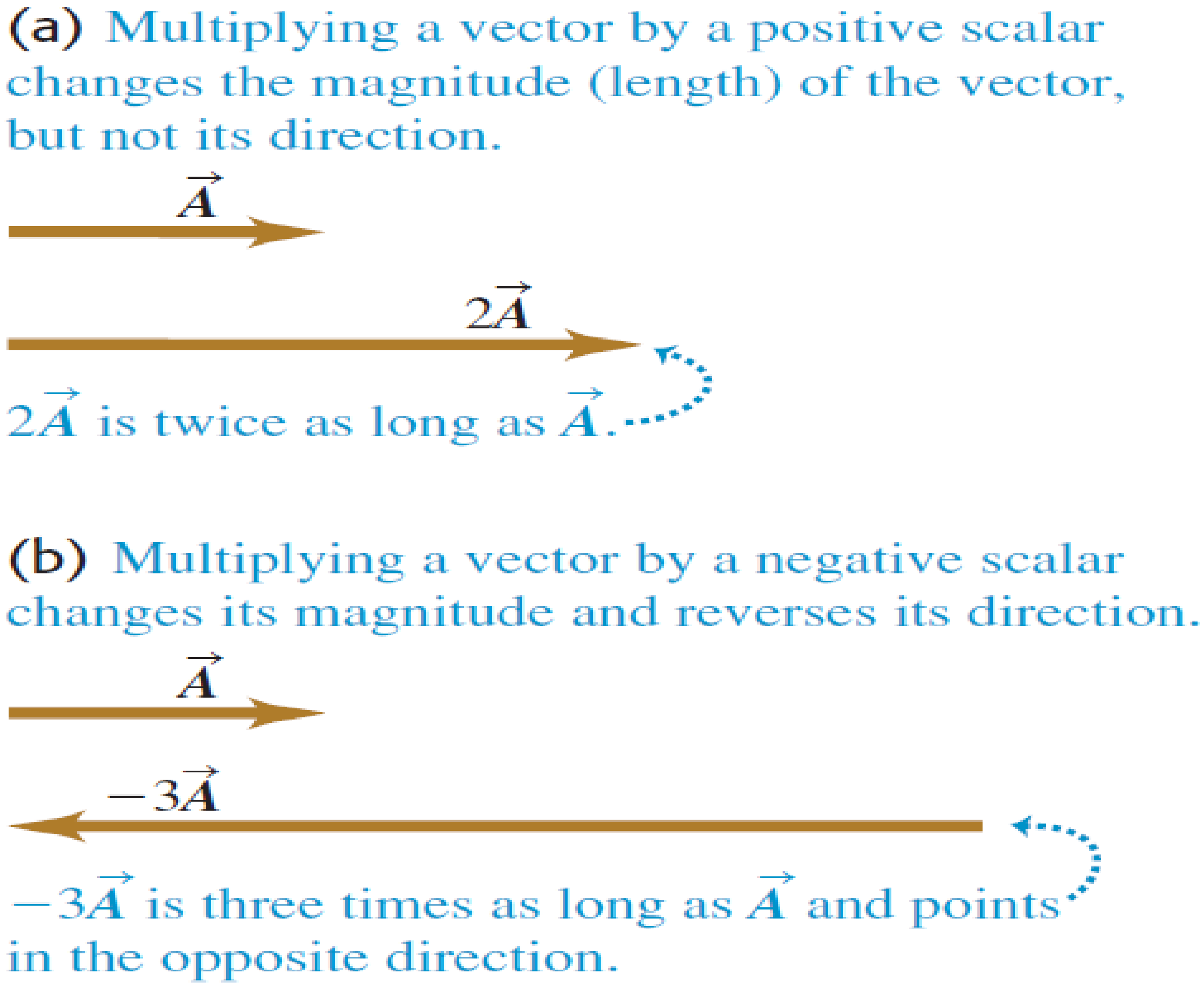
- **The magnitude |bB| = |b||B|**
- **The direction of bB depends on the algebraic sign of b**

If b > 0 then bB has the same direction as B

If b < 0 then bB has the opposite direction of B



Multiplying a vector (a) by a positive scalar and (b) by a negative scalar.



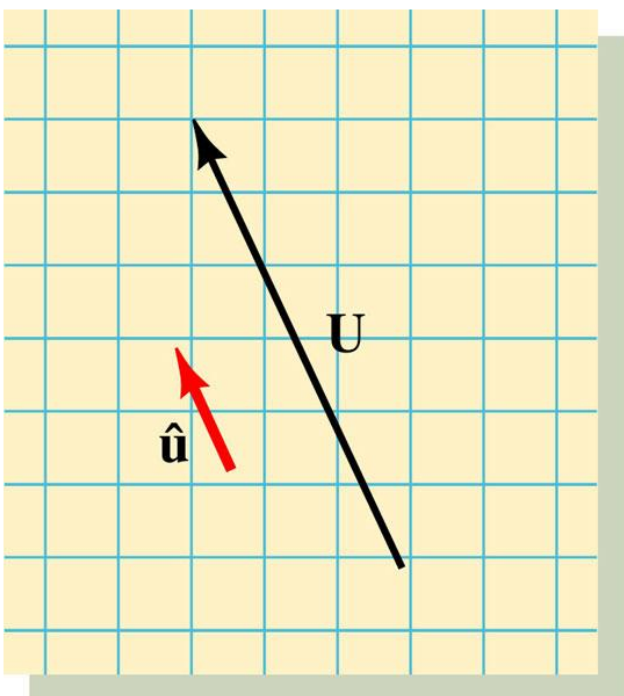
The unite vector

Unit vector is defined as any vector whose magnitude is equal to unity.

We are given a vector U and are asked to determine a unit vector $\hat{u}$ which is parallel to U

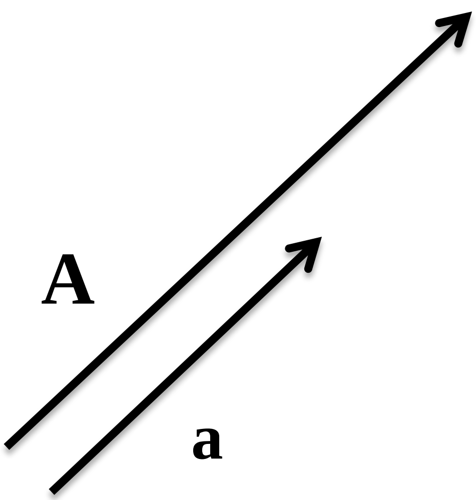
Recipe: $\hat{u} = U/|U|$

Vectors **U** and **$\hat{u}$** are parallel



على سبيل المثال المتجه $\vec{A}$ يمكن تمثيله بمقدار المتجه **A** ضرب متجه الوحدة **a** كالتالي:

$$\vec{A} = aA$$



❖The coordinates system

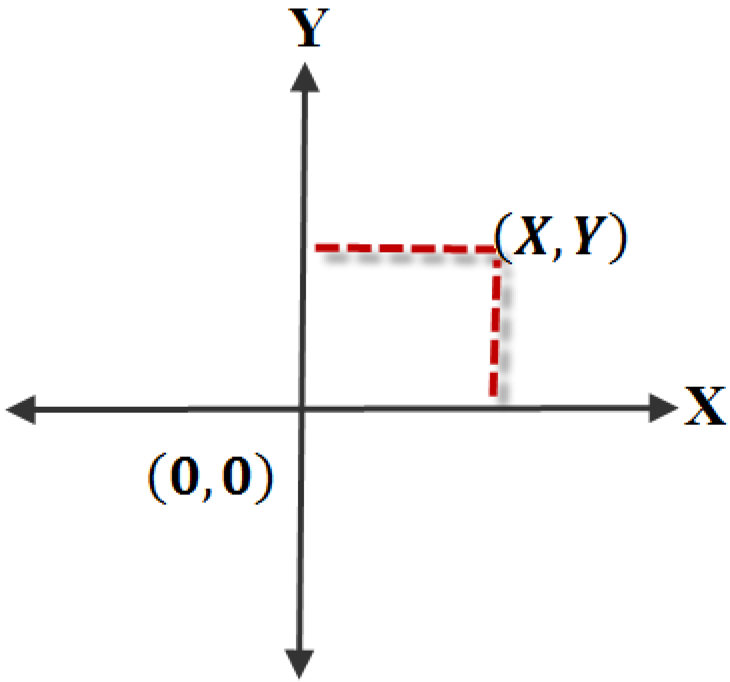
نحتاج في حياتنا الى تحديد موقع جسم ما في الفراغ سواء كان ساكنا ام متحركا ولتحديد موقع هذا الجسم فاننا نستعين **Coordinates** بما يعرف بالاحداثيات.

وهنالك نوعان من الاحداثيات التي سوف نستخدمها في هذه المحاضرة وهما

- ❑ **Rectangular coordinates** الاحداثيات المتعامدة
- ❑ **Polar coordinates** الاحداثيات القطبية

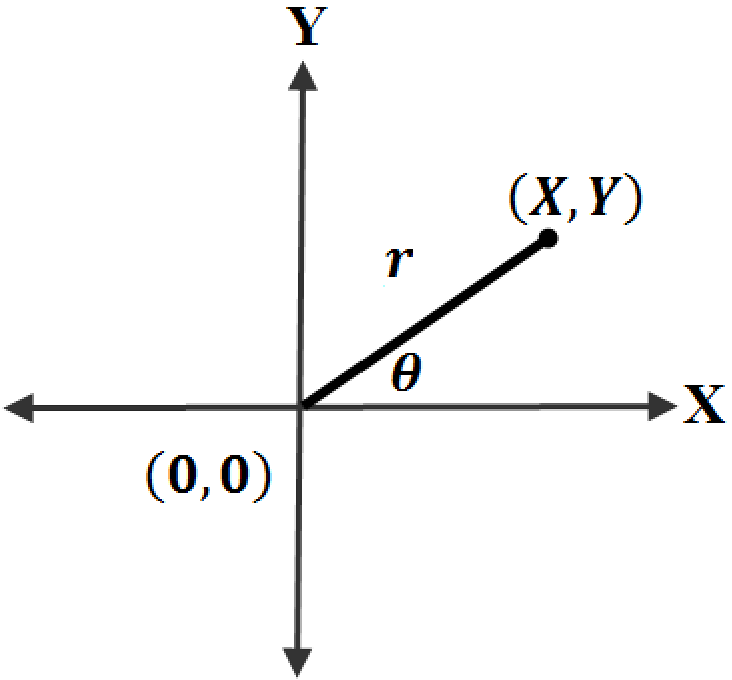
❑The rectangular coordinates

The rectangular coordinate system in two dimensions is shown in Figure. This coordinate system is consist of a fixed reference point (0,0) which called the origin point.



❑The polar coordinates

- **Some times it is more convenient to use the polar coordinate system (r, θ), where r is the distance from the origin to the point of rectangular coordinate (x, y), and θ is the angle between r and the x axis.**



❑ The relation between coordinate

- The relation between the rectangular coordinates (x, y) and the polar coordinates (r, θ) is shown in Figure where.

$$x = r\cos\theta \;\; \dots \dots \dots (1.1)$$

- And

$$y = r\sin\theta \;\; \dots \dots \dots (1.2)$$

- Squaring and adding equations (1.1) and (1.2) we get

$$r = \sqrt{x^2 + y^2 \;\; \dots \dots \dots \dots (1.3)}$$

- Dividing equation (1.1) and (1.2) we get

$$tan\,\theta = \frac{x}{y} \dots \dots \dots . (1.4)$$

➢Example

- **The polar coordinate of a point are r=5.5 m and θ=240º. What are the cartesian coordinate of this point?**

➢Solution

$$x = r\cos\theta = 5.5\cos 240^{\circ} = -2.75\, m$$

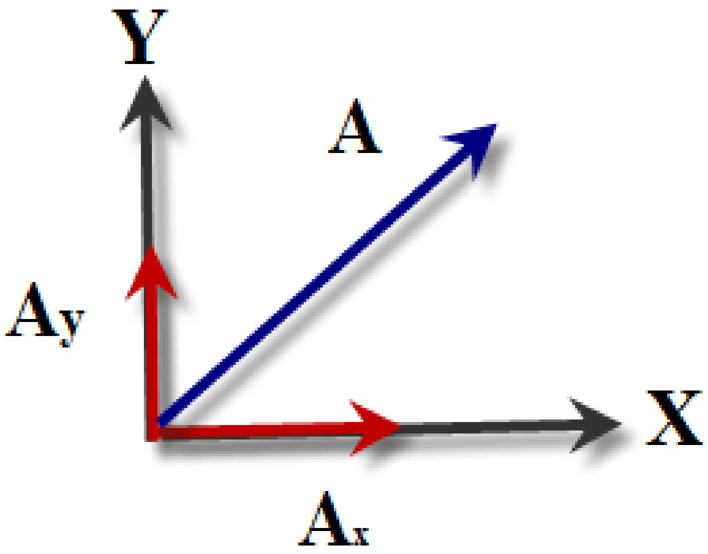
$$y = r\sin\theta = 5.5\sin 240^{\circ} = -4.76\, m$$

❖Component of a vector

Any vector $\vec{A}$ in the xy-plane can be written as a sum of two other vectors, one along the x-axis and the other along the y-axis. These are called the components of $\vec{A}$

$$A_X = A\cos\theta \ldots\ldots\ldots (1.5)$$

$$A_y = A\sin\theta \ldots\ldots\ldots (1.6)$$



عند التعامل مع عدة متجهات فاننا نحتاج الى تحليل كل متجه الى مركباته بالنسبة الى محاور الاسناد (x, y) مما يسهل ايجاد المحصلة بدلا من استخدام الطريقة البيانية لايجاد المحصلة.

The magnitude of the vector $\vec{A}$

$$A = \sqrt{A_x{}^2 + A_y{}^2} \ldots\ldots\ldots\ldots (1.7)$$

The direction of the vector to the x-axis

$$\theta = \tan^{-1}\frac{A_y}{A_x} \ldots\ldots\ldots. (1.8)$$

A vector $\vec{A}$ lying in the xy plane having rectangular components A_x and A_y can be expressed in a unit vector notation.

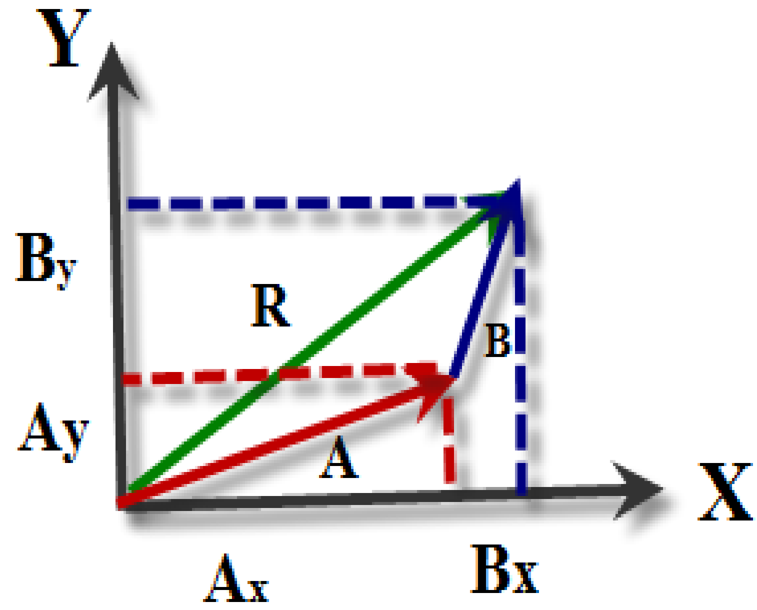
$$\vec{A} = A_x i + A_y j \ldots\ldots\ldots. (1.9)$$

يمكن استخدام طريقة تحليل المتجهات في جمع متجهين $\vec{A}$ و $\vec{B}$ كما في الشكل التالي:

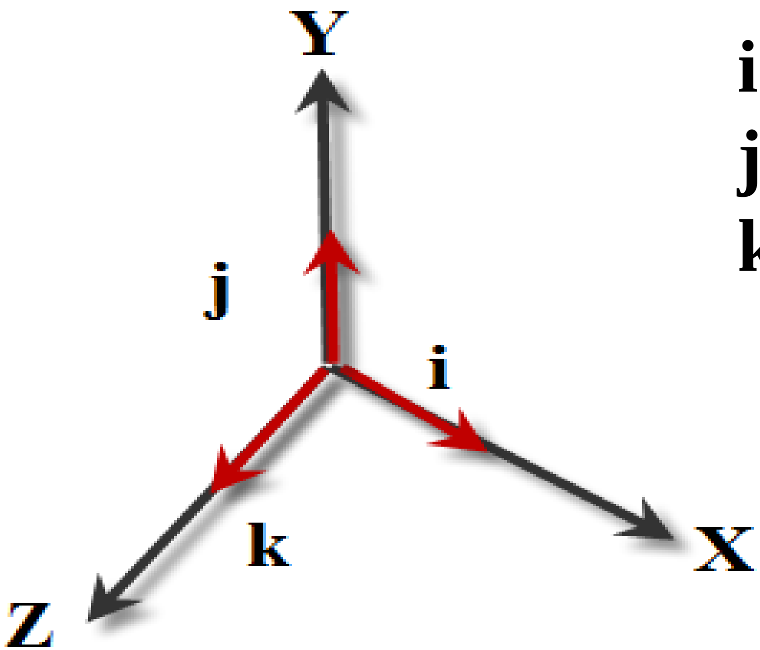
$$\vec{A} = A_x i + A_y j$$

$$\vec{B} = B_x i + B_y j$$

$$\vec{R} = \vec{A} + \vec{B} = (A_x + B_x)i + (A_y + B_y)j$$



- كذلك يمكن تمثيل متجهات وحدة (i, j, k) لمحاور الاسناد المتعامدة
- **rectangular coordinate system** كما في الشكل التالي:



i= a unit vector along the x-axis
j= a unit vector along the y-axis
k= a unit vector along the z-axis

Example

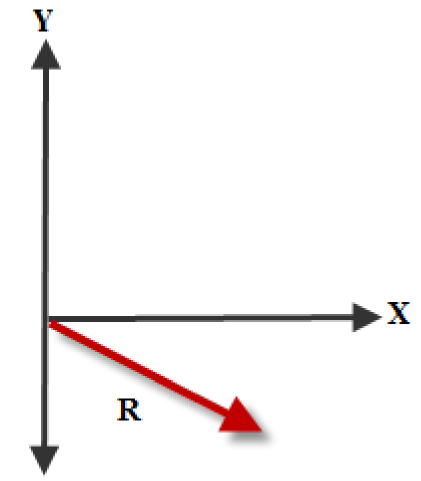
Find the sum of two vector $\vec{A}$ and $\vec{B}$ giving by:

Solution

Note that A_x =3, A_y =2, B_x =2, and B_y =-5

The magnitude of vector $\vec{R}$ is

$$\vec{R} = \vec{A} + \vec{B} = (3+2)i + (4-5)j = 5i - j$$



The direction of $\vec{R}$ with respect to x-axis is

$$R = \sqrt{{R_x}^2 + {R_y}^2} = \sqrt{25+1} = \sqrt{26} = 5.1$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{-1}{5} = -11^{\circ}$$

Questions

- The polar coordinates of a point are r=5.5m and $\theta=240^o$. What are the rectangular coordinates of this point?
- Vector $\vec{A}$ is 3 unit in length and points along the positive x-axis. Vector $\vec{B}$ is 4 unit in length and points along the negative y-axis. Use graphical methods to find the magnitude and direction of the vector (a) $\vec{A}+\vec{B}$

  (b) $\vec{A}-\vec{B}$

Multiplying a Vector by a Vector

There are two ways to multiply a vector by a vector:

- **one way produces a scalar (called the scalar product),**
- **and the other produces a new vector (called the vector product).**

The Scalar Product

- The scalar product of the vectors $\vec{a}$ and $\vec{b}$ is written as and defined to be:

$$\vec{a}.\vec{b} = ab\cos\emptyset \ldots\ldots\ldots(1.10)$$

where **a** is the magnitude of $\vec{a}$

b is the magnitude of $\vec{b}$

$\emptyset$ is the angle between and (or, more properly, between the directions of $\vec{a}$ and $\vec{b}$).

There are actually two such angles: $\emptyset$ and **360°**- $\emptyset$.

- يعرف الضرب العددي او القياسي **scalar product** بالضرب النقطي **dot product** وتكون نتيجة الضرب القياسي لمتجهين كمية قياسية.
- وتكون هذه القيمة موجبة اذا كانت الزاوية المحصورة بين المتجهين $(0\ and\ 90^{\circ})$

$$\vec{A}.\vec{B} = +ve\ when\ 0 \geq \emptyset > 90^{\circ}$$

- وتكون النتيجة سالبة اذا كانت الزاوية محصورة بين المتجهين تقع بين $(\ 90^{\circ} and\ 180^{\circ})$
- $$\vec{A}.\vec{B} = -ve\ when\ 90^{\circ} < \emptyset \leq 180^{\circ}$$
- وتساوي صفرا اذا كانت الزاوية 90°

$$\vec{A}.\vec{B} = zero\ when\ \emptyset = 0$$

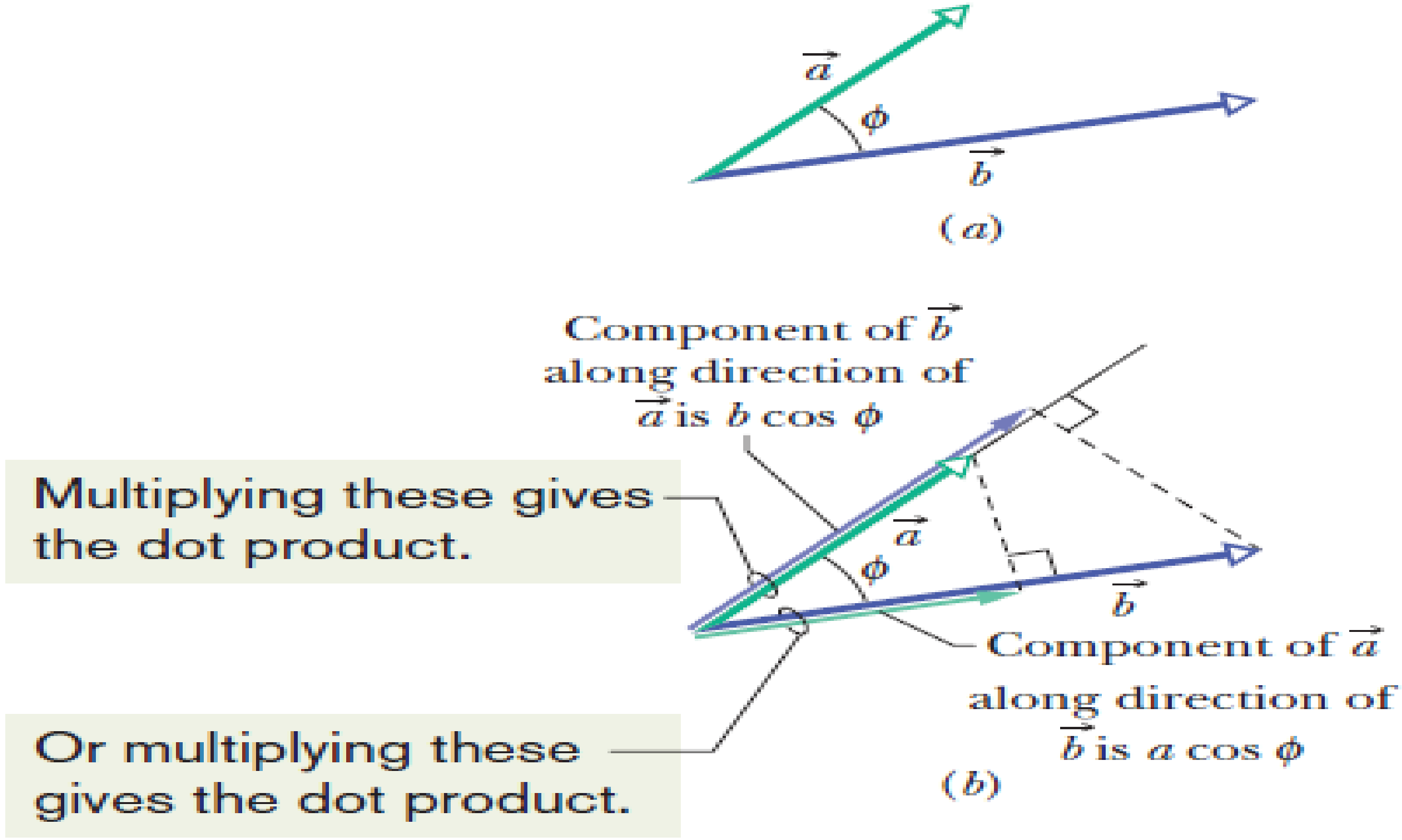


Fig. (a) Two vectors $\vec{a}$ and $\vec{b}$, with an angle $\emptyset$ between them.
(b) Each vector has a component along the direction of the other vector.

- يعرف الضرب العددي لمتجهين $\vec{A}.\vec{B}$ بحاصل ضرب مقدار المتجه الاول $\vec{A}$ في مقدار المتجه الثاني $\vec{B}$ في جيب تمام الزاوية المحصورة بينهما θ

$$\vec{A}.\vec{B} = |A||B|\cos\theta \dots\dots\dots\dots (1.11)$$

- يمكن ايجاد قيمة الضرب العددي لمتجهين باستخدام مركبات كل متجه كما يلي:

$$\vec{A} = A_x i + A_y j + A_z k \dots\dots\dots\dots (1.12)$$

$$\vec{B} = B_x i + B_y j + B_z k \dots\dots\dots\dots (1.13)$$

The scalar product is:

$$\vec{A}.\vec{B} = \left(A_x i + A_y j + A_z k\right).\left(B_x i + B_y j + B_z k\right) \dots (1.14)$$

بضرب مركبات المتجه $\vec{A}$ في مركبات المتجه $\vec{B}$ ينتج التالي:

$$\vec{A}.\vec{B} = (\boldsymbol{A_x i.B_x i + A_x i B_y j + A_x i.B_z k + A_y j.B_x i} \\ \boldsymbol{+ A_y j.B_y j + A_y j.B_z k + A_z k.B_x i + A_z k.B_y j} \\ \boldsymbol{+ A_z k.B_z k})$$

Therefore

$$\therefore \vec{A}.\vec{B} = A_x B_x + A_y B_y + A_z B_z \; \dots (1.15)$$

The angle between the two vectors is:

$$\cos\theta = \frac{\vec{A}.\vec{B}}{|A||B|} = \frac{A_x B_x + A_y B_y + A_z B_z}{|A||B|} \dots \dots (1.16)$$

Also note that:

$$\vec{A}.\vec{A} = |\vec{A}||\vec{A}| = A^2 \dots \dots \dots . (1.17)$$

Another note:

$$\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$$

$$\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\jmath} = 0$$

The vector product

يعرف الضر الاتجاهي **vector product** بـ **cross product**

وتكون نتيجة الضرب الاتجاهي لمتجهين كمية متجهة وقيمة هذا المتجه

$$\vec{C} = \vec{A} \times \vec{B}$$

واتجاهه عمودي على كل من المتجهين $\vec{A}$ و $\vec{B}$ وفي اتجاه دوران بريمة من المتجه $\vec{A}$ الى المتجه $\vec{B}$ كما في الشكل التالي:

$$\vec{A} \times \vec{B} = AB\sin\theta \ldots\ldots\ldots(1.18)$$

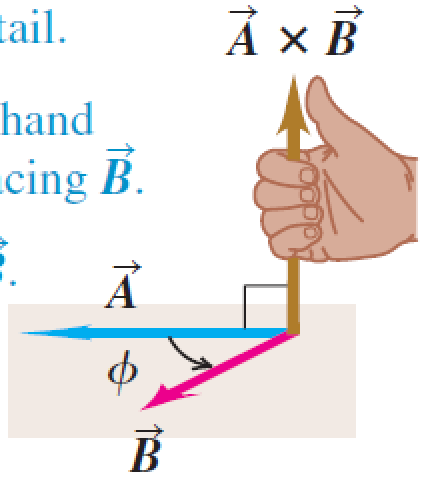
$$\vec{A} \times \vec{B} = \left(A_x i + A_y j + A_z k\right) \times \left(B_x i + B_y j + B_z k\right) \ldots\ldots(1.19)$$

To evaluate this product we use the fact that the angle between the unit vectors $i.j.k \;\; is \; 90^{\circ}$

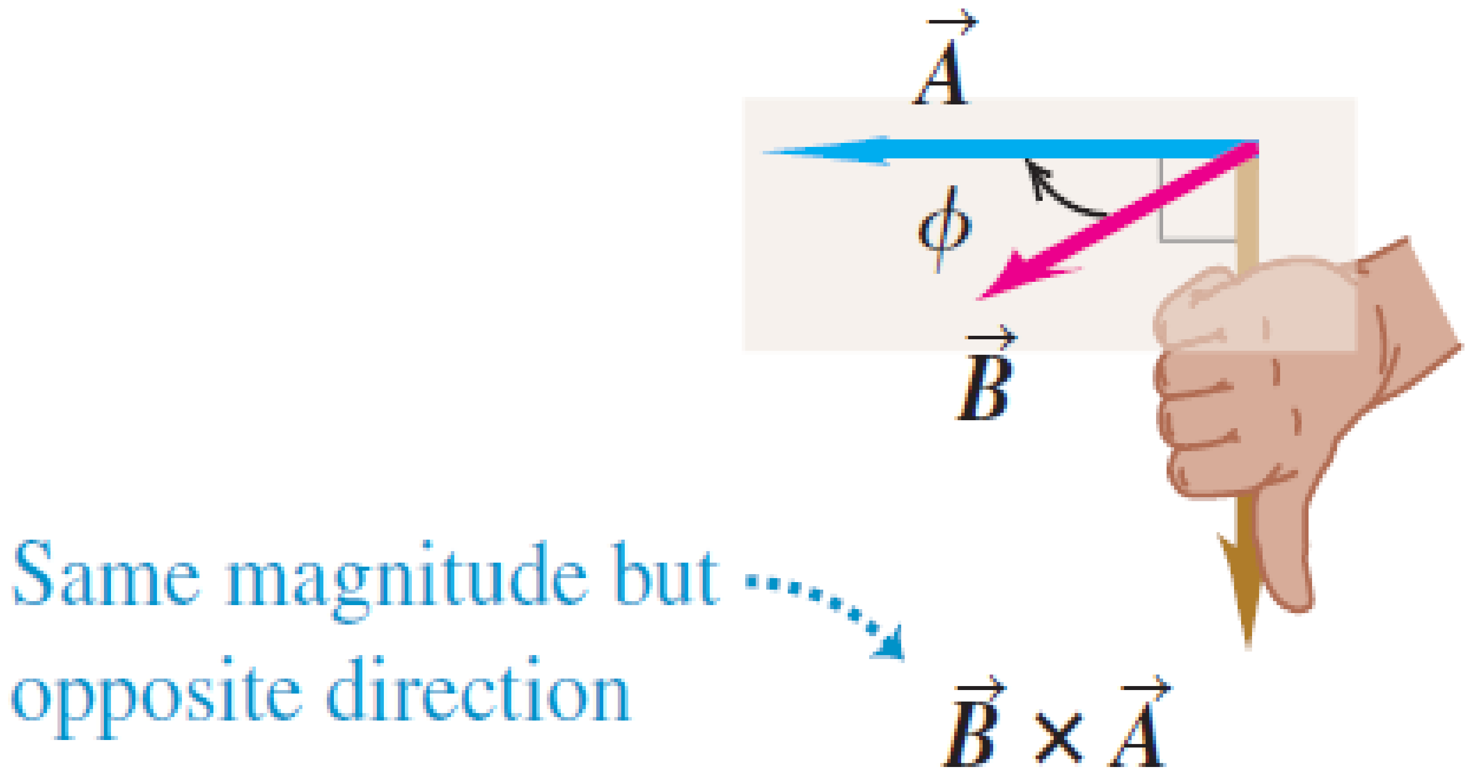
(a) The vector product determined by the right-hand rule.

(a) Using the right-hand rule to find the direction of $\vec{A} \times \vec{B}$

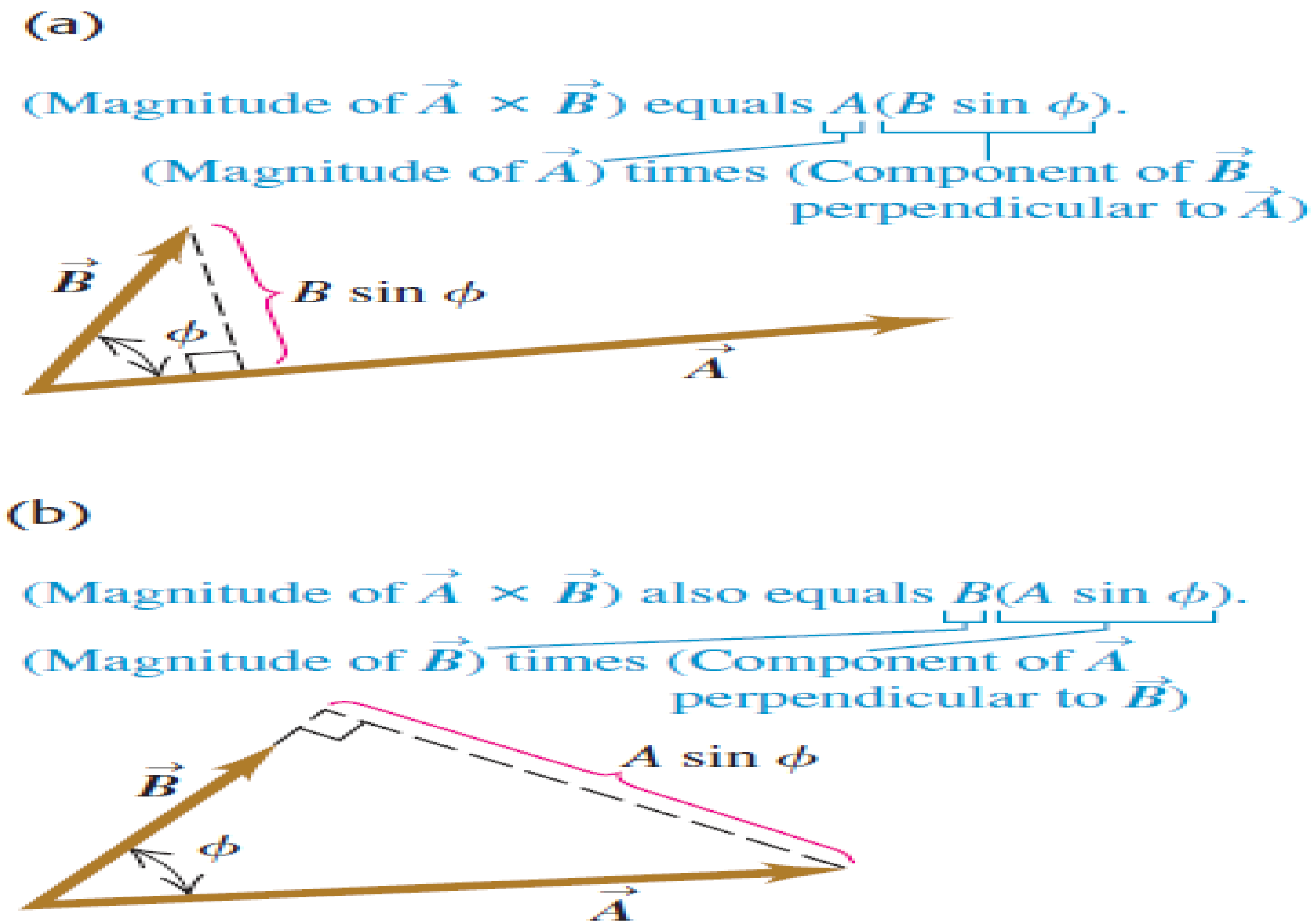
1. Place $\vec{A}$ and $\vec{B}$ tail to tail.
2. Point fingers of right hand along $\vec{A}$, with palm facing $\vec{B}$.
3. Curl fingers toward $\vec{B}$.
4. Thumb points in direction of $\vec{A} \times \vec{B}$.



(b) $\vec{B} \times \vec{A} = -\vec{A} \times \vec{B}$ (the vector product is anticommutative)



Calculating the magnitude $AB \sin \phi$ of the vector product of two vectors $\vec{A} \times \vec{B}$



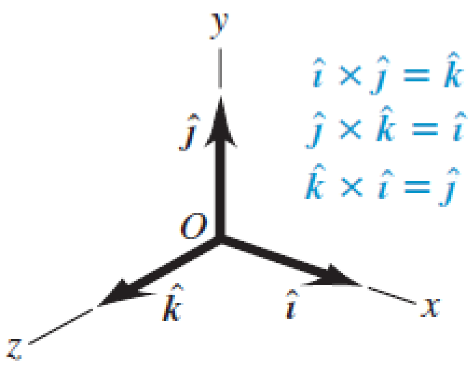
Calculating the Vector Product Using Components

If we know the components of $\vec{A}$ and $\vec{B}$ we can calculate the components of the vector product using a procedure similar to that for the scalar product. First we work out the multiplication table for the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ all three of which are perpendicular to each other as shown in Figure. The vector product of any vector with itself is zero, so

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$$

$$\hat{\imath} \times \hat{\jmath} = -\hat{\jmath} \times \hat{\imath} = \hat{k}$$
$$\hat{\jmath} \times \hat{k} = -\hat{k} \times \hat{\jmath} = \hat{\imath}$$
$$\hat{k} \times \hat{\imath} = -\hat{\imath} \times \hat{k} = \hat{\jmath}$$

(a) A right-handed coordinate system



Next we express $\vec{A}$ and $\vec{B}$ in terms of their components and the corresponding unit vectors, and we expand the expression for the vector product:

$$\begin{aligned}
\vec{A} \times \vec{B} &= (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}) \\
&= A_x\hat{\imath} \times B_x\hat{\imath} + A_x\hat{\imath} \times B_y\hat{\jmath} + A_x\hat{\imath} \times B_z\hat{k} \\
&\quad + A_y\hat{\jmath} \times B_x\hat{\imath} + A_y\hat{\jmath} \times B_y\hat{\jmath} + A_y\hat{\jmath} \times B_z\hat{k} \\
&\quad + A_z\hat{k} \times B_x\hat{\imath} + A_z\hat{k} \times B_y\hat{\jmath} + A_z\hat{k} \times B_z\hat{k}
\end{aligned}$$

$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$$

Thus the components of $\vec{C} = \vec{A} \times \vec{B}$ are given by

$$C_x = A_yB_z - A_zB_y \qquad C_y = A_zB_x - A_xB_z \qquad C_z = A_xB_y - A_yB_x$$

(components of $\vec{C} = \vec{A} \times \vec{B}$)

The vector product can also be expressed in determinant form as

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Note that:

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

But

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

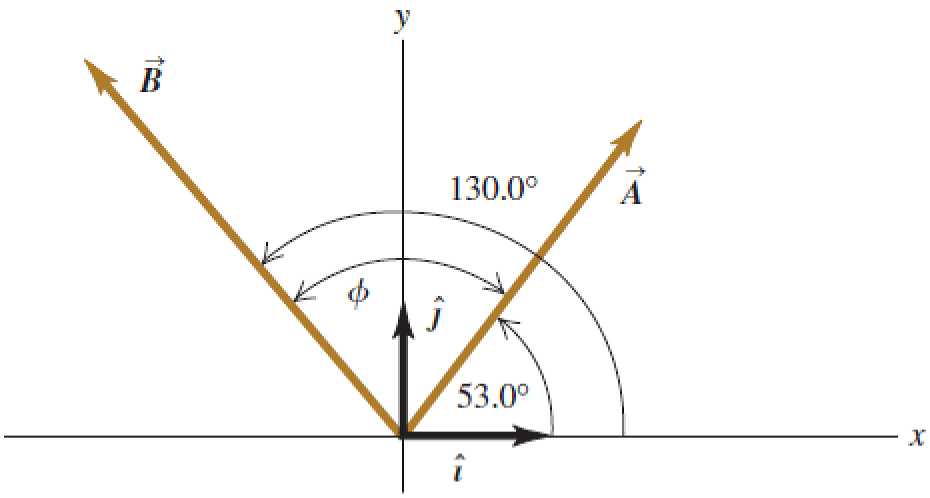
Also

$$\begin{aligned} \vec{C} \times (\vec{A} + \vec{B}) &= \vec{C} \times \vec{A} + \vec{C} \times \vec{B} \\ &= \boldsymbol{i}(A_y B_z - A_z B_y) + \boldsymbol{j}(A_z B_x - A_x B_z) \\ &+ \boldsymbol{k}(A_x B_y - A_y B_x) \end{aligned}$$

➢ Questions

1- If $\vec{C} = \vec{A} \times \vec{B}$ where $\vec{A} = 3i - 4j$ and $\vec{B} = -2i + 3k$ what is $\vec{C}$?

2- Find the scalar product $\vec{A}.\vec{B}$ of the two vectors in Figure. The magnitudes of the vectors A=4 and B=5.



Lecture (3)

Motion in Two Dimensions and Three Dimensions

Dr. Hind I. Al-Shaikh

Out lines of study

- **Position Vector and Displacement Vector**
- **Average Velocity and Instantaneous Velocity**
- **Average Acceleration and Instantaneous Acceleration**
- **One dimensional motion with constant acceleration**
- **Relative Motion in Two Dimensions**
- **Motion in two dimensions with constant acceleration**
- **Projectile Motion**
- **Trajectory of Projectile Motion**
- **Horizontal range and maximum height of a projectile**
- **Projectile Motion Analyzed**
- **Uniform Circular Motion**

❖Position Vector and Displacement Vector

➢من اساسيات دراسة علم وصف الحركة الكينماتيكا **Kinematics** للاجسام المادية هو دراسة كل من الازاحة **Displacement** والسرعة **Velocity** والتعجيل **Acceleration** ونحتاج هنا الى اعتماد محاور اسناد لتحديد موضع الجسم المتحرك عند ازمنة مختلفة ومن المناسب اعتماد محاور الاسناد الكارتيزية او ما تسمى

Rectangular coordinate (x,y,z)

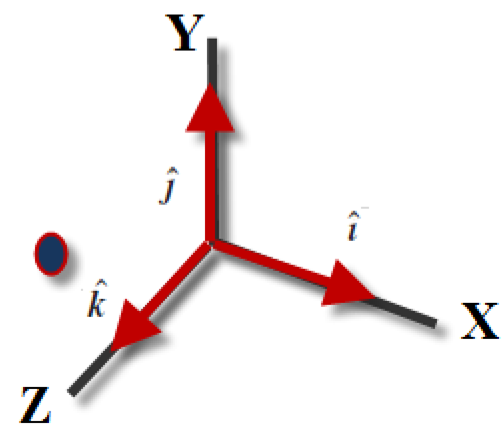
➢**One general way of locating a particles with a position vector , which is a vector that extends from a reference point (usually the origin) to the particle.**

❑Kinematic variables in one dimension

Position:	$x(t)\ m$
Velocity:	$v(t)\ m/s$
Acceleration:	$a(t)\ m/s^2$

❑Kinematic variables in three dimensions

Position:	$\vec{r}(t) = x\hat{i} + y\hat{j} + z\hat{k}$	m
Velocity:	$\vec{v}(t) = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$	m/s
Acceleration:	$\vec{a}(t) = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$	$a(t)\ m/s^2$



➢**All are vectors: have direction and magnitudes**

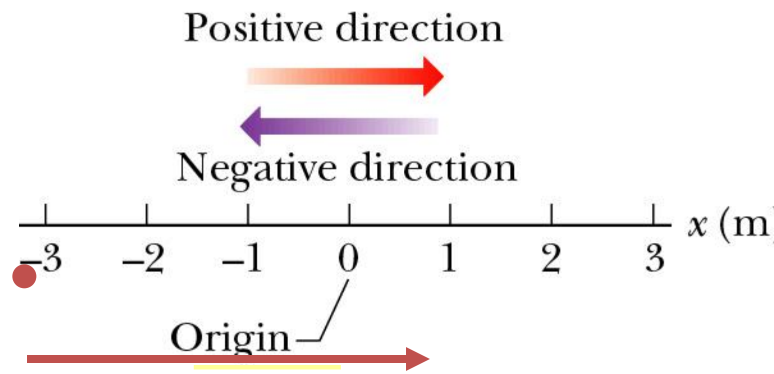
Position and Displacement

- In one dimension

$$\boxed{\Delta x = x_2(t_2) - x_1(t_1)}$$

$x_1(t_1)$ = - 3.0 m, $x_2(t_2)$ = + 1.0 m

Δx = +1.0 m + 3.0 m = +4.0 m

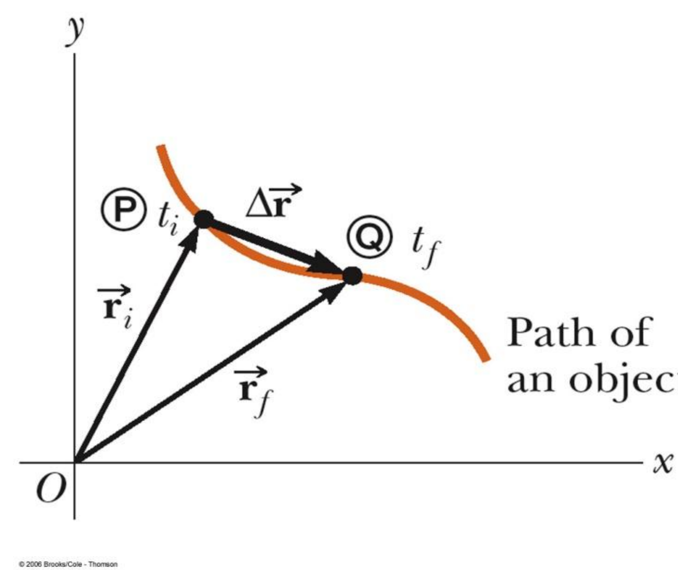


- In two dimensions
  - Position: the position of an object is described by its position vector $\vec{r}(t)$ always points to particle from origin.

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 \quad \text{......(1)}$$

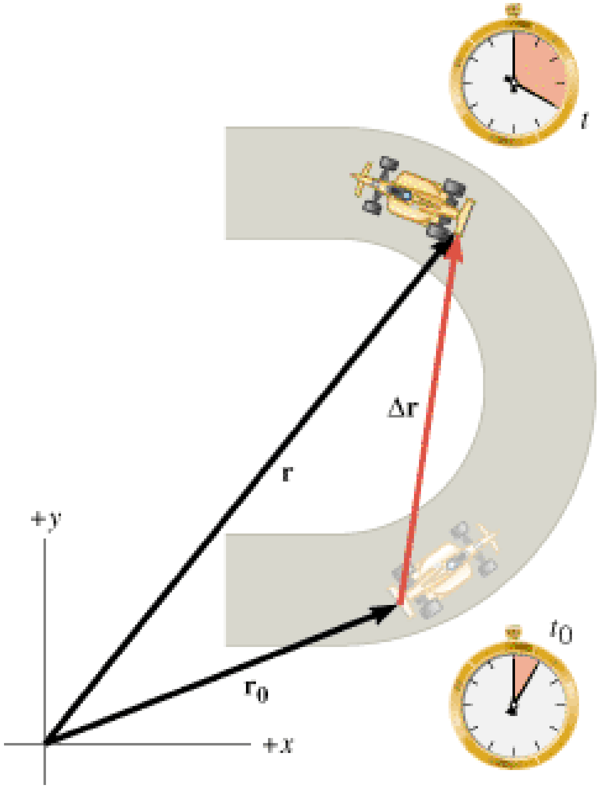
  - Displacement:

$$\Delta\vec{r} = (x_2\hat{i} + y_2\hat{j}) - (x_1\hat{i} + y_1\hat{j})$$
$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$
$$= \Delta x\hat{i} + \Delta y\hat{j}$$



الشكل يوضح متجه $r1$ الموضع يحدد موضع الجسم عند بداية الحركة ومتجه الموضع $r2$ يحدد موقع الجسم النهائي بعد زمن وقدره $\Delta t = t_2 - t_1$ وهنا فان الازاحة للجسم تعطى بالمعادلة

❑Position and Displacement in Three dimention



Position vector:

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k},$$

Displacement :

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1.$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k},$$

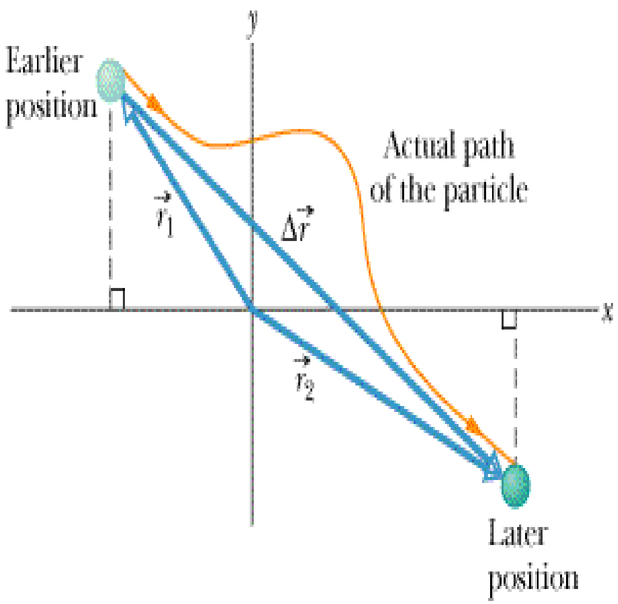
$$\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}.$$

Question 1: Write the position vector for a particale in the rectangular coordinate (x,y,z) for points (5, -6, 0), (5, -4), and (-1, 3, 6).

Question 1: Write the position vector for a particale in the rectangular coordinate (x,y,z) for points (5, -6, 0), (5, -4), and (-1, 3, 6).

- **For the point (5,-6,0) the position vector** $\vec{r} = 5i - 6j$
- **For the point (5,-4) the position vector** $\vec{r} = 5i - 4j$
- **For the point (-1, 3,6) the position vector** $\vec{r} = -i + 3j + 6k$

Question 2 : Displacement



In Fig., the position vector for a particle is initially at

$$\vec{r}_1 = (-3.0\,\mathrm{m})\hat{\mathrm{i}} + (4.0\,\mathrm{m})\hat{\mathrm{j}}$$

and then later is

$$\vec{r}_2 = (9.0\,\mathrm{m})\hat{\mathrm{i}} + (-3.5\,\mathrm{m})\hat{\mathrm{j}}.$$

What is the particle's displacement from $\vec{r}_1$ to $\vec{r}_2$?

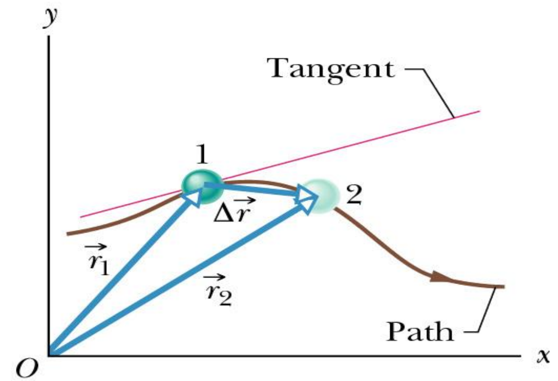
❖Average & Instantaneous Velocity

- عند انتقال الجسم من موضع البداية عند الزمن t_1 الى موضع النهاية t_2 فان حاصل قسمة الازاحة على فرق الزمن $\Delta t(t_2 - t_1)$ يعرف بالسرعة *Velocity* وحيث ان الجسم الجسم يقطع المسافة بسرعات مختلفة فان السرعة المحسوبة تسمى بمتوسط السرعة او معدل السرعة *Average velocity*. ويمكن تعريف السرعة عند اي لحظة بالسرعة اللحظية *Instantaneous Velocity*.

The Average velocity **of a particle is defined as the ratio of the displacement to the time interval. The unit of the velocity is** (m/s)

$$\text{average velocity} \equiv \frac{\text{displacement}}{\text{time interval}},$$

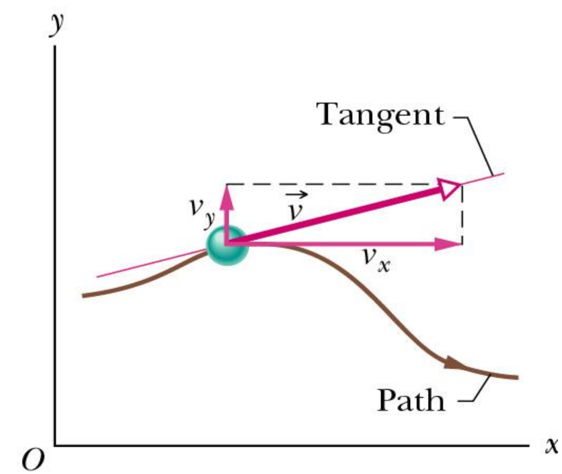
$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}.$$



The instantaneous Velocity **of a particle is defined as the limit of the average velocity as the time interval approaches zero.**

$$\vec{v} \equiv \lim_{t\to 0} \vec{v}_{avg} = \lim_{t\to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

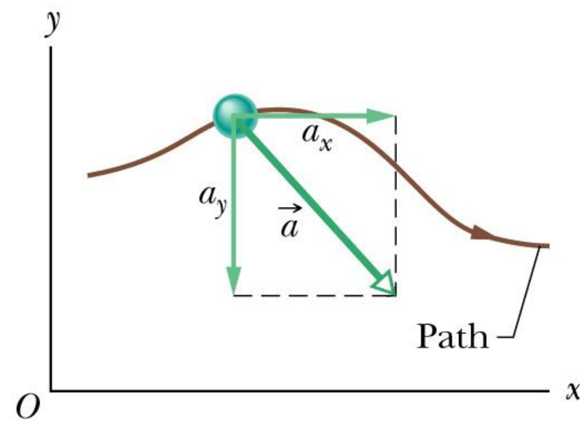


❖Average & Instantaneous Acceleration

عند انتقال الجسم من موضع البداية عند الزمن t_1 الى موضع النهاية t_2 بسرعة ابتدائية وعند النهاية كانت السرعة v_2 فان معدل تغيير السرعة بالنسبة للزمن يعرف باسم التعجيل *Acceleration* او متوسط التعجيل *Average Acceleration* ويكون التعجيل اللحظي *Instantaneous Acceleration* هو السرعة اللحظية على الزمن.

***The average acceleration** of particle is defined as the ratio of the change in the instantaneous velocity to the time interval.*

The unit of the acceleration is (m/s^2)



$$\vec{a}_{avg} \equiv \frac{\Delta \vec{v}}{\Delta t}$$

$$\vec{a}_{avg} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} = a_{avg,x}\hat{i} + a_{avg,y}\hat{j}$$

- ***The Instantaneous acceleration** is defined as the limiting value of the ratio of the average velocity to the time interval as the time approaches zero.*

$$\vec{a} \equiv \lim_{t \to 0} \vec{a}_{avg} = \lim_{t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j}$$

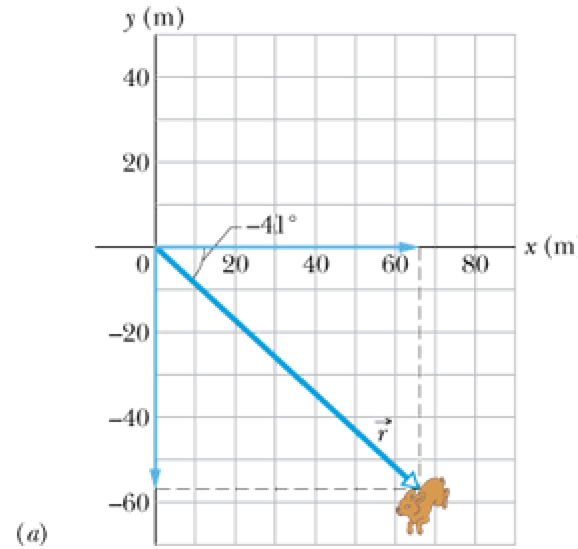
- **The magnitude of the velocity (the speed) can change**
- **The direction of the velocity can change, even though the magnitude is constant**
- **Both the magnitude and the direction can change**

Example

A rabbit runs across a parking lot on which a set of coordinate axes has, strangely enough, been drawn. The coordinates of the rabbit's position as functions of time *t* (second) are given by

$$x = -0.31t^2 + 7.2t + 28$$

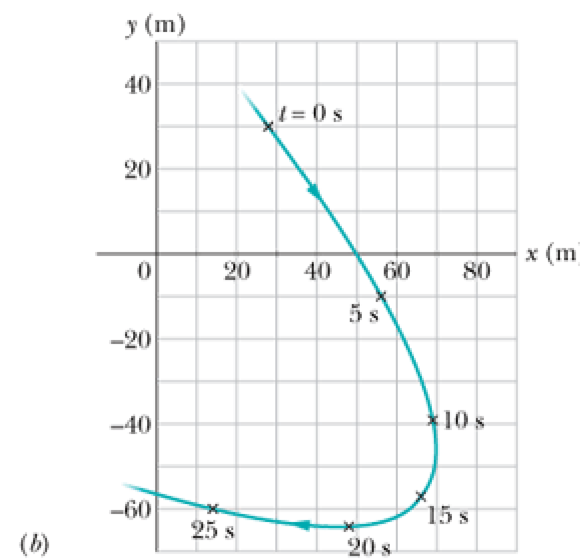
$$y = 0.22t^2 - 9.1t + 30,$$



A- At *t = 15 s*, what is the rabbit's position vector in unit vector notation and in magnitude-angle notation?

B- What is the rabbit's velocity vector in unit-vector notation and in magnitude-angle notation?

C- At t=15 s, what is the rabbit's acceleration vector in unit-vector notation and in magnitude-angle notation?



Calculations: We can write

$$x = -0.31t^2 + 7.2t + 28$$

$$y = 0.22t^2 - 9.1t + 30,$$

$$\vec{r}(t) = x(t)\hat{\mathrm{i}} + y(t)\hat{\mathrm{j}}.$$

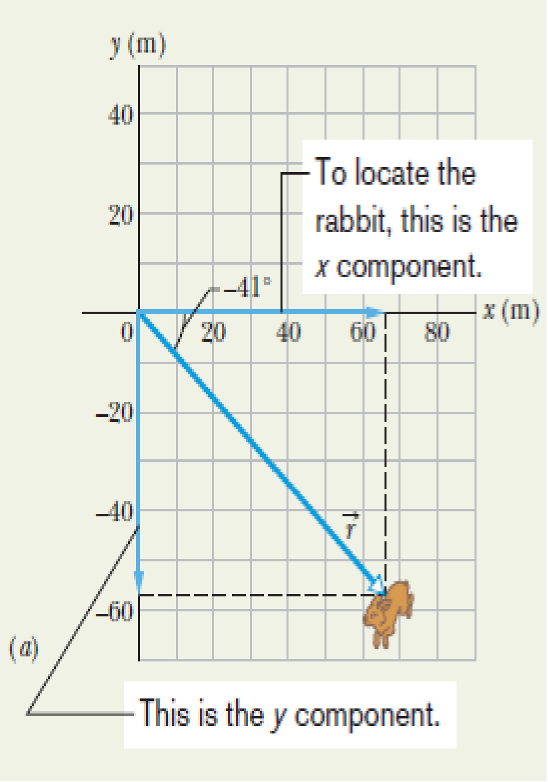
(We write $\vec{r}(t)$ rather than $\vec{r}$ because the components are functions of t, and thus $\vec{r}$ is also.)

At $t = 15$ s, the scalar components are

$$x = (-0.31)(15)^2 + (7.2)(15) + 28 = 66 \text{ m}$$

and $$y = (0.22)(15)^2 - (9.1)(15) + 30 = -57 \text{ m},$$

so $$\vec{r} = (66 \text{ m})\hat{\mathrm{i}} - (57 \text{ m})\hat{\mathrm{j}},$$



To get the magnitude and angle of $\vec{r}$, we use :

$$r = \sqrt{x^2 + y^2} = \sqrt{(66 \text{ m})^2 + (-57 \text{ m})^2}$$
$$= 87 \text{ m},$$

and $$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\left(\frac{-57 \text{ m}}{66 \text{ m}}\right) = -41^\circ.$$

We can find $\vec{v}$ by taking derivatives of the components of the rabbit's position vector.

$$x = -0.31t^2 + 7.2t + 28$$

$$y = 0.22t^2 - 9.1t + 30,$$

Calculations: Applying the v_x part we find the x component of $\vec{v}$ to be

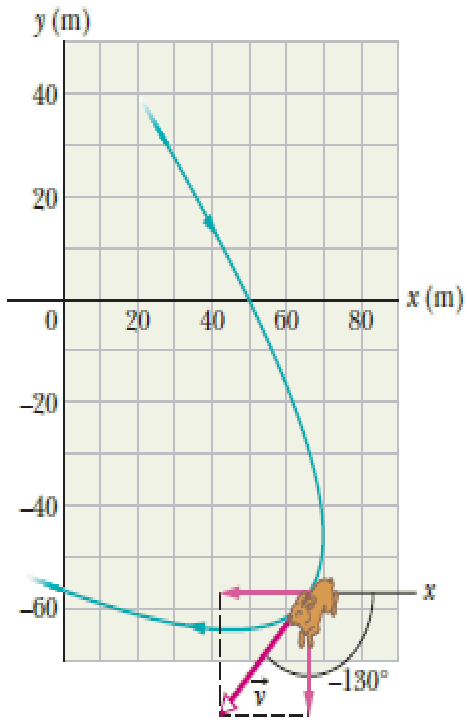
$$v_x = \frac{dx}{dt} = \frac{d}{dt}(-0.31t^2 + 7.2t + 28)$$
$$= -0.62t + 7.2.$$

At $t = 15$ s, this gives $v_x = -2.1$ m/s. Similarly, applying the v_y part

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(0.22t^2 - 9.1t + 30)$$
$$= 0.44t - 9.1.$$

At $t = 15$ s, this gives $v_y = -2.5$ m/s. Equation 4-11 then yields

$$\vec{v} = (-2.1\ \text{m/s})\hat{i} + (-2.5\ \text{m/s})\hat{j}$$



These are the x and y components of the vector at this instant.

The rabbit's velocity $\vec{v}$ at $t = 15$ s.

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-2.1\ \text{m/s})^2 + (-2.5\ \text{m/s})^2}$$
$$= 3.3\ \text{m/s}$$

and $$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\left(\frac{-2.5\ \text{m/s}}{-2.1\ \text{m/s}}\right)$$
$$= \tan^{-1} 1.19 = -130^\circ.$$

Summary in two dimension

- **Position** $\vec{r}(t) = x\hat{i} + y\hat{j}$

- **Average velocity** $\vec{v}_{avg} = \dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{\Delta x}{\Delta t}\hat{i} + \dfrac{\Delta y}{\Delta t}\hat{j} = v_{avg,x}\hat{i} + v_{avg,y}\hat{j}$

- **Instantaneous velocity** $v_x \equiv \dfrac{dx}{dt} \qquad v_y \equiv \dfrac{dy}{dt}$

$$\vec{v}(t) = \lim_{t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

- **Acceleration** $a_x \equiv \dfrac{dv_x}{dt} = \dfrac{d^2x}{dt^2} \qquad a_y \equiv \dfrac{dv_y}{dt} = \dfrac{d^2y}{dt^2}$

$$\vec{a}(t) = \lim_{t\to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j}$$

- $\vec{r}(t)$, $\vec{v}(t)$, and $\vec{a}(t)$ **are not necessarily same direction.**

One Dimensional Motion with Constant Acceleration

- سندرس الان الحركة في بعد واحد وذلك فقط عندما يكون التعجيل ثابتا acceleration constant وفي هذه الحالة يكون التعجيل اللحظي Instantaneous acceleration يساوي متوسط التعجيل Average acceleration. ونتيجة لذلك فان السرعة اما ان تتزايد او تتناقص بمعدلات متساوية خلال الحركة. ويعبر عن ذلك رياضيا على النحو التالي:

Instantaneous acceleration=Average acceleration

$$a = a_{ave} = \frac{v - v_\circ}{t - t_\circ} \dots\dots\dots (1)$$

- Let $t_\circ = 0$ then the acceleration

$$a = \frac{v - v_\circ}{t} \dots\dots\dots.. (2)$$

or

$$v = v_\circ + at \dots\dots\dots.. (3)$$

- من المعادلة (3) يمكن ايجاد السرعة v عند اي زمن t اذا عرفنا السرعة الابتدائية $v_\circ$ والتعجيل الثابت a الذي يتحرك به الجسم. واذا كان التعجيل يساوي صفرا فان السرعة لا تعتمد على الزمن وهذا يعني ان السرعة النهائية تساوي السرعة الابتدائية. لاحظ ان كل حد من حدود المعادلة السابقة له بعد سرعة (m/s^2).

Eq. (3) **is true only for constant acceleration and with** $v = v_\circ$ **at the initial time** $t = 0$**. Also, since** $v = dx/dt$**, we can integrate to get our second equation:**

$$\int_{x_\circ}^{x} dx' = \int_{t_\circ}^{t} v(t')dt' = \int_{t_\circ}^{t} (v_\circ + at')dt' \;\ldots\ldots (4)$$

$$x - x_\circ = v_\circ t + \frac{1}{2}at^2 \;\ldots\ldots\ldots.. (5)$$

Assuming $t_\circ = 0$**. I often write this as:**

$$x(t) = x_\circ + v_\circ t + \frac{1}{2}at^2 \;\ldots\ldots\ldots.. (6)$$

What if we don't know t?

We can eliminate it by solving the first equation for t and putting it into the second equation:

$$t = \frac{v - v_\circ}{a} \;\ldots\ldots..(7)$$

$$x - x_\circ = v_\circ\left(\frac{v - v_\circ}{a}\right) + \frac{1}{2}a\left(\frac{v - v_\circ}{a}\right)^2 \;\ldots\ldots..(8)$$

$$2a(x - x_\circ) = 2v_\circ(v - v_\circ) + (v - v_\circ)^2 \;\ldots\ldots..(9)$$

Changing the order of the terms gives us:

$$v(t)^2 = v_\circ^2 + 2a(x - x_\circ) \;\ldots\ldots\ldots(10)$$

To use these equation, we first identify the known and unknown quantities

- كذلك نلاحظ من هذه المعادلة ان المسافة المقطوعة $(x - x_\circ)$ تساوي المسافة المقطوعة نتيجة السرعة الابتدائية وهو الحد $v_\circ t$ بالاضافة الى المسافة نتيجة العجلة الثابتة. وهذا يظهر في الحد الاخير من المعادلة $1/2at^2$ وان كل حد من حدود المعادلة له بعد مسافة (m).

اذا كانت السرعة الابتدائية تساوي صفرا تكون المسافة المقطوعة تساوي:

$$x - x_\circ = v_\circ t$$

$$x - x_\circ = \frac{1}{2}at^2$$

Example: Ali applies the brake in a car, starting at 100 km/hr and slowing to 80 km/hr in 88 m at a constant acceleration.

A- What is a?

B- How long did this take?

Solution: First, identify knows and unknowns and convert ti SI units.

knowns	**unknowns**
$v_\circ = 100\,km/hr = 27.8\ m/s$	$t_f = ???$
$v = 80\,km/hr = 22.2\ m/s$	$a = constant = ???$
$t_\circ = 0$	
$x - x_\circ = 88m$	

A- $v^2 = v_\circ^2 + 2a(x - x_\circ) \rightarrow a = \dfrac{v^2 - v_\circ^2}{2(x - x_\circ)} = -1.6\,m/s^2$

B- $v = v_\circ + at \rightarrow t = \dfrac{v - v_\circ}{a} = 3.5\ s$

Question: A body moving with uniform acceleration has a velocity of 12 cm/s when its *x* coordinate is 3 cm. If its *x* coordinate 2 s later is -5 cm, what is the magnitude of its acceleration?

❖Motion in two dimensions

Motion in two dimensions like the motion of projectiles and satellites and the motion of charged particles in electric fields. Here we shall treat the motion in plane with constant acceleration and uniform circular motion.

درسنا سابقا الحركة في بعد واحد اي عندما يتحرك الجسم في خط مستقيم على محور x او ان يسقط الجسم سقوطا حرا في محور y سندرس الان حركة الجسم في بعدين اي في كل من (x, y) مثل حركة المقذوفات حيث يكون للازاحة والسرعة مركبتان في اتجاه المحور x والمحور y .

❖Motion in two dimensions with constant acceleration

- **Assume that the magnitude and direction of the acceleration remain unchanged the motion.**
- **The position vector for a particle in two dimensions (*x,y* plane) can be written as:**

$$\vec{r}(t) = x\hat{i} + y\hat{j}$$

Where x, y and r change with tune as the particle moves the velocity of particle is given by:

$$v = \frac{dr}{dt} = \frac{dx}{dt} i + \frac{dy}{dt} j$$

$$\vec{v} = v_x i + v_y j$$

Since the acceleration is constant then we can substitute

$$v_x = v_{0x} + a_x t \qquad v_y = v_{0y} + a_y t$$

This give

$$\begin{aligned} \boldsymbol{v} &= (\boldsymbol{v}_{x^\circ} + \boldsymbol{a}_x \boldsymbol{t})\boldsymbol{i} + (\boldsymbol{v}_{y^\circ} + \boldsymbol{a}_y \boldsymbol{t})\boldsymbol{j} \\ &= (\boldsymbol{v}_{x^\circ}\boldsymbol{i} + \boldsymbol{v}_{y^\circ}\boldsymbol{j}) + (\boldsymbol{a}_{x^\circ}\boldsymbol{i} + \boldsymbol{a}_y\boldsymbol{j})\boldsymbol{t} \end{aligned}$$

Then

$$\boldsymbol{v} = \boldsymbol{v}_\circ + \boldsymbol{a}\boldsymbol{t} \ldots\ldots\ldots\ldots(\mathbf{11})$$

من المعادلة (11) نستنتج ان سرعة الجسم عند زمن محدد t يساوي الجمع الاتجاهي للسرعة الابتدائية والسرعة الناتجة من العجلة المنتظمة.

Since our particle moves in two dimension x and y with constant acceleration then

$$x = x_{\circ} + v_{x^{\circ}}t + \frac{1}{2}a_x t^2 \quad and \quad y = y_{\circ} + v_{y^{\circ}}t + \frac{1}{2}a_y t^2$$

But

$$r = xi + yj$$

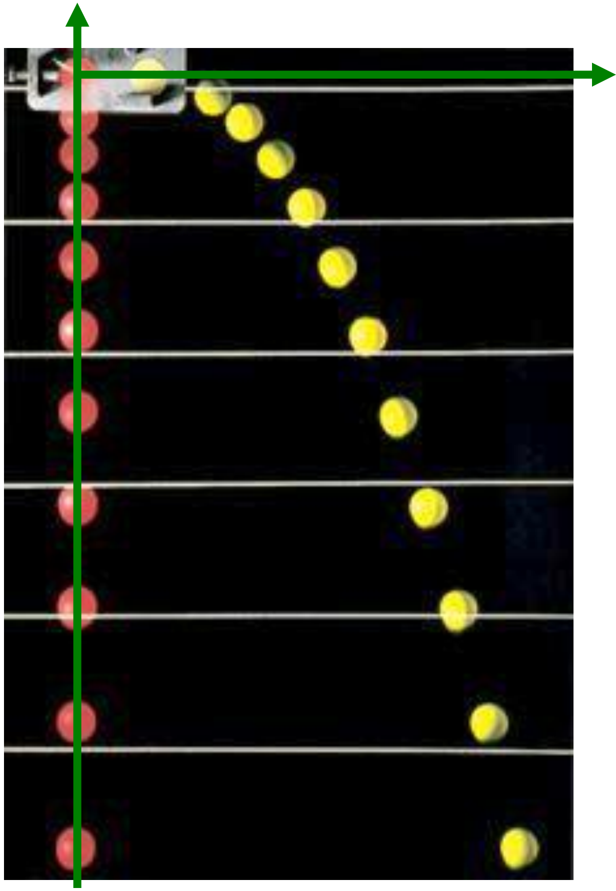
Solve $r = r_{\circ} + v_{x^{\circ}}t + \frac{1}{2}at^2 \ldots\ldots\ldots(12)$

من المعادلة(12) نستنتج ان متجه الازاحة $r - r_{\circ}$ هو عبارة عن الجمع الاتجاهي لمتجه الازاحة الناتج عن السرعة الابتدائية $v_{\circ}t$ والازاحة الناتجة عن العجلة المنتظمة $\frac{1}{2}at^2$

❖Projectile Motion حركة المقذوفات

تعتبر حركة المقذوفات من الامثلة على الحركة في بعدين وسوف نقوم بايجاد معادلات الحركة للمقذوفات لتحديد الازاحة الافقية والراسية والسرعة والعجلة من خلال بعض الامثلة.

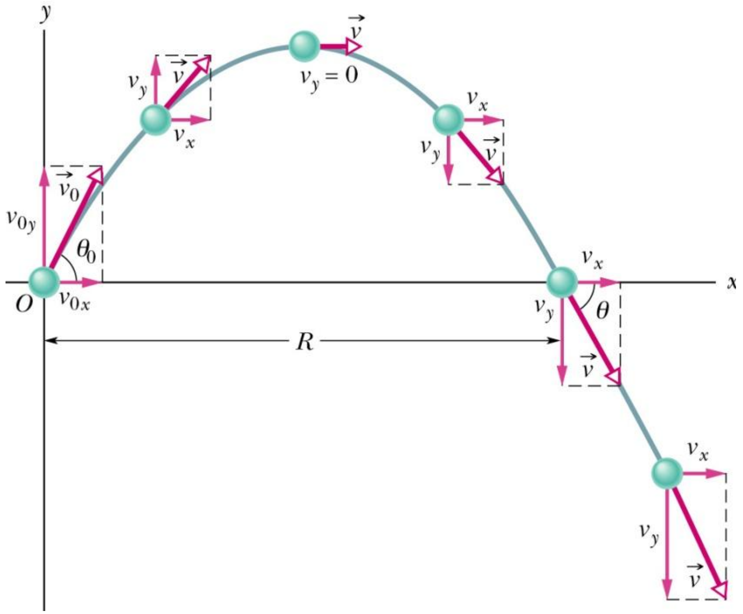
- 2-D problem and define a coordinate system: x- horizontal, y- vertical (up +)
- Try to pick $x_0 = 0, y_0 = 0$ at $t = 0$
- Horizontal motion + Vertical motion
- Horizontal: $a_x = 0$, constant velocity motion
- Vertical: $a_y = -g = -9.8$ m/s², $v_{0y} = 0$
- Equations:



Horizontal	Vertical
$v_x = v_{0x} + a_x t$	$v_y = v_{0y} + a_y t$
$x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2$	$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$	$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

Projectile Motion

- 2-D problem and define a coordinate system.
- Horizontal: $a_x = 0$ and vertical: $a_y = -g$.
- Try to pick $x_0 = 0, y_0 = 0$ at $t = 0$.
- Velocity initial conditions:
  - v_0 can have x, y components.
  - v_{0x} is constant usually. $v_{0x} = v_0 \cos\theta_0$
  - v_{0y} changes continuously. $v_{0x} = v_0 \sin\theta_0$
- Equations:



Horizontal

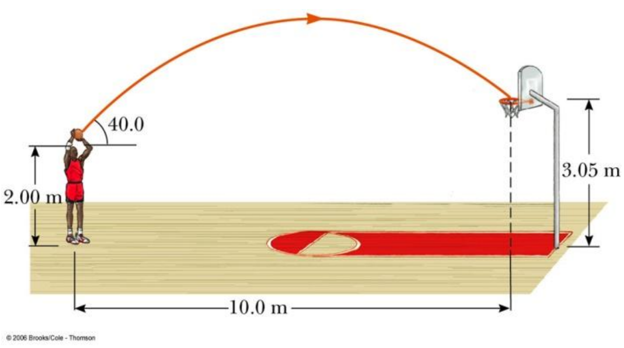
$$v_x = v_{0x}$$

$$x = x_0 + v_{0x}t$$

Vertical

$$v_y = v_{0y} - gt$$

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$



❖Trajectory of Projectile Motion

- **Initial conditions (*t = 0*): $x_0 = 0, y_0 = 0$ $v_{0x} = v_0 \cos\theta_0$ and $v_{0y} = v_0 \sin\theta_0$**
- **Horizontal motion:**

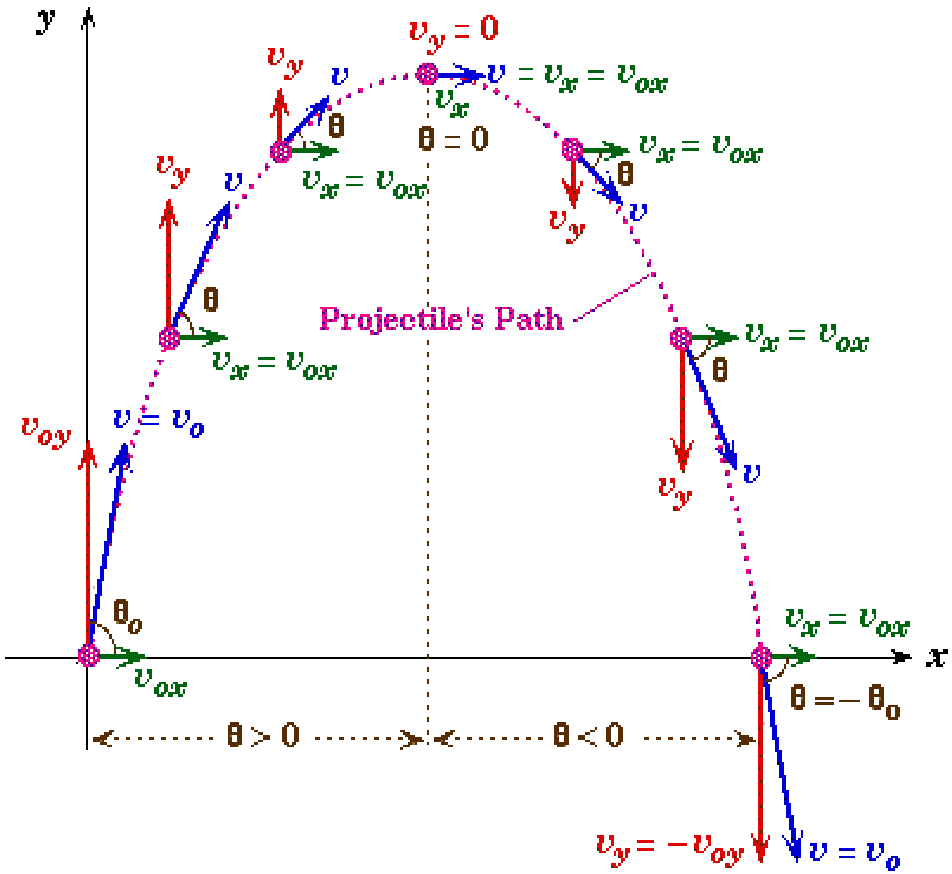
$$x = 0 + v_{0x}t \quad \Rightarrow \quad t = \frac{x}{v_{0x}}$$

- **Vertical motion:**

$$y = 0 + v_{0y}t - \tfrac{1}{2}gt^2$$

$$y = v_{0y}\left(\frac{x}{v_{0x}}\right) - \frac{g}{2}\left(\frac{x}{v_{0x}}\right)^2$$

$$y = x\tan\theta_0 - \frac{g}{2{v_0}^2\cos^2\theta_0}x^2$$

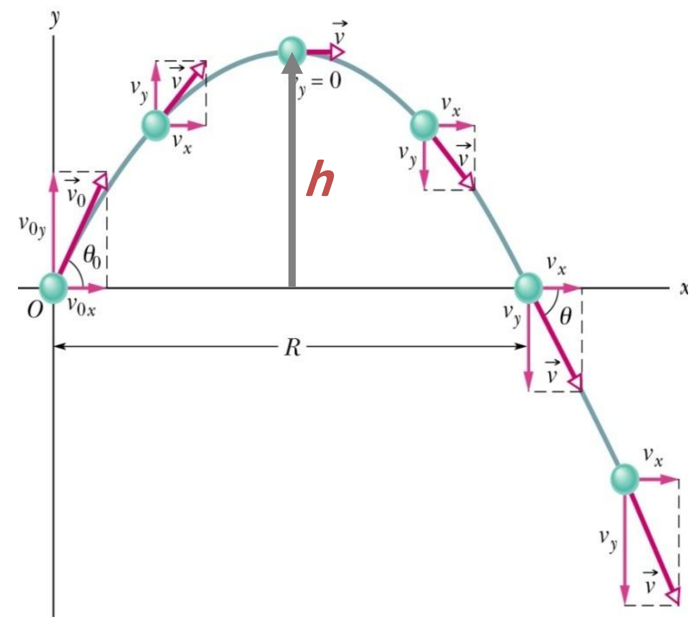


❖Horizontal range and maximum height of a projectile

The horizontal range ***R*** of the projectile is the horizontal distance the projectile has travelled when it returns to its initial height (the height at which it is launched). To find range h, let us put $x - x_\circ = R$ in eq. 1, and $y - y_\circ = o$ in eq. 2

$$x - x_0 = (v_0 \cos \theta_0)t \qquad \text{.......... 1}$$

$$0 = (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2 \qquad \text{...........2}$$



By substituting the time t in the above equation

$$R = (v_\circ \sin \theta_\circ) \frac{v_\circ \sin \theta_\circ}{g} - \frac{1}{2} g \left[\frac{v_\circ \sin \theta_\circ}{g}\right]^2$$

$$R = \frac{v_\circ^2 \sin^2 \theta_\circ}{2g} \quad \dots\dots\dots\dots.3$$

$$R = \frac{v_\circ^2}{g} \sin 2\theta \quad \dots\dots\dots 4$$

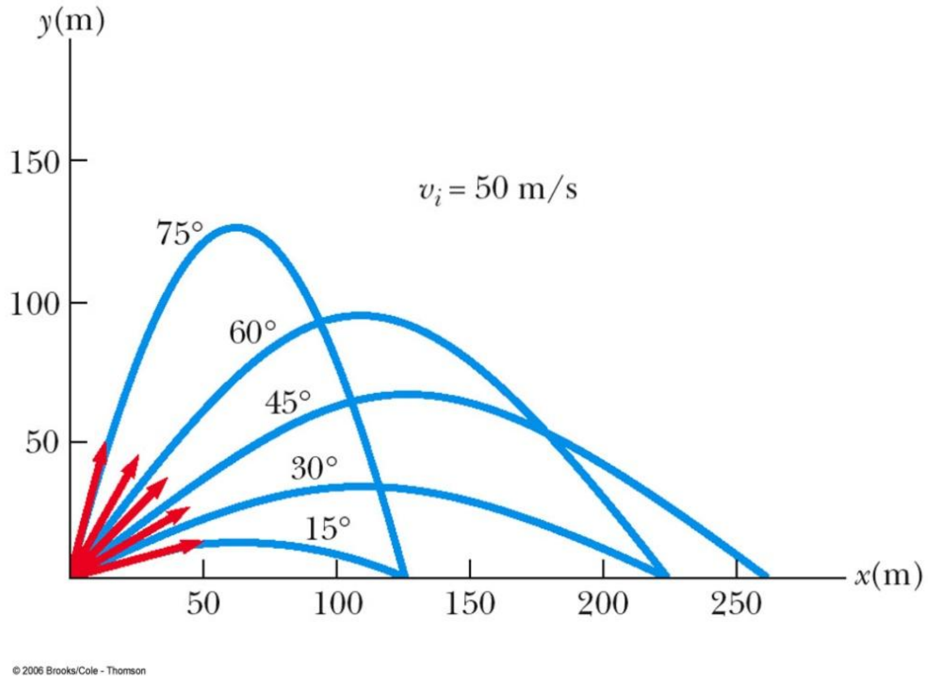
من المعادلة 3 نلاحظ اقصى ارتفاع يصل اليه الجسم المتحرك في بعدين كحركة المقذوفات على عجلة الجاذبية.

- 90 or 45°.

➢Projectile Motion at Various Initial Angles

- **Complementary values of the initial angle result in the same range**
  - **The heights will be different**
- **The maximum range occurs at a projection angle of 45º**

$$R = \frac{v_0^{\ 2}\sin 2\phi}{g}$$



Example: A long jumper leaves the ground at an angle of 20^o to the horizontal and a speed of 11 *ms*. A- How far does he jump? B- The maximum reached?

A- $x = (v_\circ \cos\theta_\circ)t = (11 \times \cos 20^\circ)t$

X can be found if t is known from the equation

$$v_y = v_\circ \sin\theta^\circ - gt_1$$

$$0 = 11\sin 20 - gt_1$$

t_1 =0.384 s where t_1 is the time required to reach the top then

$t = 2t_1$

T=0.768 s

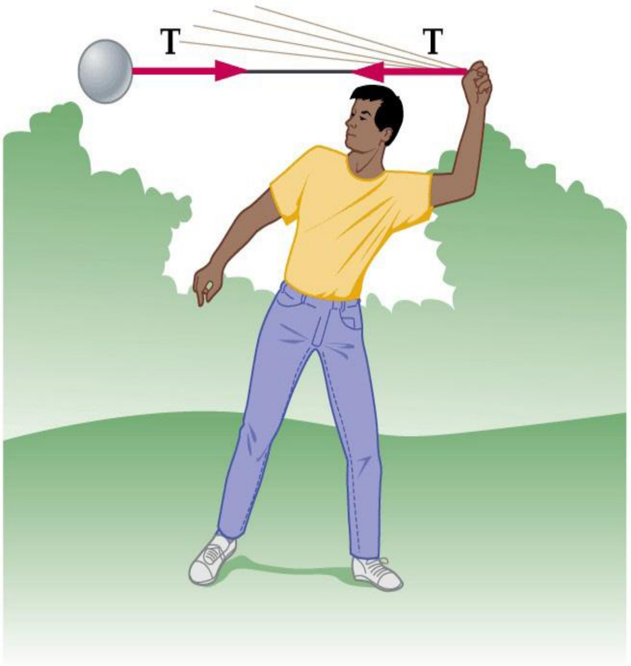
Therefore x= 7.94 m

B- The maximum height reached is found using the value of $t_1 = 0.384\ s$

$$y_{max} = (v_\circ \sin\theta^\circ)t_1 - \frac{1}{2}gt_1^2$$

$$y_{max} = 0.722\ m$$

Uniform Circular Motion



- من الممكن ان يتحرك جسم على مسار دائري بسرعة خطية ثابتة **linear constant speed** قد يخطر لنا الان ان العجلة في هذه الحالة تساوي صفرا. وذلك لان السرعة ثابتة, وهذا غير صحيح لان الجسم يتحرك على مسار دائري لذا توجد عجلة.
- نحن نعلم ان السرعة كمية متجهة. والعجلة هي عبارة عن كمية متجهة لانها تساوي معدل التغير في السرعة بالنسبة للزمن, والتغيير في السرعة قد يكون في المقدار او في الاتجاه.
- في حالة حركة الجسم على مسار دائري فان العجلة لا تؤثر على على مقدار السرعة وانما تغير من اتجاه السرعة. لهذا فان الجسم يتحرك على مسار دائري وبسرعة ثابتة.
- يكون متجه السرعة عموديا على نصف القطر وفي اتجاه المماس عند اية نقطة على المسار الدائري كما في الشكل.

Uniform Circular Motion

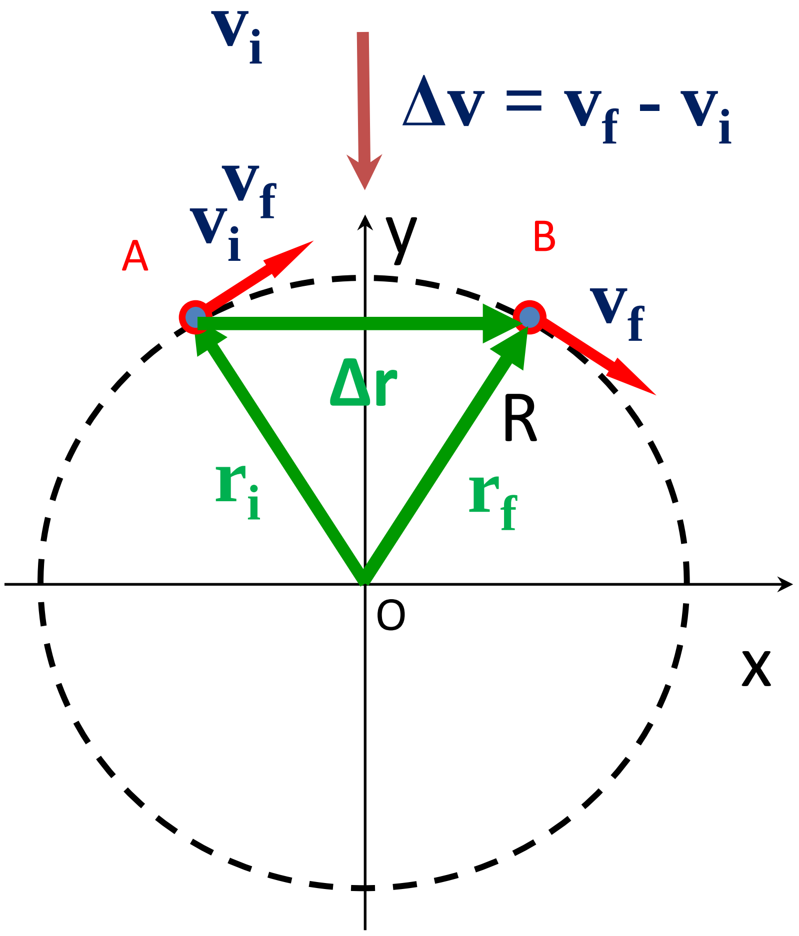
- **Centripetal acceleration**

$$\frac{\Delta v}{v} = \frac{\Delta r}{r} \quad \text{so,} \quad \Delta v = \frac{v\Delta r}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{\Delta r}{\Delta t}\frac{v}{r} = \frac{v^2}{r}$$

$$a_r = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

- **Direction: Centripetal**

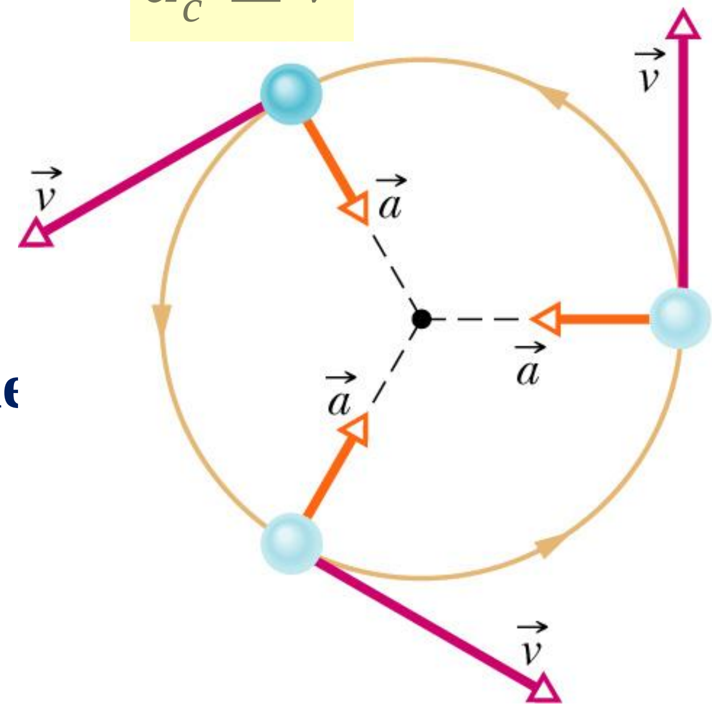


❖Uniform Circular Motion

- **Velocity:**
  - Magnitude: constant v
  - The direction of the velocity is tangent to the circle
- **Acceleration:**
  - Magnitude: $$a_c = \frac{v^2}{r}$$
  - directed toward the center of the circle of motion
- **Period:**
  - time interval required for one complete revolution of the particle

$$T = \frac{2\pi r}{v}$$

$$\vec{a}_c \perp \vec{v}$$



Summary

- **Position** $\vec{r}(t) = x\hat{i} + y\hat{j}$
- **Average velocity** $\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} = v_{avg,x}\hat{i} + v_{avg,y}\hat{j}$
- **Instantaneous velocity** $v_x \equiv \frac{dx}{dt}$ $v_y \equiv \frac{dy}{dt}$

$$\vec{v}(t) = \lim_{t\to 0}\frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

- **Acceleration** $a_x \equiv \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$ $a_y \equiv \frac{dv_y}{dt} = \frac{d^2y}{dt^2}$

$$\vec{a}(t) = \lim_{t\to 0}\frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j}$$

- $\vec{r}(t)$, $\vec{v}(t)$, and $\vec{a}(t)$ **are not necessarily in the same direction.**

Summary

- If a particle moves with constant acceleration *a*, motion equations are

$$\boxed{\vec{r}_f = \vec{r}_i + \vec{v}_i t + \tfrac{1}{2}\vec{a}t^2}$$

$$\vec{r}_f = x_f\hat{i} + y_f\hat{j} = (x_i + v_{xi}t + \tfrac{1}{2}a_{xi}t^2)\hat{i} + (y_i + v_{yi}t + \tfrac{1}{2}a_{yi}t^2)\hat{j}$$

$$\boxed{\vec{v} = \vec{v}_i + \vec{a}t}$$

$$\boxed{\vec{v}_f(t) = v_{fx}\hat{i} + v_{fy}\hat{j} = (v_{ix} + a_x t)\hat{i} + (v_{iy} + a_y t)\hat{j}}$$

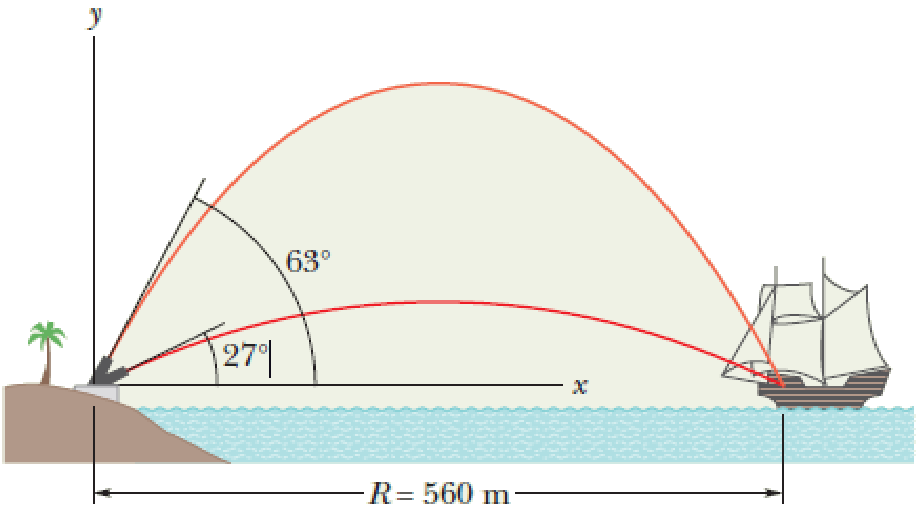
- Projectile motion is one type of 2-D motion under constant acceleration, where $a_x = 0$, $a_y = -g$.

Question1

- **Figure shows a pirate ship 560 m from a fort defending a harbour entrance. A defense cannon, located at sea level, fires balls at initial speed *v0 82 m/s.***

(a) At what angle u0 from the horizontal must a ball be fired to hit the ship?

(b) What is the maximum range of the cannonballs?



A pirate ship under fire.

Question2

- **A particle moves in a circular path 0.4m in radius with speed. If the particle makes five revolution in each second of its motion. Find**
- **a- the speed of the particle?**
- **b- its acceleration.**

Lecture ﴾4﴿

Newton's Law Of Motion

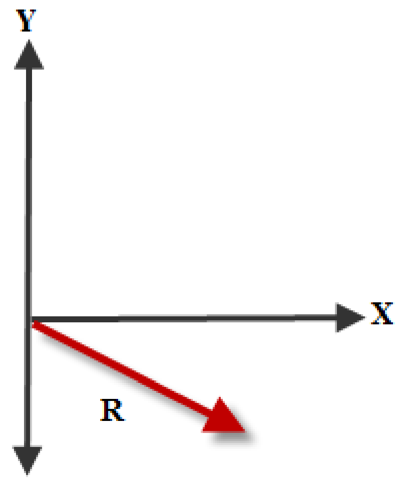
Dr. Hind I. Al-Shaikh

Q-Find the sum of two vector $\vec{A}$ and $\vec{B}$ giving by:

Solution

Note that A_x =3, A_y=2,B_x =2, and B_y=-5

1-The magnitude of vector $\vec{R}$.



2-The direction of $\vec{R}$ with respect to x-axis.

Four Forces Known in the Universe

- **Electromagnetic- caused from electric and magnetic interactions**

- **Strong Nuclear- Responsible for holding nucleus together in the atom; strongest force; acts over the shortest distance**

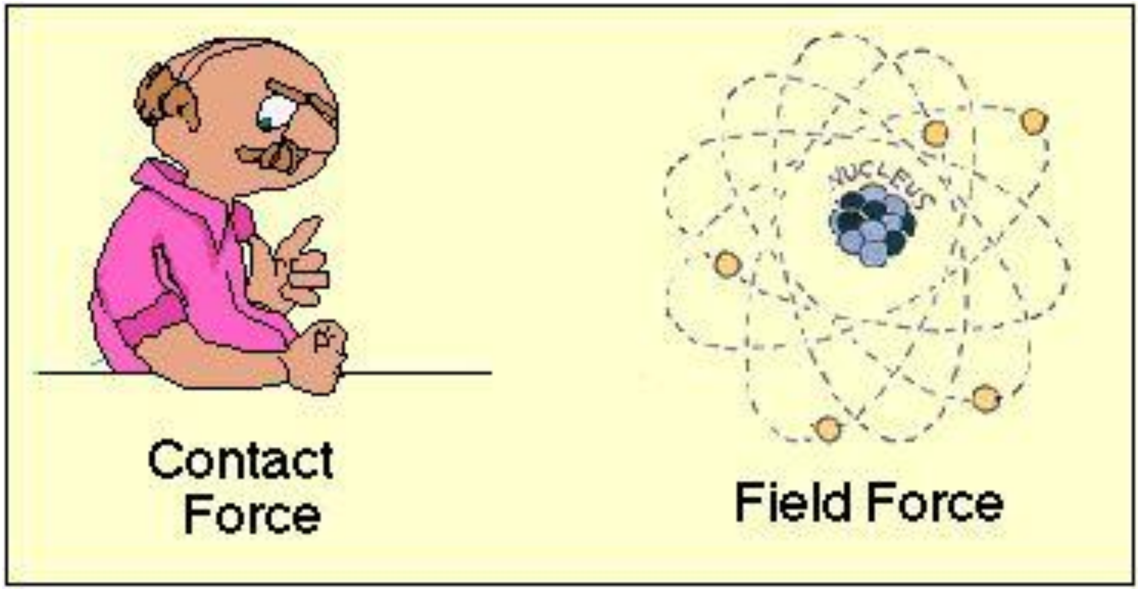
- **Gravitation- weakest force; acts over the longest distance**

- **Weak Nuclear- Responsible for radioactivity in atoms**

❖The concept of force مفهوم القوة

- نتعامل في حياتنا مع العديد من انواع القوى المختلفة التي قد تؤثر على الاجسام المتحركة فتغير من سرعتها مثل شخص يدفع عربة او يسحبها.
- او ان تؤثر القوة على الاجسام الساكنة لتبقيها ساكنة مثل الكتاب على الطاولة او الصورة المعلقة على الحائط.
- قد يكون تاثير القوة مباشر مثل عملية سحب او دفع صندوق وتسمى القوة **Contact force**.
- او ان يكون تاثير القوة عن بعد مثل تجاذب او تنافر قطبي مغناطيس وتسمى القوة **Field force**.

- **Types of Forces**
  - **Contact**
  - **Field**



❑The concept of force

It is not always force needed to move object from one place to another but force are also exist when object do not move. For example , when you read a book you exert force holding the book against the force of gravitation.

يعرف الجسم الساكن بانه في حالة اتزان عندما تكون محصلة القوى المؤثرة عليه تساوي صفرا The body in equilibrium.

It is very important to know that when a body is at rest or when moving at constant speed we say that the net force on the body is zero, the body in equilibrium.

Two Types of Forces

- **Example of Contact Forces**
  - Friction
  - Tension

- **Examples of Field Forces**
  - Gravitational
  - Electric
  - Magnetic
  - Applied
  - Spring

❖Adding Forces

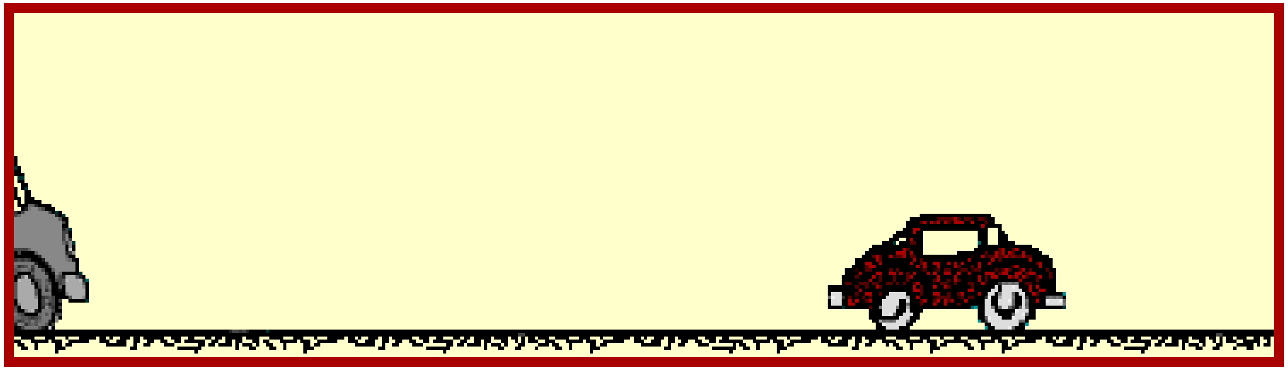
- **Forces are vectors (They have both magnitude and direction) and so add as follows:**
- **In one dimension, note direction using a + or – sign then add like scalar quantities (regular numbers with no direction associated with them)**
- **Examples:**

$$\xrightarrow[+3\text{ N}]{} \; + \; \xrightarrow[+3\text{ N}]{} \; = \; \xrightarrow[+6\text{ N}]{\qquad\qquad}$$

$$\xrightarrow[+3\text{ N}]{} \; + \; \xleftarrow[-3\text{ N}]{} \; = \; 0\text{ N}$$

❖Force and mass

- Mass – measurement of how difficult it is to change the objects velocity
- Inertia العطالة – resistance to change in velocity
- So mass is a measurement of an object's inertia



Newton's Laws

Sir Isaac Newton (1643-1727)

an English scientist and mathematician famous for

his discovery of the law of

gravity also discovered the

three laws of motion.

Today these laws are known as Newton's Laws of Motion and describe the motion of all objects on the scale we experience in our everyday lives.

Newton's Laws of Motion

1. An object in motion tends to stay in motion and an object at rest tends to stay at rest unless acted upon by an unbalanced force.
2. Force equals mass times acceleration (F = ma).
3. For every action there is an equal and opposite reaction.

Newton's First Law

An object at rest tends to stay at rest and an object in motion tends to stay in motion unless acted upon by an unbalanced force.

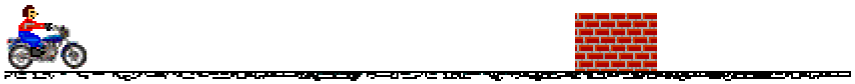
يفسر القانون الاول لنيوتن حالة الاجسام التي تؤثر عليها مجموعة قوى محصلتها تساوي صفرا, حيث يبقى الجسم الساكن ساكنا والجسم المتحرك يبقى متحركا بسرعة ثابتة مالم يؤثر عليه قوى غير متوازنة.

What does this mean?

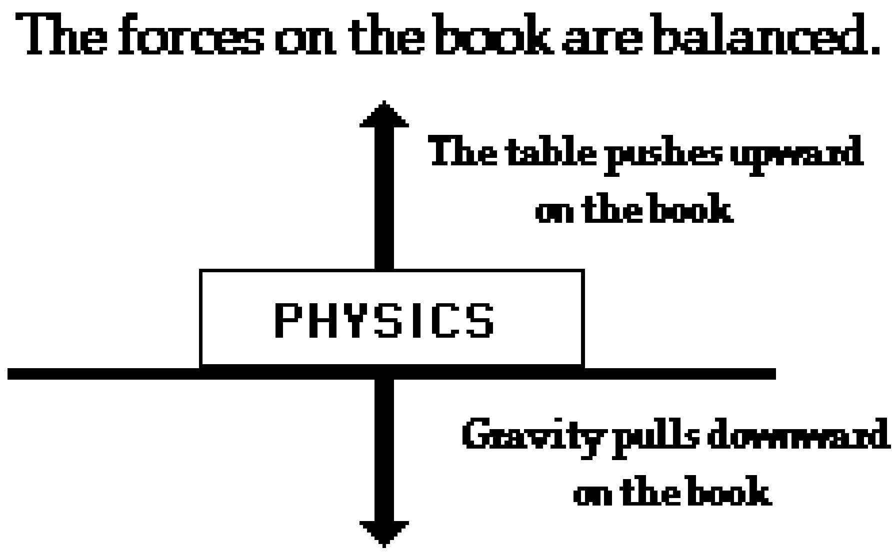
Basically, an object will “keep doing what it was doing” unless acted on by an unbalanced force.

If the object was sitting still, it will *remain stationary*. If it was moving at a constant velocity, it will *keep moving*.

It takes *force* to change the motion of an object.



What is meant by *unbalanced force?*



- **If the forces on an object are equal and opposite, they are said to be balanced, and the object experiences no change in motion.**

- **If they are not equal and opposite, then the forces are unbalanced and the motion of the object changes.**

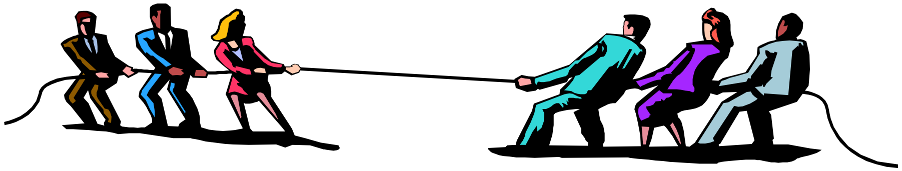
What is this unbalanced force that acts on an object in motion?

Friction!

- There are four main types of friction:
  - Sliding friction: ice skating
  - Rolling friction: bowling
  - Fluid friction (air or liquid): air or water resistance
  - Static friction: initial friction when moving an object

Some Examples from Real Life

A soccer ball is sitting at rest. It takes an unbalanced force of a kick to change its motion.



Two teams are playing tug of war. They are both exerting equal force on the rope in opposite directions. This balanced force results in no change of motion.

Question

- **What is the relationship between mass and inertia?**
- **Mass is a measure of how much inertia something has.**
- **Is inertia a force?**
- **No, inertia is a *property* of matter. Something has inertia. Inertia does not act on something.**
- **A force of gravity between the sun and its planets holds the planets in orbit around the sun. If that force of gravity suddenly disappeared, in what kind of path would the planets move?**
- **Each planet would move in a straight line at constant speed.**

Newton’s Second Law

The law of acceleration , state that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

قانون نيوتن الثاني يختص بالاجسام التي تؤثر عليها قوة خارجية تؤدي الى تحريكها بعجلة او ان تغير من سرعتها اذا كانت الاجسام متحركة .

Force equals mass times acceleration.

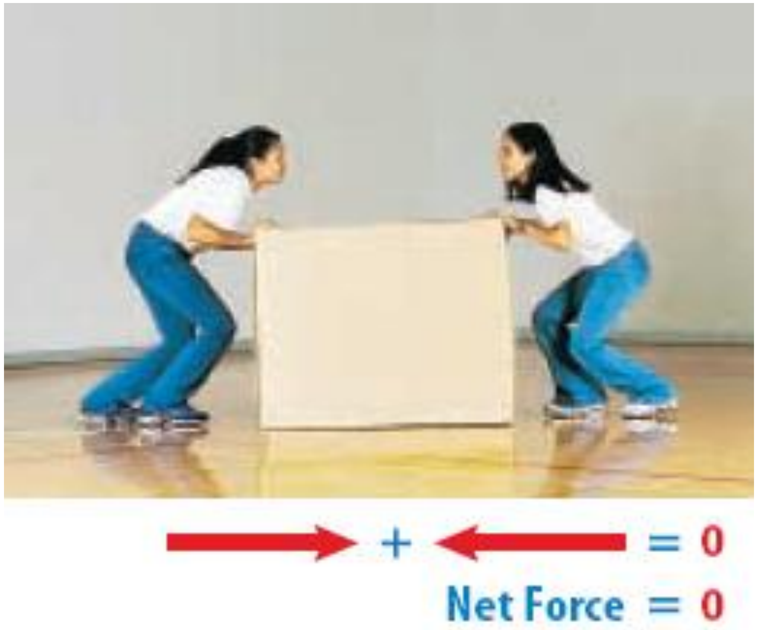
F = ma

The net force of an object is equal to the product of its mass and acceleration, or F=ma.

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}, \qquad \vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m}$$

<u>Acceleration</u>: a measurement of how quickly an object is changing speed.

Balanced Versus Unbalanced



Balanced forces cause no acceleration.

Unbalanced forces cause acceleration.

➢What does *F = ma* mean?

Force is *directly proportional* to mass and acceleration. Imagine a ball of a certain mass moving at a certain acceleration. This ball has a certain force.

Now imagine we make the ball twice as big (double the mass) but keep the acceleration constant. F = ma says that this new ball has *twice the force* of the old ball.

- When mass is in kilograms and acceleration is in m/s^2, the unit of force is in newtons (N).

In other words…..using the same amount of force….

F

Large Mass

Small acceleration

a

F

Small Mass

Large acceleration

a

More about F = ma

F = ma basically means that the force of an object comes from its mass and its acceleration.

Force is measured in

Newtons (N) = mass (kg) x acceleration (m/s^2)

$$1 \text{ Newton} = 1 \text{ kg} * \frac{\text{m}}{\text{s}^2}$$

If you *double* the mass, you *double* the force. If you *double* the acceleration, you *double* the force.

What if you double the mass *and* the acceleration?

(2m)(2a) = 4F

Using Newton's 2^{nd} Law to Solve Problems

1. **Identify all forces acting on the object**

   -Pushes or Pulls -Frictional forces -Tension in a string
   -Gravitational Force (or weight = mg where g is $9.8\ m/s^2$)
   - "Normal forces" (one object touching another).

2. **Draw a "Freebody Diagram"**
   -draw the object, show all forces acting <u>on that object</u> as vectors pointing in the correct direction. Show the direction of the acceleration.

3. **Chose a coordinate system.**

4. **Translate the freebody diagram into an algebraic expression based on Newton's second law.**

2nd Law (F = m x a)

- How much force is needed to accelerate a 1400 kilogram car 2 meters per second/per second?
- Write the formula
- F = m x a
- Fill in given numbers and units
- F = 1400 kg x 2 meters per second/second
- Solve for the unknown
- 2800 kg-meters/second/second or 2800 N

Newton's Third Law

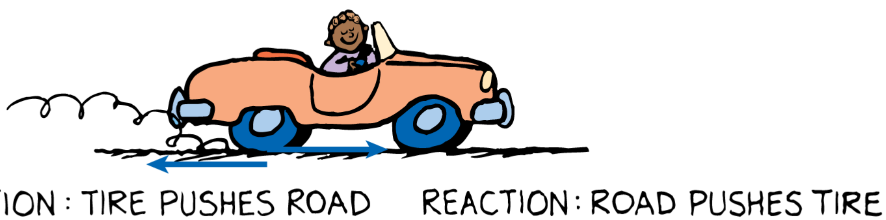
For every action there is an equal and opposite reaction.

يختص القانون الثالث لنيوتن على القوة المتبادلة بين الاجسام حيث انه اذا ثرت بقوة على جسم ما وليكن كتاب ترفعه بيدك فان الكتاب بالمقابل يؤثر بنفس القوة على يدك وفي الاتجاه المعاكس.

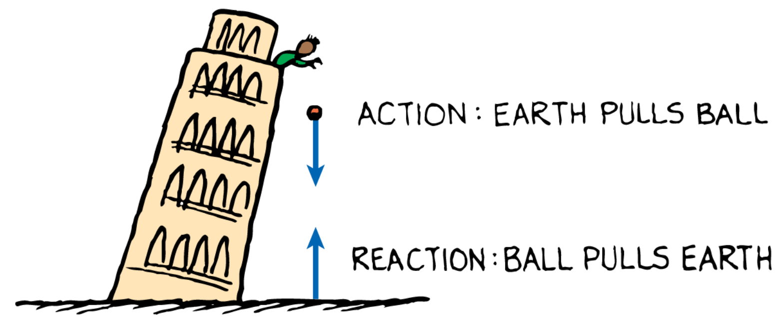
Identifying Action and Reaction Pairs

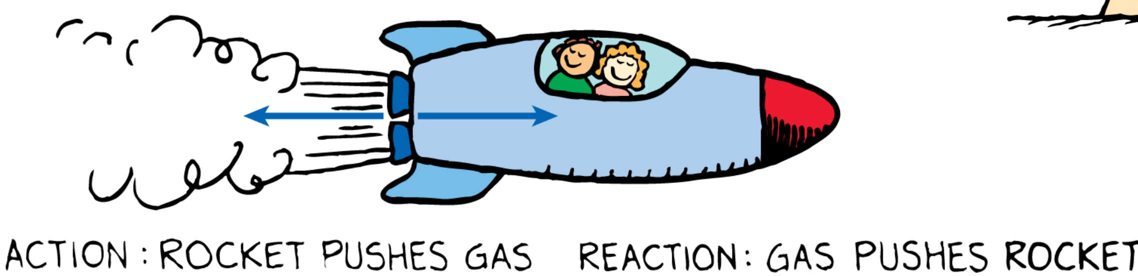
$$F_{12} = -F_{21}$$

يتضح من الاشكال مفهوم قانون نيوتن الثالث لقوة الفعل ورد الفعل.



F_{12} $\qquad$ $-F_{21}$





Example

- **A ball is hold in a persons hand. (a)Identify all the external forces acting on the ball and the reaction to each of these forces. (b)If the ball is dropped , what force is exerted on it while it is flight? Identify the reaction force in this case.**
- **(a)The external forces acting on the ball are:**
- **1- *FH the force which the hand exerts on the ball.***
- ***2- W the force of gravity exerted on the ball by the earth.***
- ***The reaction forces are:***
- ***1- FH : The force which the ball exerts on the hand.***
- ***2- W: The gravitational force which the ball exerts on the earth.***
- **(b)When the ball is in free fall the only force exerted on it it is its weight *W* which is exerted by the earth. The reaction force is the gravitational force which the ball exerts on the earth.**

EXAMPLE: Parts A, B, and C of Fig. 1 show three situations in which one or two forces act on a puck that moves over frictionless ice along an *x axis, in one-dimensional motion. The puck's* mass is *m = 0.20 kg. Forces and are directed along the* axis and have magnitudes *F1 4.0 N and F2 2.0 N. Force* is directed at angle θ = 30° and has magnitude *F3 = 1.0* N. In each situation, what is the acceleration of the puck?

In each situation we can relate the acceleration $\vec{a}$ to the net force $\vec{F}_{net}$ acting on the puck with Newton's second law, $\vec{F}_{net} = m\vec{a}$. However, because the motion is along only the x axis, we can simplify each situation by writing the second law for x components only:

$$F_{net,x} = ma_x.$$

Situation A: For Fig. 1-b where only one horizontal force acts, Eq. 5-4 gives us

$$F_1 = ma_x,$$

which, with given data, yields

$$a_x = \frac{F_1}{m} = \frac{4.0\text{ N}}{0.20\text{ kg}} = 20\text{ m/s}^2.$$

The positive answer indicates that the acceleration is in the positive direction of the x axis.

Situation B: In Fig. 1-d two horizontal forces act on the puck, $\vec{F}_1$ in the positive direction of x and $\vec{F}_2$ in the negative direction. Now Eq. 5-4 gives us

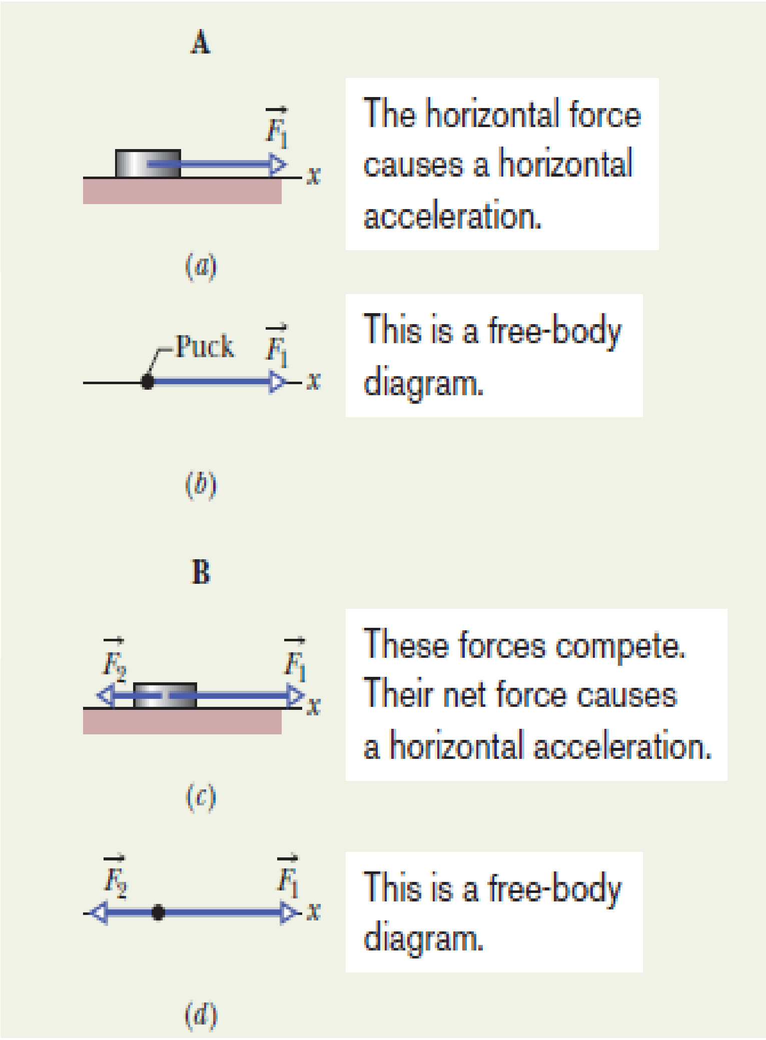
$$F_1 - F_2 = ma_x,$$

which, with given data, yields

$$a_x = \frac{F_1 - F_2}{m} = \frac{4.0\text{ N} - 2.0\text{ N}}{0.20\text{ kg}} = 10\text{ m/s}^2.$$

(Answer)

Thus, the net force accelerates the puck in the positive direction of the x axis.



Situation C: In Fig. *f*, force $\vec{F}_3$ is not directed along the direction of the puck's acceleration; only *x* component $F_{3,x}$ is. (Force $\vec{F}_3$ is two-dimensional but the motion is only one-dimensional.)

$$F_{3,x} - F_2 = ma_x.$$

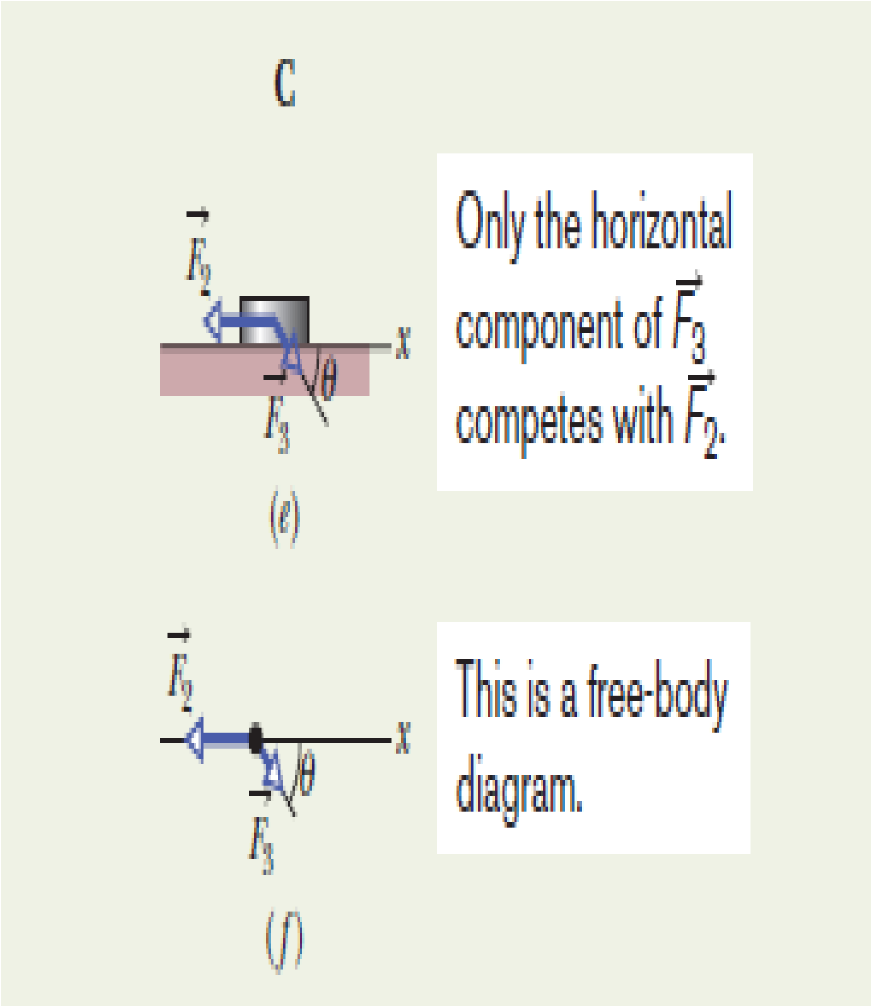
From the figure, we see that $F_{3,x} = F_3 \cos\theta$. Solving for the acceleration and substituting for $F_{3,x}$ yield

$$a_x = \frac{F_{3,x} - F_2}{m} = \frac{F_3 \cos\theta - F_2}{m}$$

$$= \frac{(1.0\text{ N})(\cos 30^\circ) - 2.0\text{ N}}{0.20\text{ kg}} = -5.7\text{ m/s}^2.$$

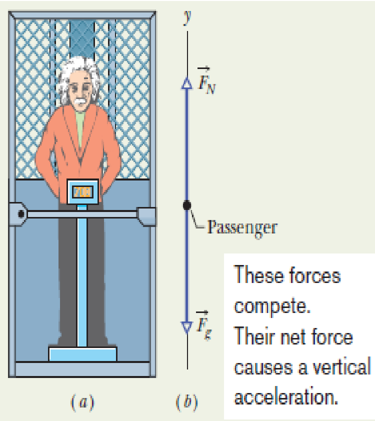
(Answer)

Thus, the net force accelerates the puck in the negative direction of the *x* axis.



❖Weight and tension

- نعلم ان الوزن Weightهو كمية فيزيائية لها وحدة القوة N وهي ناتجة من تاثير عجلة الجاذبية الارضية gوعلى كتلة الجسم m وبتطبيق قانون نيوتن الثاني على جسم موجود على بعد قريب من سطح الارض حيث يتاثر بقوة الجاذبية الارضية ومقدارها كتلة الجسم في عجلة الجاذبية الارضية وبالتالي فان الوزن $W= mg$



➢ *ان وزن شخص موجود في المصعد يتغير في حالة الصعود والهبوط بالشكل التالي:*

➢ *1- عندما يتحرك المصعد بدون عجلة (سرعة ثابتة) فان العجلة تكون صفرا ويكون الوزن المقاس هو الوزن الحقيقي للشخص.*

➢ *2- عندما يتحرك المصعد الى الاعلى تكون العجلة موجبة.* $F_N = mg + ma$

➢ *3- عندما يتحرك المصعد نحو الاسفل فان العجلة تكون سالبة.* $F_N = mg - ma$

➢ *هنا سوف نستخدم قانون نيوتن الثاني وبتحليل القوى المؤثرة على الشخص في المصعد نجد ان هناك قوتين الاولى هي وزن الشخص* $W= mg$ *والقوة الاخرى هي قوة رد فعل المصعد على الشخص* FN *بتطبيق قانون نيوتن الثاني نجد ان*

$$\sum F = F_N - mg = ma$$

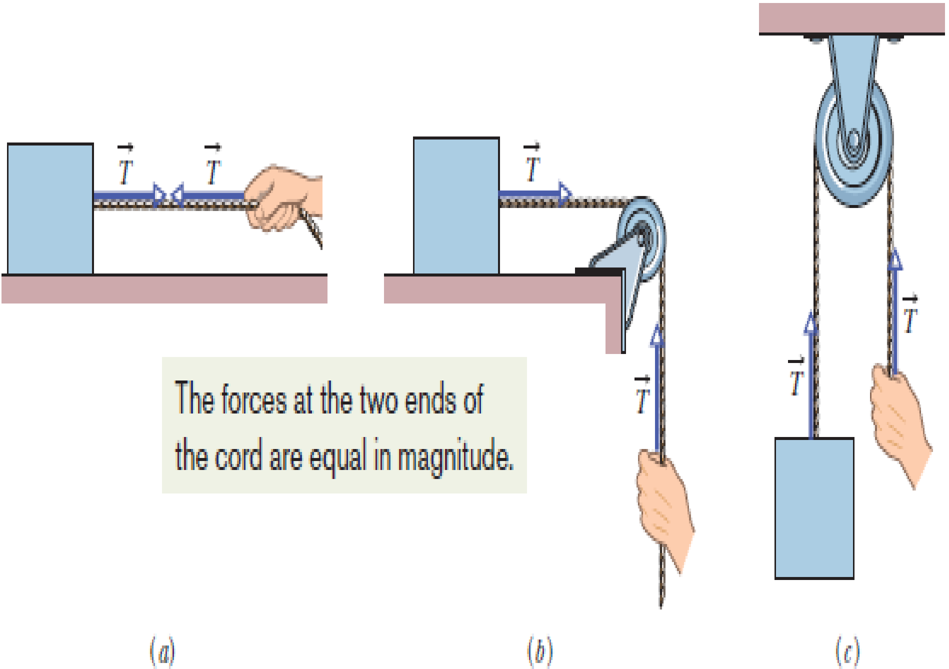
$$F_N = mg + ma$$

الوزن الظاهر
Apparent weight

True weight

Where *a is the acceleration of elevator*

❖Tension



- عند سحب جسم بواسطة حبل فان القوة المؤثرة على الجسم من خلال الحبل تدعى قوة الشد **Tension** ويرمز لها بالرمز **T** ووحدته ويظهر في الشكل صور مختلفة من قوة الشد وكيفية تحديدها.

EX. An electron of mass $9.1 \times 10^{-31} Kg$ has an initial speed of $3.0 \times 10^{5}\ m/s$ It travels in a straight line and its speed increases to $7.0 \times 10^{5}\ m/s$ in a distance of 5.0 cm. A assuming its acceleration is constant. A- determine the force on the electron . B- compare this force with the weight of the electron.

- **Solution**
- ***A-F=ma*** **and** $v^2 = v_\circ^2 + 2ax \ \ or \ \ a = \frac{(v^2 - v_\circ^2)}{2x}$

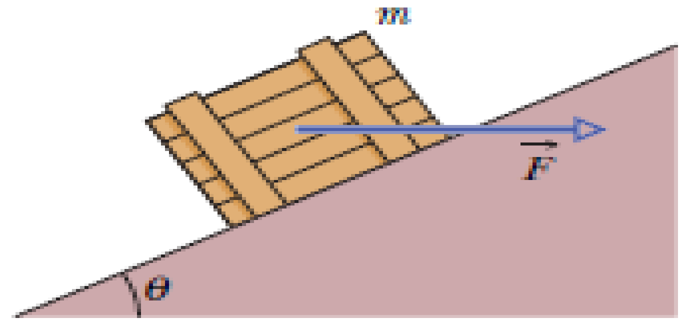
$$a = \frac{(v^2 - v_\circ^2)}{2x} = \frac{9.1 \times 10^{-31}\left((7.0 \times 10^5)^2 - (3.0 \times 10^5)^2\right)}{2 \times 10^{-2} \times 5.0} = 3.6 \times 10^{18} N$$

- **B- The weight of electron is:**

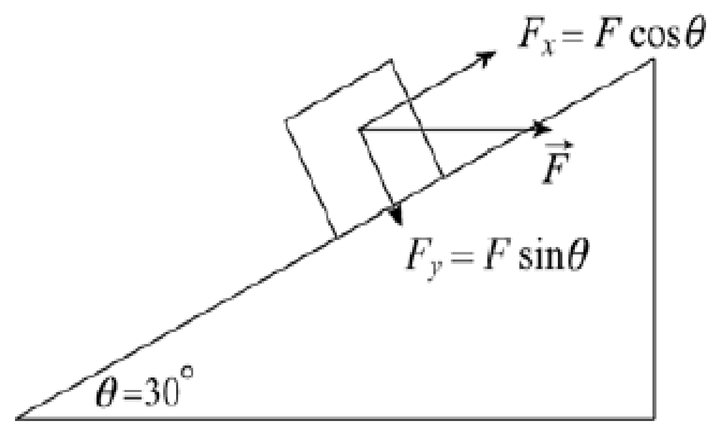
$$W = mg = (9.0 \times 10^{-31}\, kg) \times (9.8\, m/s^2) = 8.9 \times 10^{-30} N$$

- **The acceleration force is approximately 10^{11} times the weight of the electron.**

In Fig. 1, a crate of mass $m = 100$ kg is pushed at constant speed up a frictionless ramp ($\theta = 30.0°$) by a horizontal force $\vec{F}$. What are the magnitudes of (a) $\vec{F}$ and (b) the force on the crate from the ramp?



We resolve this horizontal force into appropriate components.



(a) Newton's second law applied to the x-axis produces

$$F\cos\theta - mg\sin\theta = ma.$$

For $a = 0$, this yields $F = 566$ N.

(b) Applying Newton's second law to the y axis (where there is no acceleration), we have

$$F_N - F\sin\theta - mg\cos\theta = 0$$

which yields the normal force $F_N = 1.13 \times 10^3$ N.

In Fig. 1 a block of mass $m = 5.00$ kg is pulled along a horizontal frictionless floor by a cord that exerts a force of magnitude $F = 12.0$ N at an angle $\theta = 25.0°$. (a) What is the magnitude of the block's acceleration? (b) The force magnitude F is slowly increased. What is its value just before the block is lifted (completely) off the floor? (c) What is the magnitude of the block's acceleration just before it is lifted (completely) off the floor?

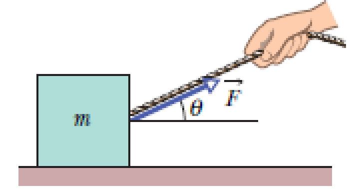
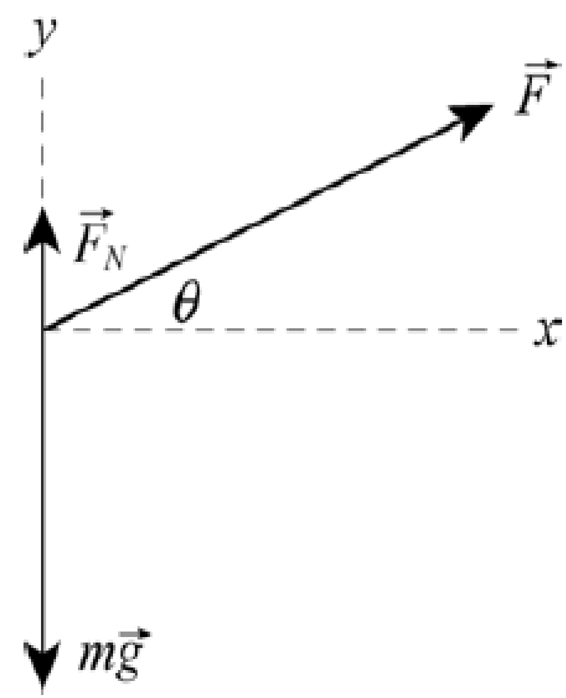


Fig. 1

The free-body diagram (not to scale) for the block is shown below. $\vec{F}_N$ is the normal force exerted by the floor and $m\vec{g}$ is the force of gravity.



(a) The x component of Newton's second law is $F\cos\theta = ma$, where m is the mass of the block and a is the x component of its acceleration. We obtain

$$a = \frac{F\cos\theta}{m} = \frac{(12.0\,\text{N})\cos 25.0°}{5.00\,\text{kg}} = 2.18\ \text{m/s}^2.$$

This is its acceleration provided it remains in contact with the floor. Assuming it does, we find the value of F_N (and if F_N is positive, then the assumption is true but if F_N is negative then the block leaves the floor). The y component of Newton's second law becomes

$$F_N + F\sin\theta - mg = 0,$$

so

$$F_N = mg - F\sin\theta = (5.00\text{ kg})(9.80\text{ m/s}^2) - (12.0\text{ N})\sin 25.0° = 43.9\text{ N}.$$

Hence the block remains on the floor and its acceleration is $a = 2.18\ \text{m/s}^2$.

(c) The acceleration is still in the x direction and is still given by the equation developed in part (a):

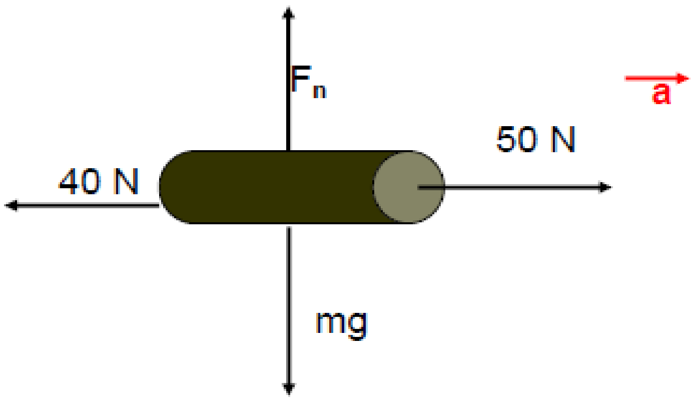
$$a = \frac{F\cos\theta}{m} = \frac{(116\text{ N})\cos 25.0°}{5.00\text{ kg}} = 21.0\,\text{m/s}^2.$$

❖Force of friction

- لقد اهملنا سابقا القوة الناتجة عن الاحتكاك وذلك بفرض ان الاجسام تتحرك على اسطح ناعمة وذلك حتى لانزيد من المعادلات الرياضية المصاحبة لحل المسائل ولكن بعد ان علمنا كيفية التعامل مع متجهات القوة بمختلف انواعها مثل الوزن والشد ورد الفعل والقوى الخارجية المؤثرة على الحركة سندخل في نوع اخر من القوة المؤثرة على الحركة وهي قوة الاحتكاك *Force of friction* ويرمز لها بالرمز f واتجاه هذه القوة دائما عكس اتجاه الحركة وهي ناتجة عن خشونة الاسطح المتحركة ويقسم الاحتكاك الى نوعين
- الاحتكاك السكوني *static friction*
- الاحتكاك الحركي *kinetic friction*
- ولقد وجد عمليا ان قوة الاحتكاك تتناسب طرديا مع قوة رد الفعل لهذا فان الاحتكاك يمكن ان يكتب كالتالي $f = \mu N$
- حيث $\mu_\circ$ تسمى معامل الاحتكاك وفي حالة الاحتكاك السكوني تسمى *coefficient of static friction*
- اما في حالة الاحتكاك الحركي μ تسمى *coefficient of kinetic friction*
- معامل الاحتكاك الحركي يكون دائما اكبر من معامل الاحتكاك السكوني ومعامل الاحتكاك ليس له وحدة.

Example

- A 50 N applied force drags an 8.16 kg log to the right across a horizontal surface. What is the acceleration of the log if the force of friction is 40.0 N?



$$F_{NET} = ma$$

$$F_a - F_f = ma$$

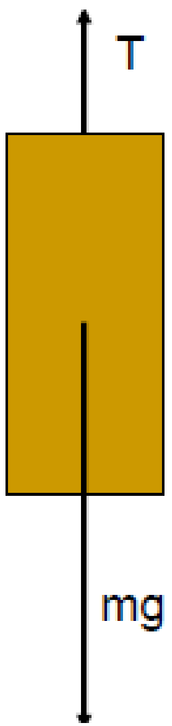
$$50 - 40 = 8.16a$$

$$10 = 8.16a$$

$a =$ 1.23 m/s/s

Example

- **An elevator with a mass of 2000 kg rises with an acceleration of 1.0 m/s/s. What is the tension in the supporting cable?**



Equation of Motion

$$F_{NET} = ma$$

$$\boxed{T - mg = ma}$$

$$T = ma + mg$$

$$T = (2000)(1) + (2000)(9.8)$$

$$T = \text{21,600 N}$$

Lecture (5)

Kinetic energy and Work

Dr. Hind I. Al-Shaikh

Q1 - A body moving with uniform acceleration has a velocity of 12 cm/s when its *x* coordinate is 3 cm. If its *x* coordinate 2 s later is -5 cm, what is the magnitude of its acceleration?

Q2- Write the projectile motion equation in horizontal and vertical motion with horizontal range and maximum height of a projectile?

Objectives

- **Define kinetic energy and potential energy, along with the appropriate units in each system.**
- **Describe the relationship between work and kinetic energy, and apply the WORK-ENERGY THEOREM.**
- **Define and apply the concept of POWER, along with the appropriate units.**

Work and Energy

- ان مفهوم الشغل والطاقة مهم جدا في علم الفيزياء حيث توجد الطاقة في الطبيعة في صور مختلفة مثل:

- ❑ الطاقة الميكانيكية *Mechanical energy*
- ❑ الطاقة الكهرومغناطيسية *Electromagnetic energy*
- ❑ الطاقة الكيميائية *Chemical energy*
- ❑ الطاقة الحرارية *Thermal energy*
- ❑ الطاقة النووية *Nuclear energy*

ان الطاقة بصورها المختلفة تتحول من شكل الى اخر ولكن في النهاية الطاقة الكلية ثابتة.

فمثلا الطاقة الكيميائية المختزنة في بطارية تتحول الى طاقة كهربائية لتتحول بدورها الى طاقة حركية ودراسة تحولات الطاقة مهم جدا في جميع العلوم.

Energy

Energy is anything that can be converted into work; i.e., anything that can exert a force through a distance. Energy is the capability for doing work.

Types of Energy

- Kinetic Energy
- Potential Energy

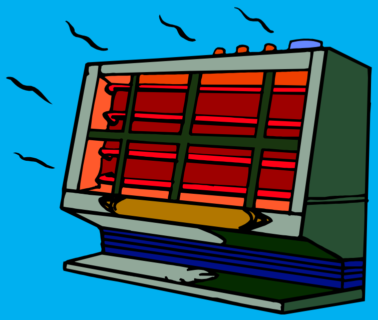
Forms of Energy

Radiant

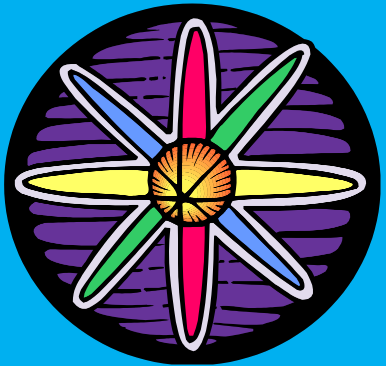
Electrical

Chemical

Sound



Thermal



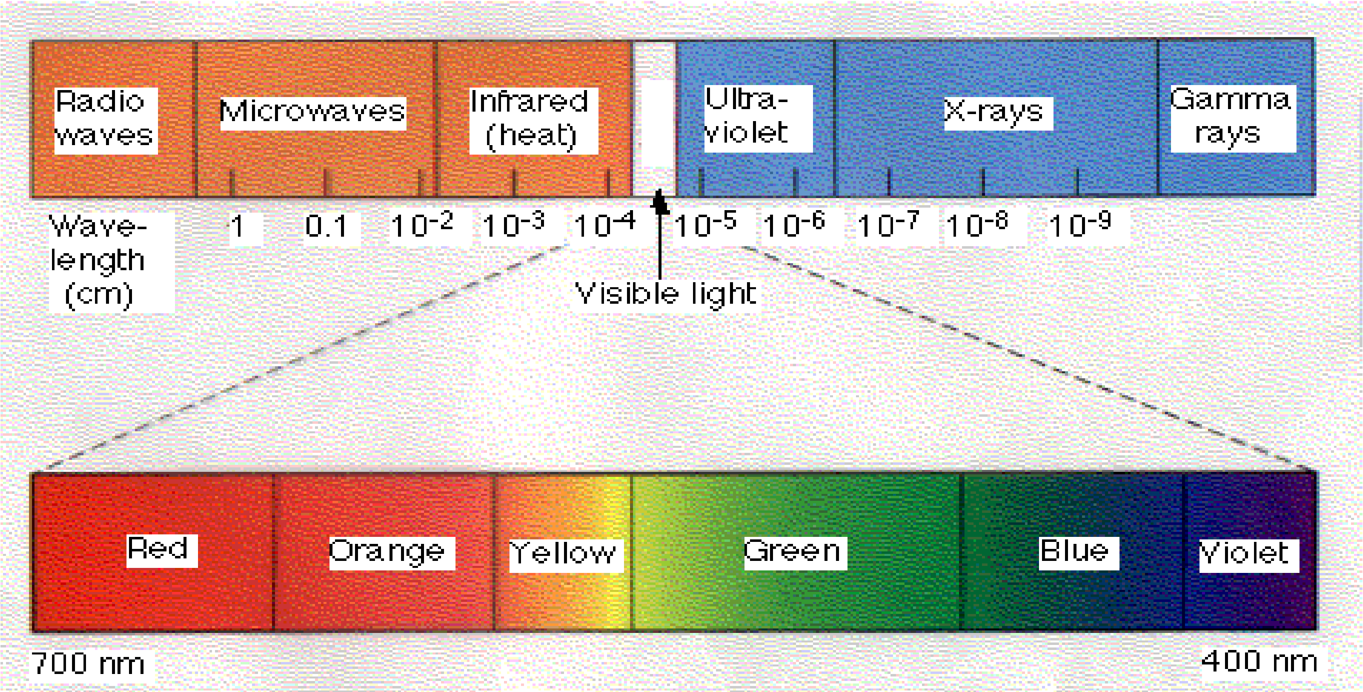
Nuclear

Mechanical

Magnetic

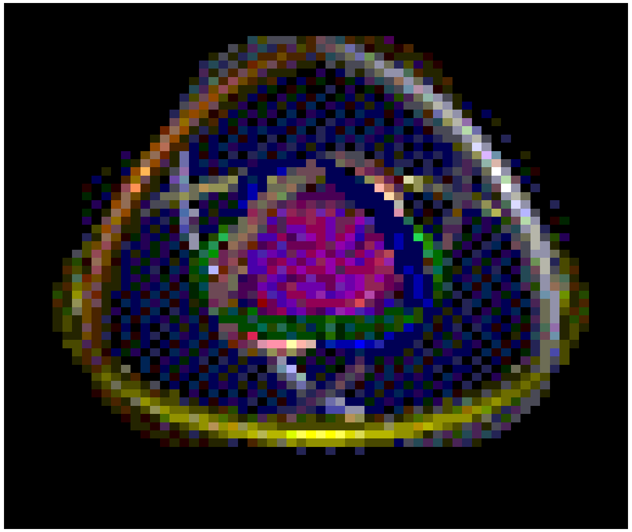
Radiant Energy

Radiant energy is also called electromagnetic energy. Radiant energy is the movement of photons. All life on earth is dependent on radiant energy from the sun. Examples of radiant energy include radio waves (AM, FM, TV), microwaves, X-rays, and plant growth. Active solar energy uses photovoltaic panels and light to turn radiant energy into chemical energy.



Chemical Energy

Chemical energy is the energy stored in the bonds of atoms and molecules. This a form of potential energy until the bonds are broken.



Electrical Energy

Electrical Energy traveling as the flow of charged particles (i.e. electrons) Lightning and static electricity are examples of electrical energy that occur naturally.

Nuclear Energy

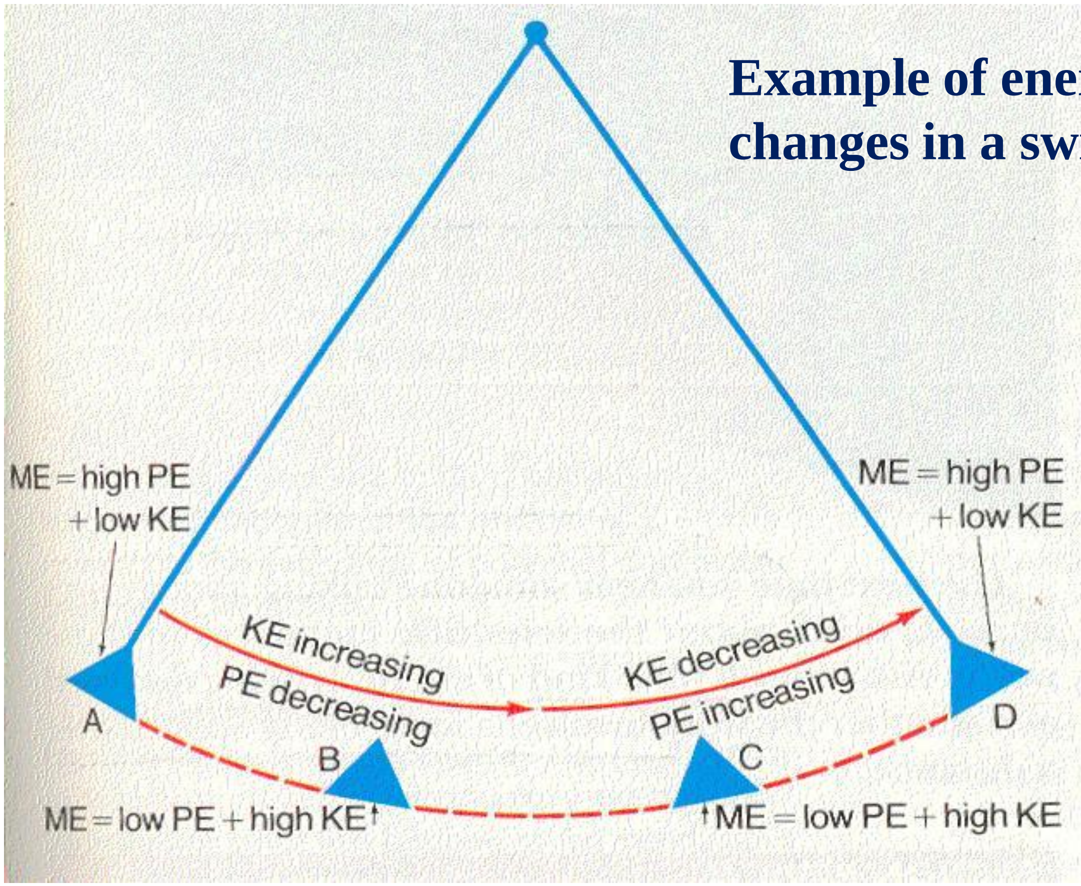
Nuclear energy Energy produced from the splitting of atoms. Submarines, power plants, and smoke detectors all use nuclear energy. Nuclear power plants use uranium, a radioactive element, to create electricity.

Thermal Energy

Thermal energy is the internal energy in substances-the vibration and movement of atoms and molecules within substance. Thermal energy is created in the movement of atoms. Boiling water, burning wood, and rubbing your hands together really fast are all examples of heat energy.

Mechanical Energy

Mechanical energy is the movement of machine parts. Mechanical energy is also the total amount of kinetic and potential energy in a system.



Example of energy chanExample of energy changes in a swing or pendulum.

Potential energy + Kinetic energy = Mechanical energy

Mechanical Energy

- **<u>Mechanical Energy:</u> Energy of movement and position**
- **There are two major types of mechanical energy:**
  - **<u>Potential Energy:</u> Energy of position**
  - **<u>Kinetic Energy:</u> Energy of motion**

Potential Energy

Potential energy exists whenever an object which has mass has a position within a force field. The most everyday example of this is the position of objects in the earth's gravitational field. The potential energy of an object in this case is given by the relation:

$$PE = mgh$$

PE = Energy (in Joules)

m = mass (in kilograms)

g = gravitational acceleration of the earth (9.8 m/sec2)

h = height above earth's surface (in meters)

Example Problem: **What is the potential energy of a 50-kg person in a skyscraper if he is 480 m above the street below?**

Gravitational Potential Energy

What is the P.E. of a 50-kg person at a height of 480 m?

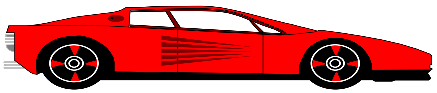
$$PE = mgh = (50\text{ kg})(9.8\text{ m/s}^2)(480\text{ m})$$

$$\boxed{PE = 235\text{ kJ}}$$

Kinetic Energy

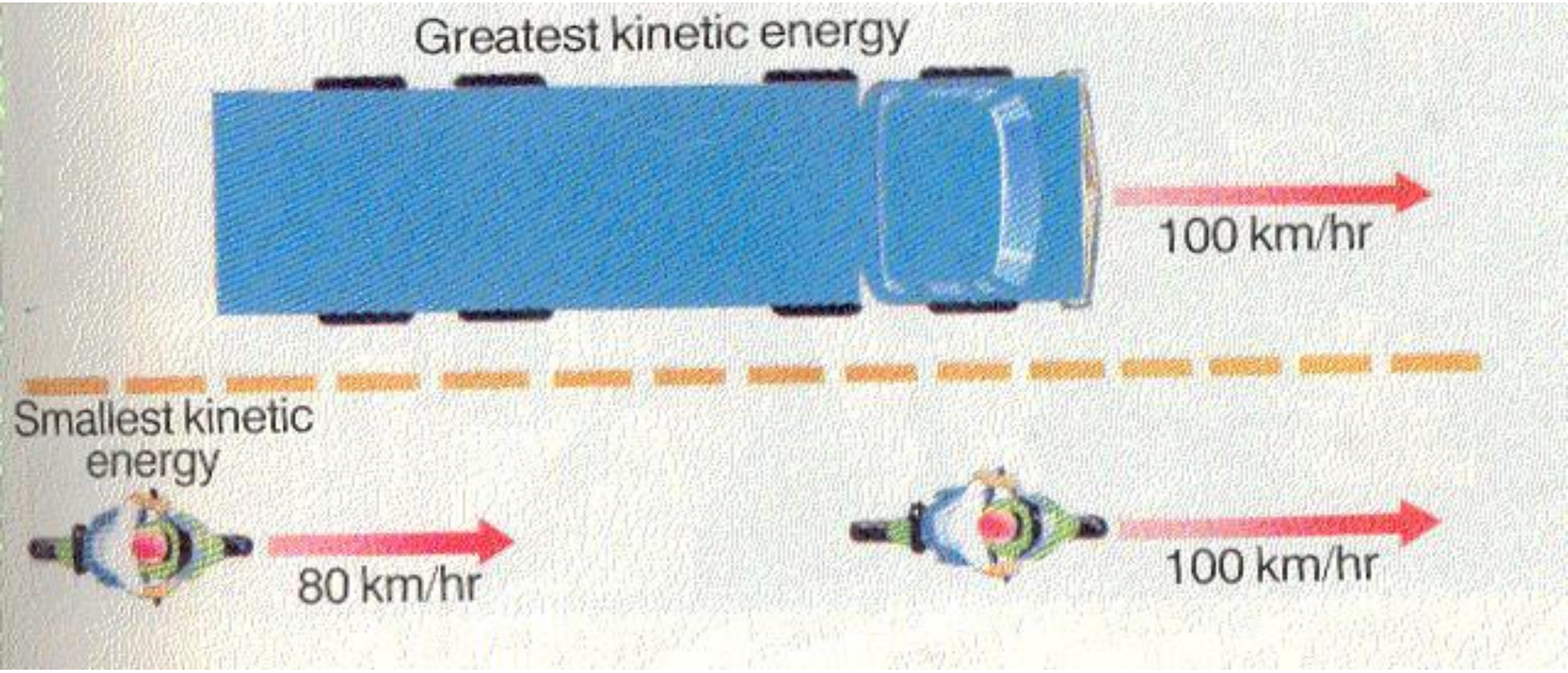
- Kinetic energy exists whenever an object which has mass is in motion with some velocity. (Mass with velocity)
- Everything you see moving about has kinetic energy. The kinetic energy of an object in this case is given by the relation:
- $KE = (1/2)mv^2$
- *m*=mass of the object
- *V*=velocity of the object

A speeding car



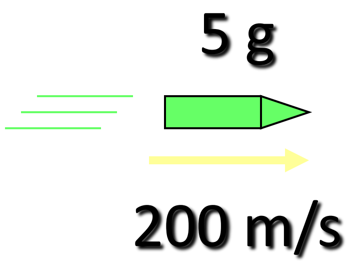
- The greater the mass or velocity of a moving object, the more kinetic energy it has.
- The faster the object moves, the greater is its kinetic energy. When the object is stationary, its kinetic energy is zero.

Kinetic Energy



Examples of Kinetic Energy

What is the kinetic energy of a 5-g bullet traveling at 200 m/s?

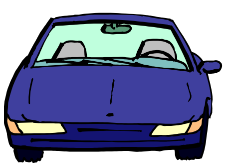


$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.005\text{ kg})(200\text{ m/s})^2$$

K = 100 J

What is the kinetic energy of a 1000-kg car traveling at 14.1 m/s?

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1000\text{ kg})(14.1\text{ m/s})^2$$



K = 99.4 J

Questions

1- Jill has a velocity of 5m/s. If she has a mass of 60kg, what is her kinetic energy?

2-If Bob, at 70kg, is standing on top of a 13m high hill. What is his potential energy?

Law of Conservation of Energy

Law of Conservation of Energy- Energy can neither be created nor destroyed. Energy is always changing from one kind to another. The total energy of an object never changes.

Potential energy + Kinetic energy = Total energy and Total energy – Kinetic energy = Potential energy and Total energy - Potential energy = Kinetic energy

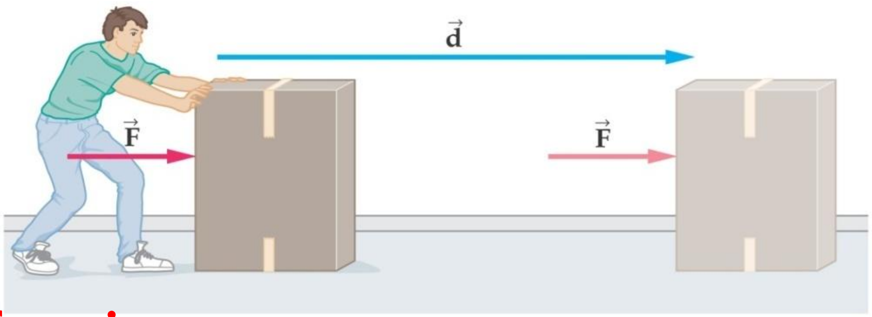
Potential and kinetic energy are <u>constantly</u> transforming back and forth. Most of the time during this transformation, some energy is turned to heat and transferred out of the system.

Work

Work *W* is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work. Work is force times distance

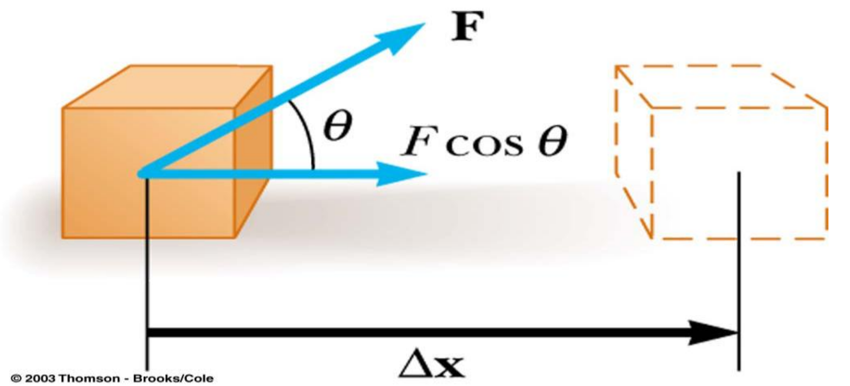
➢**Work done by a constant force**

$$W = F \cdot x ......(1)$$



➢**Work done by constant force in direction of motion**

$$W = F\Delta x \cos\theta ......(2)$$



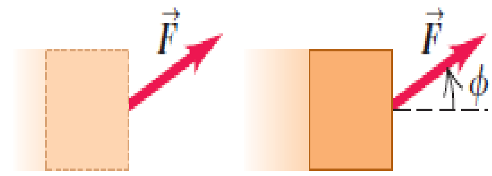
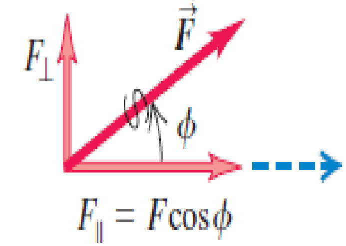
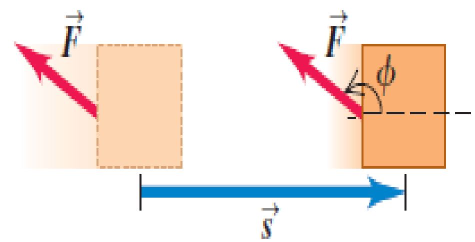
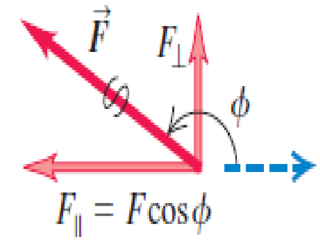
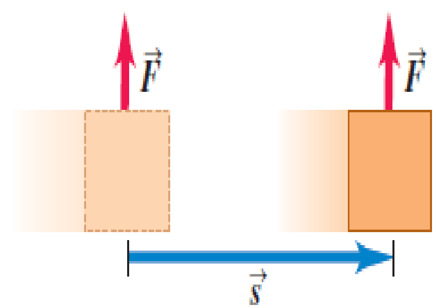
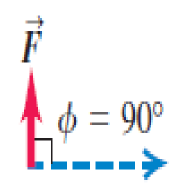
- **Scalar quantity**
- **Independent of time**

SI unit = Joule

1 J = 1 N·m = 1 kg·m^2/s^2

Work: Positive, Negative, or Zero

A force does positive work when it has a vector component in the same direction as the displacement, and it does negative work when it has a vector component in the opposite direction. It does zero work when it has no such vector component.

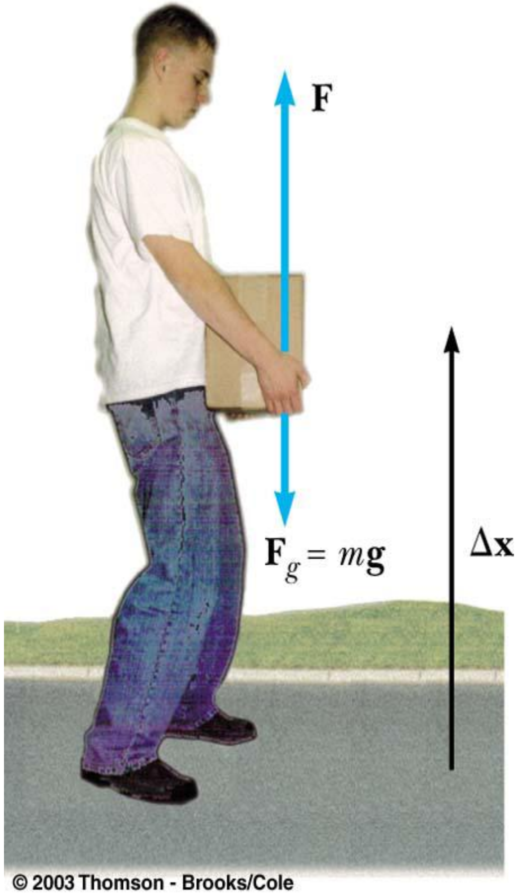
Direction of Force (or Force Component)	Situation	Force Diagram
(a) Force $\vec{F}$ has a component in direction of displacement: $W = F_{\parallel}s = (F\cos\phi)s$ Work is *positive*.		
(b) Force $\vec{F}$ has a component opposite to direction of displacement: $W = F_{\parallel}s = (F\cos\phi)s$ Work is *negative* (because $F\cos\phi$ is negative for $90° < \phi < 180°$).		
(c) Force $\vec{F}$ (or force component $F_{\perp}$) is perpendicular to direction of displacement: The force (or force component) does *no* work on the object.		

A force does positive work when it has a vector component in the same direction as the displacement, and it does negative work when it has a vector component in the opposite direction. It does zero work when it has no such vector component.

Work can be positive or negative

- **Man does positive work lifting box**
- **Man does negative work lowering box**

- **Gravity does positive work when box lowers**
- **Gravity does negative work when box is raised**



Work done by several forces

- **How do we calculate work when several forces act on a body? One way is to use Eq. 1 or 2 to compute the work done by each separate force. Then, because work is a scalar quantity, the total work done on the body by all the forces is the algebraic sum of the quantities of work done by the individual forces.**

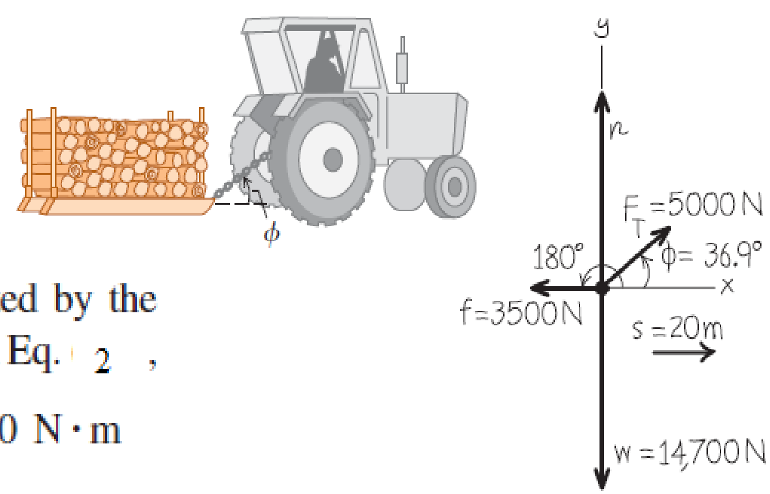
$$W = F \cdot x.....(1)$$

$$W = F \Delta x \cos\theta......(2)$$

EXP. A farmer hitches her tractor to a sled loaded with firewood and pulls it a distance of 20 m along level ground (Fig. 1 a). The total weight of sled and load is 14,700 N. The tractor exerts a constant 5000-N force at an angle of 36.9° above the horizontal. A 3500-N friction force opposes the sled's motion. Find the work done by each force acting on the sled and the total work done by all the forces.

1-The work done by the weight is zero because its direction is perpendicular to the displacement. For the same reason, the work done by the normal force is also zero.

(a) (b) Free-body diagram for sled



So $W_w = W_n = 0$

That leaves the work W_T done by the force F_T exerted by the tractor and the work W_f done by the friction force f. From Eq. 2 ,

$$W_T = F_T s \cos\phi = (5000\ \text{N})(20\ \text{m})(0.800) = 80{,}000\ \text{N}\cdot\text{m}$$
$$= 80\ \text{kJ}$$

The friction force $\vec{f}$ is opposite to the displacement, so for this force $\phi = 180°$ and $\cos\phi = -1$. Again from Eq. (6.2),

$$W_f = fs\cos 180° = (3500\ \text{N})(20\ \text{m})(-1) = -70{,}000\ \text{N}\cdot\text{m}$$
$$= -70\ \text{kJ}$$

(2) In the second approach, we first find the *vector* sum of all the forces (the net force) and then use it to compute the total work. The vector sum is best found by using components. From Fig. 1 'b,

$$\sum F_x = F_T \cos\phi + (-f) = (5000\ \text{N}) \cos 36.9° - 3500\ \text{N}$$
$$= 500\ \text{N}$$

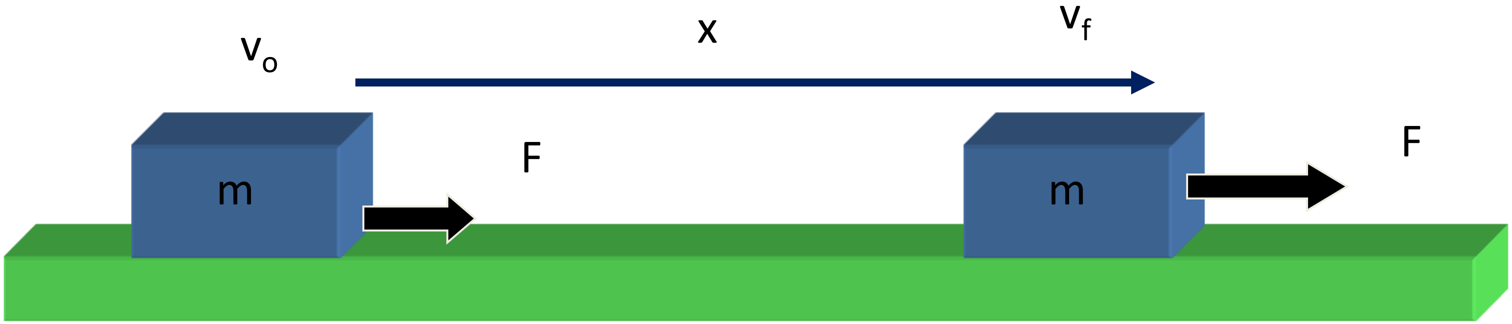
$$\sum F_y = F_T \sin\phi + n + (-w)$$
$$= (5000\ \text{N}) \sin 36.9° + n - 14{,}700\ \text{N}$$

We don't need the second equation; we know that the y-component of force is perpendicular to the displacement, so it does no work. Besides, there is no y-component of acceleration, so $\sum F_y$ must be zero anyway. The total work is therefore the work done by the total x-component:

$$W_{\text{tot}} = \left(\sum \vec{F}\right) \cdot \vec{s} = \left(\sum F_x\right)s = (500\ \text{N})(20\ \text{m}) = 10{,}000\ \text{J}$$
$$= 10\ \text{kJ}$$

Work and Kinetic Energy

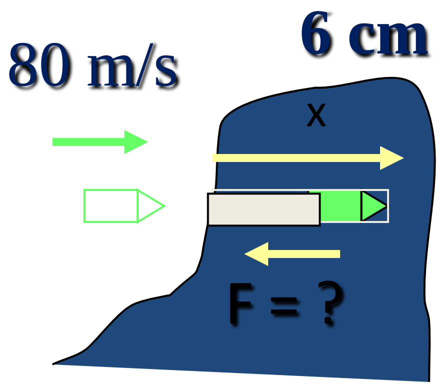
A resultant force changes the velocity of an object and does work on that object.



$$Work = Fx = (ma)x; \qquad a = \frac{v_f^2 - v_0^2}{2x}$$

$$\boxed{Work = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_0^2}$$

Example 1: A **20-g** projectile strikes a mud bank, penetrating a distance of **6 cm** before stopping. Find the stopping force ***F*** if the entrance velocity is **80 m/s**.



$$\text{Work} = \cancelto{0}{\tfrac{1}{2} m v_f^2} - \tfrac{1}{2} m v_o^2$$

$$F\,x = -\tfrac{1}{2} m v_o^2$$

$$F\,(0.06\text{ m})\cos 180^0 = -\tfrac{1}{2}\,(0.02\text{ kg})(80\text{ m/s})^2$$

$$F\,(0.06\text{ m})(-1) = -64\text{ J}$$

$$\boxed{F = 1067\text{ N}}$$

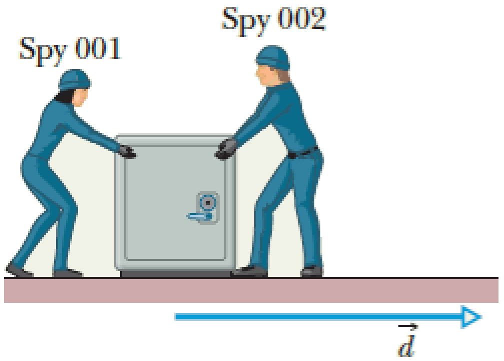
Work to stop bullet = change in K.E. for bullet

Work done by two constant forces

Figure 2 *a* shows two industrial spies sliding an initially stationary 225 kg floor safe a displacement $\vec{d}$ of magnitude 8.50 m, straight toward their truck. The push $\vec{F}_1$ of spy 001 is 12.0 N, directed at an angle of 30.0° downward from the horizontal; the pull $\vec{F}_2$ of spy 002 is 10.0 N, directed at 40.0° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.

(a) What is the net work done on the safe by forces $\vec{F}_1$ and $\vec{F}_2$ during the displacement $\vec{d}$?

(b) During the displacement, what is the work W_g done on the safe by the gravitational force $\vec{F}_g$ and what is the work W_N done on the safe by the normal force $\vec{F}_N$ from the floor?



(*a*)

(1) The net work W done on the safe by the two forces is the sum of the works they do individually. (2) Because we can treat the safe as a particle and the forces are constant in both magnitude and direction, we can use either Eq. 1 ($W = Fd \cos \phi$) or Eq. 2 ($W = \vec{F} \cdot \vec{d}$) to calculate those works. Since we know the magnitudes and directions of the forces, we choose Eq. 7-7.

Calculations: From Eq. 1 and the free-body diagram for the safe in Fig. 2 *b*, the work done by $\vec{F}_1$ is

$$W_1 = F_1 d \cos \phi_1 = (12.0 \text{ N})(8.50 \text{ m})(\cos 30.0°)$$
$$= 88.33 \text{ J},$$

and the work done by $\vec{F}_2$ is

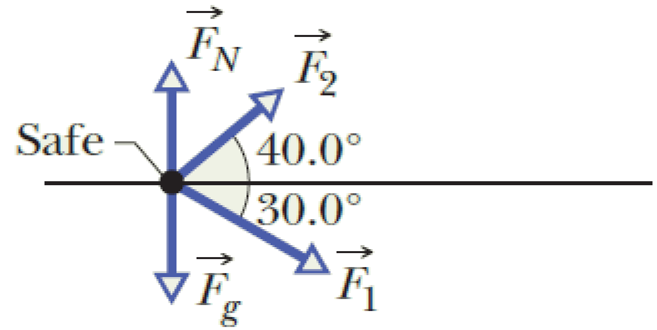
$$W_2 = F_2 d \cos \phi_2 = (10.0 \text{ N})(8.50 \text{ m})(\cos 40.0°)$$
$$= 65.11 \text{ J}.$$

Thus, the net work W is

$$W = W_1 + W_2 = 88.33 \text{ J} + 65.11 \text{ J}$$
$$= 153.4 \text{ J} \approx 153 \text{ J}.$$

During the 8.50 m displacement, therefore, the spies transfer 153 J of energy to the kinetic energy of the safe.

Only force components parallel to the displacement do work.



(*b*)

(b) During the displacement, what is the work W_g done on the safe by the gravitational force $\vec{F}_g$ and what is the work W_N done on the safe by the normal force $\vec{F}_N$ from the floor?

Because these forces are constant in both magnitude and direction, we can find the work they do with Eq. 1 .

Calculations: Thus, with mg as the magnitude of the gravitational force, we write

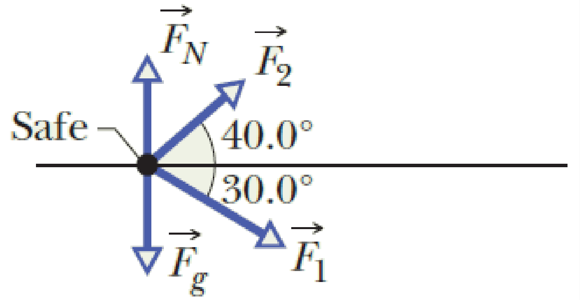
$$W_g = mgd \cos 90° = mgd(0) = 0$$

and

$$W_N = F_N d \cos 90° = F_N d(0) = 0.$$

We should have known this result. Because these forces are perpendicular to the displacement of the safe, they do zero work on the safe and do not transfer any energy to or from it.

Only force components parallel to the displacement do work.



(*b*)

(c) The safe is initially stationary. What is its speed v_f at the end of the 8.50 m displacement?

The speed of the safe changes because its kinetic energy is changed when energy is transferred to it by $\vec{F}_1$ and $\vec{F}_2$.

Calculations: We relate the speed to the work done by:

$$W = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2.$$

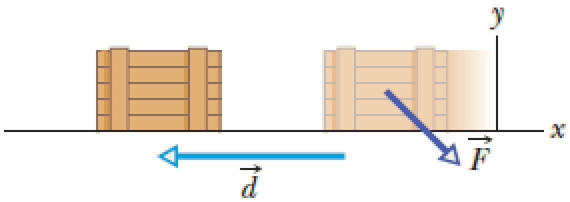
The initial speed v_i is zero, and we now know that the work done is 153.4 J. Solving for v_f and then substituting known data, we find that

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(153.4\text{ J})}{225\text{ kg}}} = 1.17\text{ m/s}.$$

Work done by a constant force in unit-vector notation

During a storm, a crate of crepe is sliding across a slick, oily parking lot through a displacement $\vec{d} = (-3.0\text{ m})\hat{\imath}$ while a steady wind pushes against the crate with a force $\vec{F} = (2.0\text{ N})\hat{\imath} + (-6.0\text{ N})\hat{\jmath}$. The situation and coordinate axes are shown in Fig. 3 .

The parallel force component does *negative* work, slowing the crate.



(a) How much work does this force do on the crate during the displacement?

Because we can treat the crate as a particle and because the wind force is constant ("steady") in both magnitude and direction during the displacement, we can use either Eq. 1 ($W = Fd\cos\phi$) or Eq. 2 ($W = \vec{F}\cdot\vec{d}$) to calculate the work. Since we know $\vec{F}$ and $\vec{d}$ in unit-vector notation, we choose Eq. 2 .

Calculations: We write

$$W = \vec{F}\cdot\vec{d} = [(2.0\text{ N})\hat{\imath} + (-6.0\text{ N})\hat{\jmath}]\cdot[(-3.0\text{ m})\hat{\imath}].$$

Of the possible unit-vector dot products, only $\hat{\imath}\cdot\hat{\imath}$, $\hat{\jmath}\cdot\hat{\jmath}$, and $\hat{k}\cdot\hat{k}$ are =1 . Here we obtain

$$W = (2.0\text{ N})(-3.0\text{ m})\hat{\imath}\cdot\hat{\imath} + (-6.0\text{ N})(-3.0\text{ m})\hat{\jmath}\cdot\hat{\imath}$$
$$= (-6.0\text{ J})(1) + 0 = -6.0\text{ J}.$$

Thus, the force does a negative 6.0 J of work on the crate, transferring 6.0 J of energy from the kinetic energy of the crate.

(b) If the crate has a kinetic energy of 10 J at the beginning of displacement $\vec{d}$, what is its kinetic energy at the end of $\vec{d}$?

Because the force does negative work on the crate, it reduces the crate's kinetic energy.

Calculation: Using the work–kinetic energy theorem in the form of Eq. 7-11, we have

$$K_f = K_i + W = 10\text{ J} + (-6.0\text{ J}) = 4.0\text{ J}. \qquad \text{(Answer)}$$

Less kinetic energy means that the crate has been slowed.

Work Done by the Gravitational Force

We next examine the work done on an object by the gravitational force acting on it. Figure 4 shows a particle-like tomato of mass m that is thrown upward with initial speed v_0 and thus with initial kinetic energy $K_i = \frac{1}{2}mv_0^2$. As the tomato rises, it is slowed by a gravitational force $\vec{F}_g$; that is, the tomato's kinetic energy decreases because $\vec{F}_g$ does work on the tomato as it rises. Because we can treat the tomato as a particle, we can use Eq. 1 ($W = Fd\cos\phi$) to express the work done during a displacement $\vec{d}$. For the force magnitude F, we use mg as the magnitude of $\vec{F}_g$. Thus, the work W_g done by the gravitational force $\vec{F}_g$ is

$$W_g = mgd\cos\phi \qquad \text{(work done by gravitational force).}$$

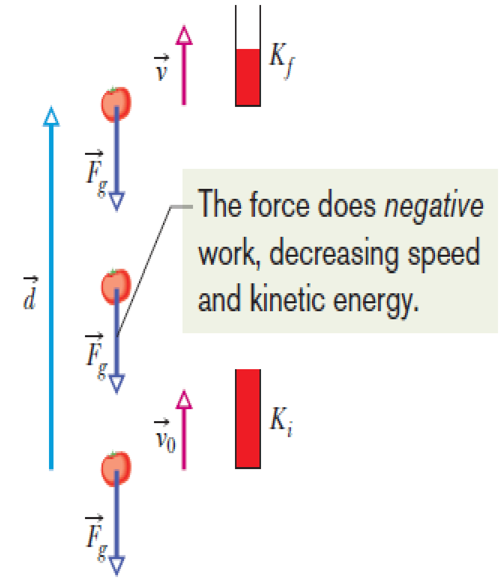
For a rising object, force $\vec{F}_g$ is directed opposite the displacement $\vec{d}$, as indicated in Fig. 4. Thus, $\phi = 180°$ and

$$W_g = mgd\cos 180° = mgd(-1) = -mgd.$$

The minus sign tells us that during the object's rise, the gravitational force acting on the object transfers energy in the amount mgd from the kinetic energy of the object. This is consistent with the slowing of the object as it rises.

After the object has reached its maximum height and is falling back down, the angle ϕ between force $\vec{F}_g$ and displacement $\vec{d}$ is zero. Thus,

$$W_g = mgd\cos 0° = mgd(+1) = +mgd.$$



Exp. An elevator cab of mass $m = 500$ kg is descending with speed $v_i = 4.0$ m/s when its supporting cable begins to slip, allowing it to fall with constant acceleration $\vec{a} = \vec{g}/5$ (Fig. 5a).

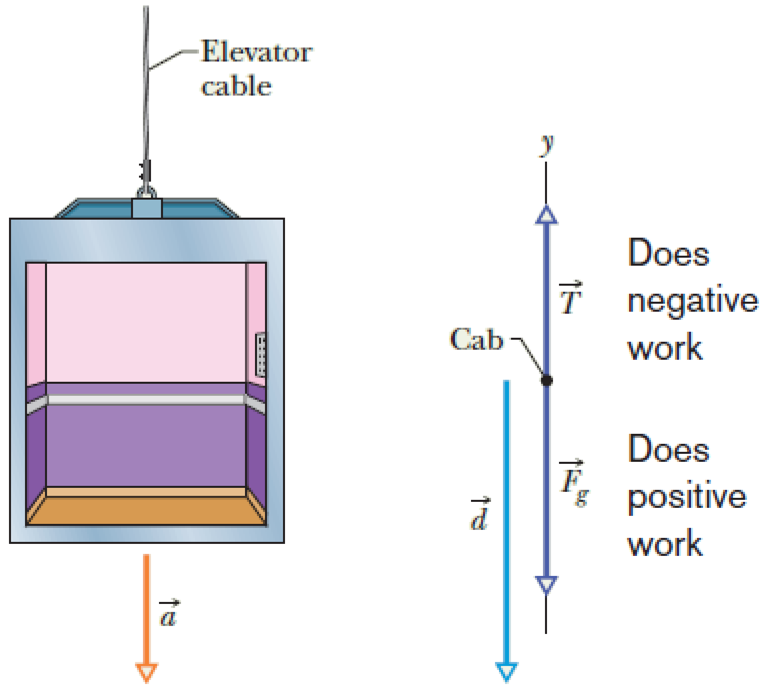
(a) During the fall through a distance $d = 12$ m, what is the work W_g done on the cab by the gravitational force $\vec{F}_g$?

We can treat the cab as a particle and thus use Eq. ($W_g = mgd \cos \phi$) to find the work W_g.

Calculation: From Fig. 5b , we see that the angle between the directions of $\vec{F}_g$ and the cab's displacement $\vec{d}$ is 0°. Then we find

$$W_g = mgd \cos 0° = (500 \text{ kg})(9.8 \text{ m/s}^2)(12 \text{ m})(1)$$
$$= 5.88 \times 10^4 \text{ J} \approx 59 \text{ kJ}.$$

(b) During the 12 m fall, what is the work W_T done on the cab by the upward pull $\vec{T}$ of the elevator cable?



(*a*) (*b*)

(1) We can calculate work W_T with Eq. 1 ($W = Fd\cos\phi$) if we first find an expression for the magnitude T of the cable's pull. (2) We can find that expression by writing Newton's second law for components along the y axis in Fig. 5b ($F_{net,y} = ma_y$).

Calculations: We get

$$T - F_g = ma. \qquad (2)$$

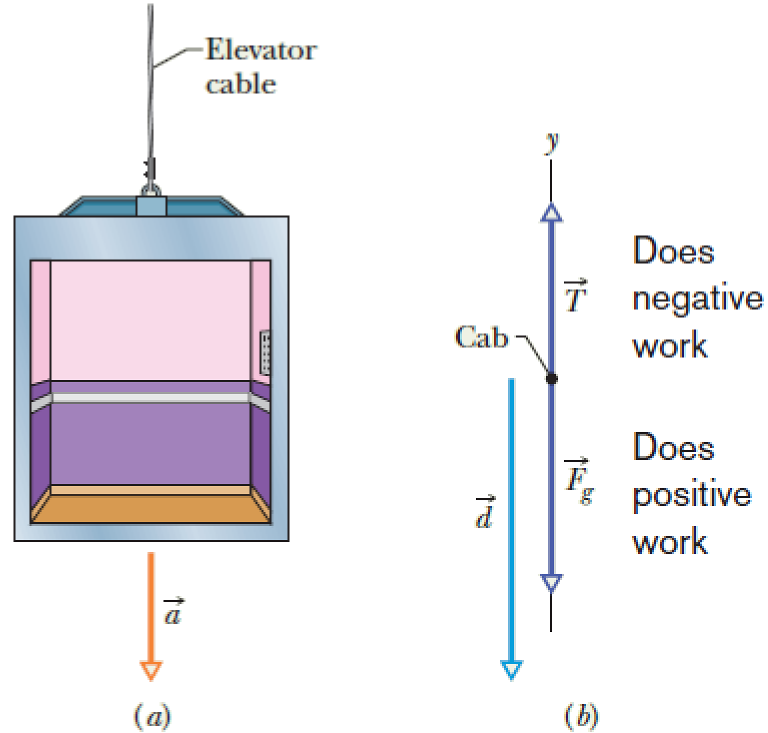
Solving for T, substituting mg for F_g, and then substituting the result in Eq. 7-7, we obtain

$$W_T = Td\cos\phi = m(a+g)d\cos\phi. \qquad (3)$$

Next, substituting $-g/5$ for the (downward) acceleration a and then 180° for the angle ϕ between the directions of forces $\vec{T}$ and $m\vec{g}$, we find

$$W_T = m\left(-\frac{g}{5}+g\right)d\cos\phi = \frac{4}{5}mgd\cos\phi$$
$$= \frac{4}{5}(500\text{ kg})(9.8\text{ m/s}^2)(12\text{ m})\cos 180°$$
$$= -4.70\times10^4\text{ J} \approx -47\text{ kJ}. \qquad \text{(Answer)}$$

Caution: Note that W_T is not simply the negative of W_g. The reason is that, because the cab accelerates during the fall, its speed changes during the fall, and thus its kinetic energy also changes. Therefore, Eq. 7-16 (which assumes that the initial and final kinetic energies are equal) does *not* apply here.



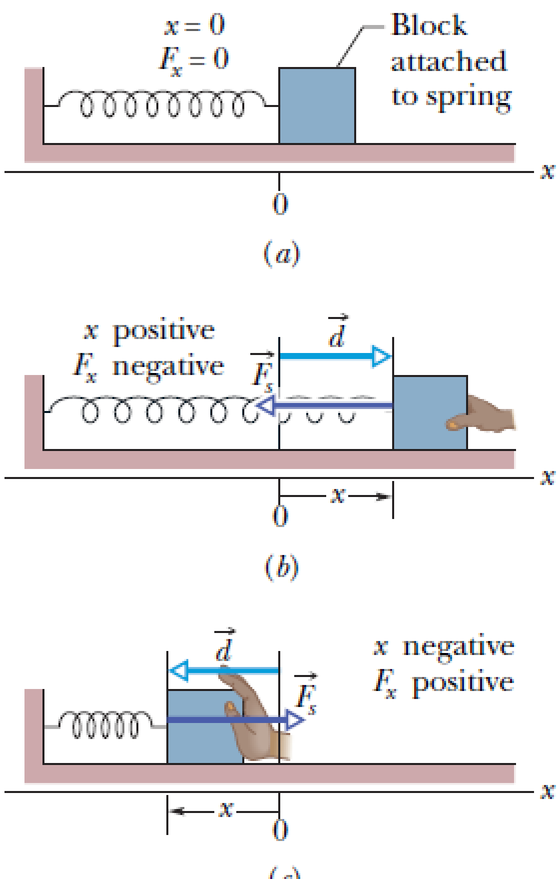
(c) What is the net work W done on the cab during the fall?

Calculation: The net work is the sum of the works done by the forces acting on the cab:

$$W = W_g + W_T = 5.88\times10^4\text{ J} - 4.70\times10^4\text{ J}$$
$$= 1.18\times10^4\text{ J} \approx 12\text{ kJ}.$$

Work Done by a Spring Force

- **Figure 6-*a shows a spring in its relaxed state.* One end is fixed, and a particle-like object—a block, say—is attached to the other, free end.**
- **If we stretch the spring by pulling the block to the right as in Fig. 6-b, the spring pulls on the block toward the left.**
- **If we compress the spring by pushing the block to the left as in Fig. 6-c, the spring now pushes on the block toward the right.**



The force from a spring is proportional to the displacement of the free end from its position when the spring is in the relaxed state.The spring force is given by

$$\vec{F}_s = -k\vec{d} \qquad \text{(Hooke's law),}$$

Which is known as Hooke's law indicates that the direction of the spring force is always opposite the direction of the displacement of the spring's free end. The constant *k* is called the spring constant (or force constant) and is a measure of the stiffness of the spring.

$$F_x = -kx \qquad \text{(Hooke's law)},$$

If x is positive (the spring is stretched toward the right on the x axis) then Fx is negative (it is a pull toward the left).

If x is negative (the spring is compressed toward the left), then Fx is positive (it is a push toward the right).

The Work Done by a Spring Force

Work W_s is positive if the block ends up closer to the relaxed position ($x = 0$) than it was initially. It is negative if the block ends up farther away from $x = 0$. It is zero if the block ends up at the same distance from $x = 0$.

$$W_s = \sum -F_{xj}\,\Delta x.$$

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \qquad \text{(work by a spring force).}$$

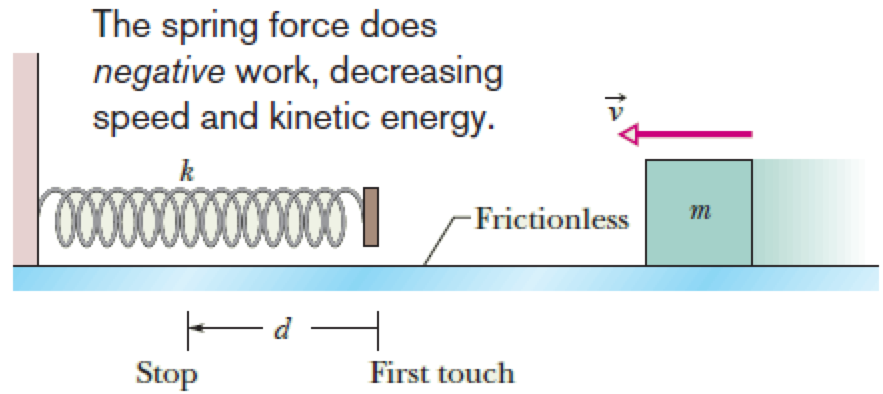
If $x_i = 0$ and if we call the final position x, then Eq becomes

$$W_s = -\tfrac{1}{2}kx^2 \qquad \text{(work by a spring force).}$$

In Fig. 7 , a cumin canister of mass $m = 0.40$ kg slides across a horizontal frictionless counter with speed $v = 0.50$ m/s. It then runs into and compresses a spring of spring constant $k = 750$ N/m. When the canister is momentarily stopped by the spring, by what distance d is the spring compressed?

$$W_s = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2 \qquad \text{(work by a spring force).}$$

$$W_s = -\tfrac{1}{2}kx^2$$



Putting the first two of these ideas together, we write the work–kinetic energy theorem for the canister as

$$K_f - K_i = -\tfrac{1}{2}kd^2.$$

Substituting according to the third key idea gives us this expression

$$0 - \tfrac{1}{2}mv^2 = -\tfrac{1}{2}kd^2.$$

Simplifying, solving for d, and substituting known data then give us

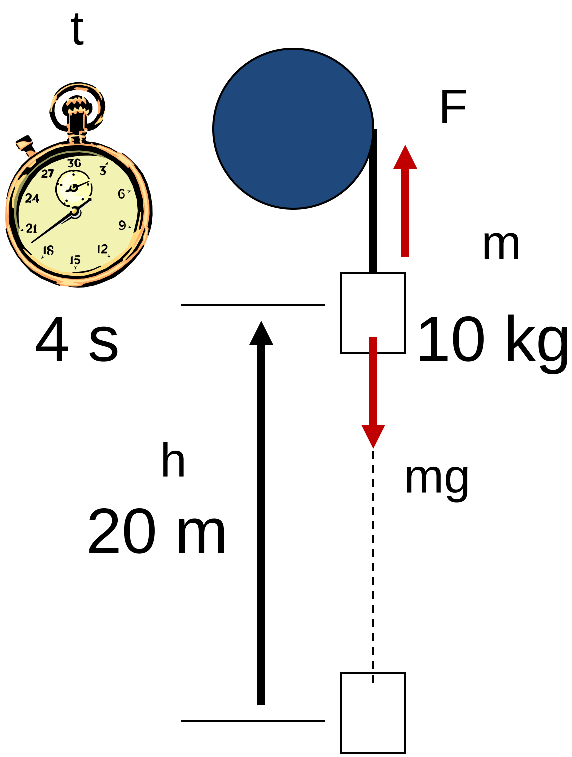
$$d = v\sqrt{\frac{m}{k}} = (0.50\text{ m/s})\sqrt{\frac{0.40\text{ kg}}{750\text{ N/m}}}$$

$$= 1.2 \times 10^{-2}\text{ m} = 1.2\text{ cm}.$$

Power

Power is defined as the rate at which work is done: (P = dW/dt)

One watt (W) is work done at the rate of one joule per second.



$$Power = \frac{Work}{time} = \frac{Fr}{t}$$

$$P = \frac{mgr}{t} = \frac{(10\text{kg})(9.8\text{m/s}^2)(20\text{m})}{4\text{ s}}$$

$$\boxed{P = 490\,\text{J/s} \quad \text{or} \quad 490 \text{ watts (W)}}$$

Power of 1 W is work done at rate of 1 J/s

<u>1 W = 1 J/s</u> and <u>1 kW = 1000 W</u>

Lecture (6)

Potential Energy and Conservation of Energy

Dr. Hind I. Al-Shaikh

Q1 - A body moving with uniform acceleration has a velocity of 12 cm/s when its *x* coordinate is 3 cm. If its *x* coordinate 2 s later is -5 cm, what is the magnitude of its acceleration?

Q2- Write the projectile motion equation in horizontal and vertical motion with horizontal range and maximum height of a projectile?

Key contents

- Gravitational Potential Energy
- Elastic Potential Energy
- Work-Energy Theorem
- Conservative and Non-conservative Forces
- Conservation of Energy

Potential energy and conservation of energy

- درسنا في الفصل السابق مفهوم الطاقة الحركية لجسم متحرك ووجدنا ان طاقة الجسم تتغير عندما يبذل شغل على الجسم.
- سندرس في هذا الفصل نوعا اخر من انواع الطاقة الميكانيكية وهي الطاقة الكامنة **Potential energy** وهنا يمكن ان تتحول الطاقة الى طاقة حركية او الى بذل شغل .
- وهنا تجدر الاشارة الى انواع القوى التي درسناها وهي:
- قوة التعجيل الارضي **Fg**
- قوة الاحتكاك **f**
- قوة الشد **T**
- القوى المؤثرة الخارجية **F**
- وهنا سوف نقسم القوى الى نوعين:

❑ قوى محافظة ***Conservative forces***

❑ قوى غير محافظة ***Nonconservative forces***

❖Conservative and Nonconservative Forces

Conservative force: the forces that do path independent work.

Example of a conservative force: gravity

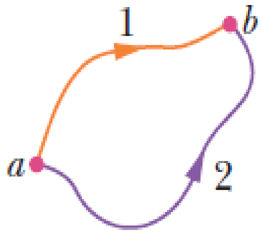
Non-conservative forces: the forces that do path dependent work

Example of a nonconservative force: friction

Also: the work done by a conservative force moving an object around a closed path is zero; this is not true for a nonconservative force

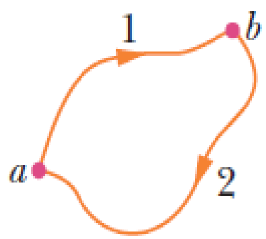
❖ Conservative Forces

The net work done by a conservative force on a particle moving around any closed path is zero.



(a)

The force is conservative. Any choice of path between the points gives the same amount of work.



(b)

And a round trip gives a total work of zero.

write this result as

$$W_{ab,1} = W_{ab,2}.$$

Without worrying about where positive work is done and where negative work is done, let us just represent the work done from a to b along path 1 as $W_{ab,1}$ and the work done from b back to a along path 2 as $W_{ba,2}$. If the force is conservative, then the net work done during the round trip must be zero:

$$W_{ab,1} + W_{ba,2} = 0,$$

and thus

$$W_{ab,1} = -W_{ba,2}.$$

In words, the work done along the outward path must be the negative of the work done along the path back.

Let us now consider the work $W_{ab,2}$ done on the particle by the force when the particle moves from a to b along path 2, as indicated in Fig. 1a . If the force is conservative, that work is the negative of $W_{ba,2}$:

$$W_{ab,2} = -W_{ba,2}.$$

Fig. 1(*a) As a conservative force* acts on it, a particle can move from point *a to point b along either path 1* or path 2.

(*b)The particle moves in a* round trip, from point *a to point b* along path 1 and then back to point *a* along path 2

Work and potential energy

The change ΔU in potential energy (gravitational, elastic, etc) is defined as being equal to *the negative of the work done on the object by the corresponding conservative force.*

$$\Delta U = -W.$$

 # e.g. work done by gravitational force, by a spring etc.

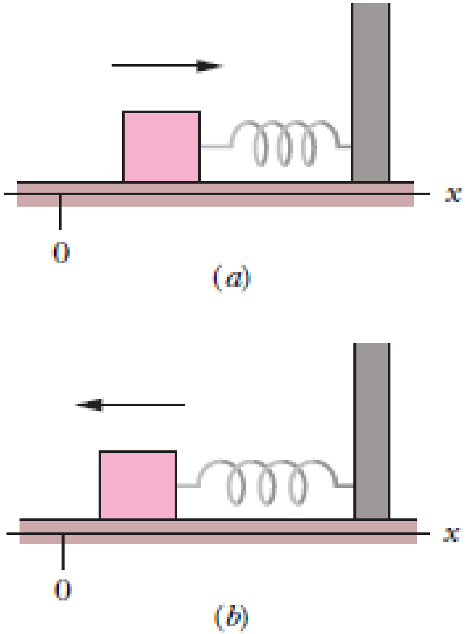


Fig. shows A block, attached to a spring and initially at rest at $x = 0$, is set in motion toward the right. (a) As the block moves rightward (as indicated by the arrow), the spring force does negative work on it. (b) Then, as the block moves back toward $x = 0$, the spring force does positive work on it.

Exp. **Figure 2a shows a 2.0 kg block of slippery cheese that slides along a frictionless track from point a to point b. The cheese travels through a total distance of 2.0 m along the track, and a net vertical distance of 0.80 m. How much work is done on the cheese by the gravitational force during the slide?**

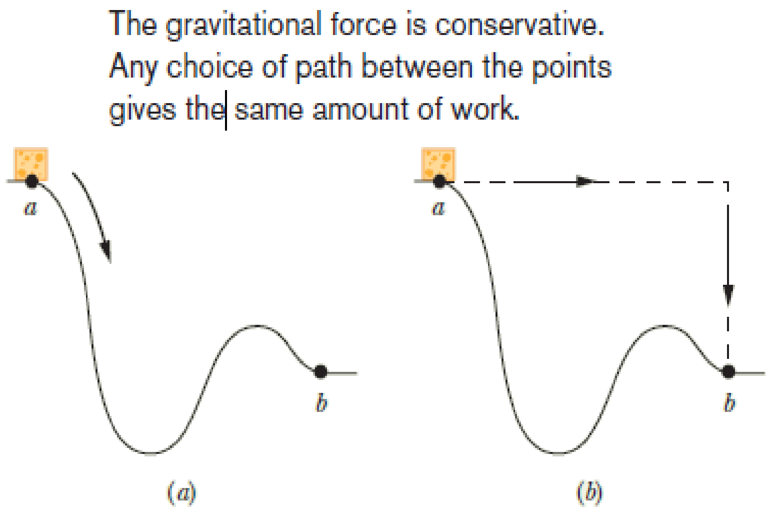


Fig. 2 (*a*) A block of cheese slides along a frictionless track from point *a* to point *b*. (*b*) Finding the work done on the cheese by the gravitational force is easier along the dashed path than along the actual path taken by the cheese; the result is the same for both paths.

Calculations: **Let us choose the dashed path in Fig. 2b; it consists of two straight segments. Along the horizontal segment, the angle *f* is a constant 90°. Even though we do not know the displacement along that horizontal segment, the work W_h done there is**

$$W_h = mgd \cos 90^\circ = 0.$$

Along the vertical segment, the displacement *d is 0.80 m* and, with and both downward, the angle is a constant = 0°. Thus, for the work W_v *done along the* vertical part of the dashed path,

$$W_v = mgd \cos 0^\circ$$
$$= (2.0\ \text{kg})(9.8\ \text{m/s}^2)(0.80\ \text{m})(1) = 15.7\ \text{J}.$$

The total work done on the cheese by F_g as the cheese moves from point *a to point b along the dashed path is then*

$$W = W_h + W_v = 0 + 15.7\ \text{J} \approx 16\ \text{J}. \quad \text{(Answer)}$$

This is also the work done as the cheese slides along the track from *a to b.*

❖Determining Potential Energy values

For the most general case, in which the conservative force may vary with position, we may write the work *W*:

$$W = \int_{x_i}^{x_f} F(x)\, dx.$$

The change of the potential energy is defined to be the negative value of the work done by that conservative force.

$$\Delta U = -\int_{x_i}^{x_f} F(x)\, dx.$$

Gravitational Potential Energy

A particle with mass *m* moving vertically along a *y* axis (the positive direction is upward). As the particle moves from point y_i to point y_f, the gravitational force does work on it. The corresponding change in the gravitational potential energy of the particle–Earth system is:

$$\Delta U = -\int_{y_i}^{y_f} (-mg)\, dy = mg \int_{y_i}^{y_f} dy = mg \Big[y \Big]_{y_i}^{y_f},$$

$$\Big\downarrow$$

$$\Delta U = mg(y_f - y_i) = mg\,\Delta y.$$

$$\Big\downarrow$$

The gravitational potential energy associated with a particle–Earth system depends only on the vertical position y (or height) of the particle relative to the reference position y =0, not on the horizontal position.

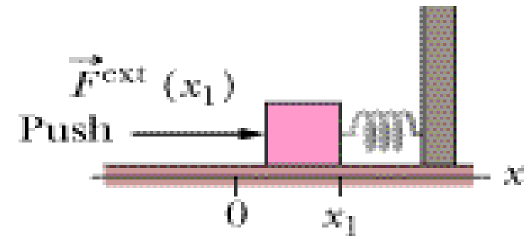
- لقد علمنا سابقا ان الشغل يساوي التغيير في الطاقة الحركية ولكن اذا تحرك جسم تحت تاثير قوة محافظة مثل قوة عجلة الجاذبية الارضية ازاحة محددة فان الشغل هنا يعتمد على نقطتي البداية والنهاية ولا يعتمد على المسار.
- وهنا لا نستطيع القول ان الشغل يساوي التغيير في طاقة الحركة. فمثلا اذا حاول شخص رفع كتلة ما من على سطح الارض الى ارتفاع معين قدره h فان هذا الشخص سيبذل شغلا موجبا مساويا ل mgh لان القوة التي بذلها باتجاه الحركة, ولكن من وجهة نظر الجسم فانه بذل شغلا سالبا قدره $-mgh$ وذلك لان قوته (وزنه) في عكس اتجاه الازاحة هذا الشغل السالب يدعى الطاقة الكامنة التي اكتسبها الجسم عند تحريكه من نقطة الى اخرى تحت تاثير قوة محافظة (قوة عجلة الجاذبية الارضية).

Elastic Potential Energy

In a block–spring system, the block is moving on the end of a spring of spring constant *k*. As the block moves from point x_i to point x_f , the spring force $F_x = -kx$ does work on the block. The corresponding change in the elastic potential energy of the block–spring system is

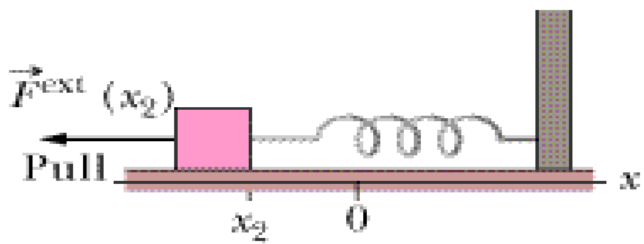
$$\Delta U = -\int_{x_i}^{x_f} (-kx)\, dx = k \int_{x_i}^{x_f} x\, dx = \tfrac{1}{2}k\left[x^2\right]_{x_i}^{x_f},$$

$$\Delta U = \tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2.$$



(a)

If the reference configuration is when the spring is at its relaxed length, and the block is at $x_i = 0$.

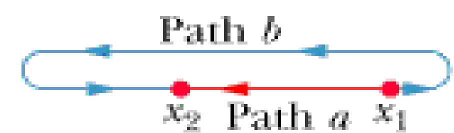


(b)

$$U - 0 = \tfrac{1}{2}kx^2 - 0,$$

$$\Downarrow$$

$$\boxed{U(x) = \tfrac{1}{2}kx^2}$$ **Elastic potential energy**



(c)

Sample problem: gravitational potential energy

A 2.0 kg sloth hangs 5.0 m above the ground (Fig. 3).

(a) What is the gravitational potential energy U of the sloth–Earth system if we take the reference point $y = 0$ to be (1) at the ground, (2) at a balcony floor that is 3.0 m above the ground, (3) at the limb, and (4) 1.0 m above the limb? Take the gravitational potential energy to be zero at $y = 0$.

Calculations: For choice (1) the sloth is at $y = 5.0$ m, and

$$U = mgy = (2.0\ \text{kg})(9.8\ \text{m/s}^2)(5.0\ \text{m})$$
$$= 98\ \text{J}. \qquad \text{(Answer)}$$

For the other choices, the values of U are

(2) $U = mgy = mg(2.0\ \text{m}) = 39\ \text{J},$

(3) $U = mgy = mg(0) = 0\ \text{J},$

(4) $U = mgy = mg(-1.0\ \text{m})$
$= -19.6\ \text{J} \approx -20\ \text{J}. \qquad \text{(Answer)}$

(b) The sloth drops to the ground. For each choice of reference point, what is the change ΔU in the potential energy of the sloth–Earth system due to the fall?

Calculation: For all four situations, we have the same $\Delta y =$

$$\Delta U = mg\,\Delta y = (2.0\ \text{kg})(9.8\ \text{m/s}^2)(-5.0\ \text{m})$$
$$= -98\ \text{J}. \qquad \text{(Answer)}$$

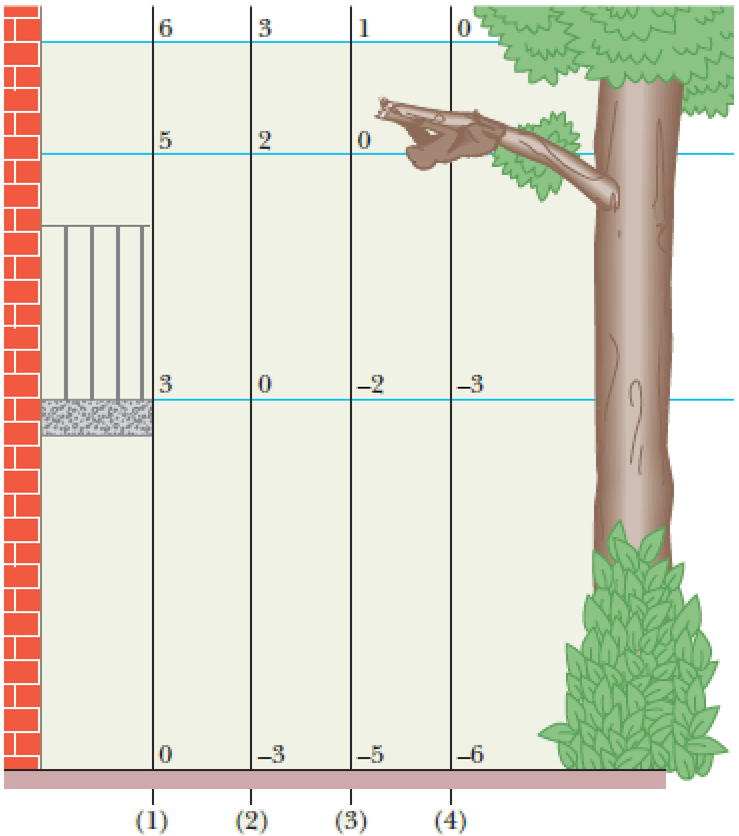


Fig. 3 Four choices of reference point $y = 0$. Each y axis is marked in units of meters. The choice affects the value of the potential energy U of the sloth–Earth system. However, it does not affect the change ΔU in potential energy of the system if the sloth moves by, say, falling.

❖ Conservation of Mechanical Energy

Principle of conservation of energy:

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

The mechanical energy E_{mec} of a system is the sum of its potential energy U and the kinetic energy K of the objects within it:

$$E_{\text{mec}} = K + U \qquad \text{(mechanical energy).}$$

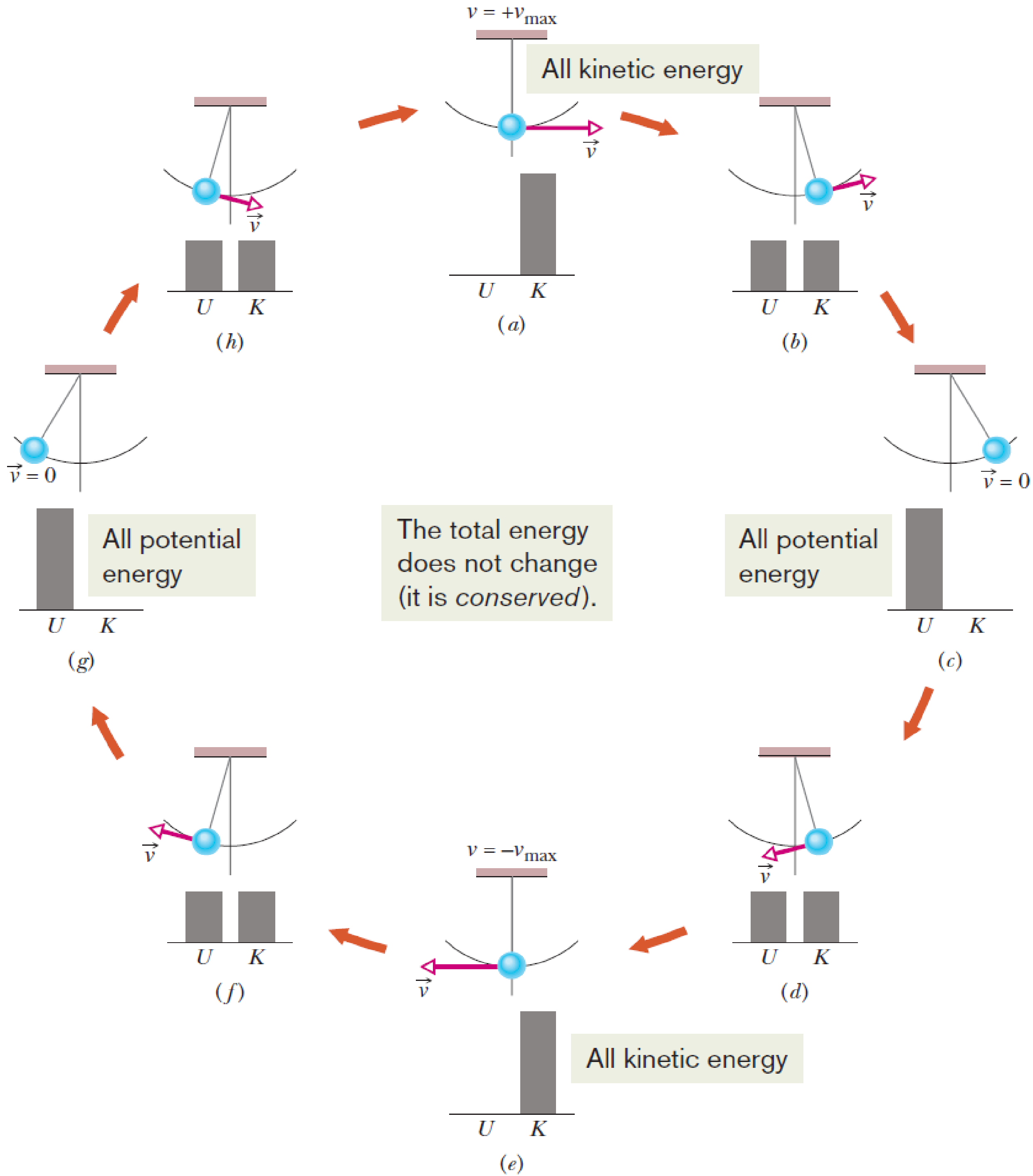
With $\Delta K = W$ **and** $\Delta U = -W.$

We have: $\Delta K = -\Delta U.$ ➡ $$\Delta E_{\text{mec}} = \Delta K + \Delta U = 0.$$

$$\begin{pmatrix}\text{the sum of } K \text{ and } U \text{ for} \\ \text{any state of a system}\end{pmatrix} = \begin{pmatrix}\text{the sum of } K \text{ and } U \text{ for} \\ \text{any other state of the system}\end{pmatrix}$$

$$E_i = E_f$$

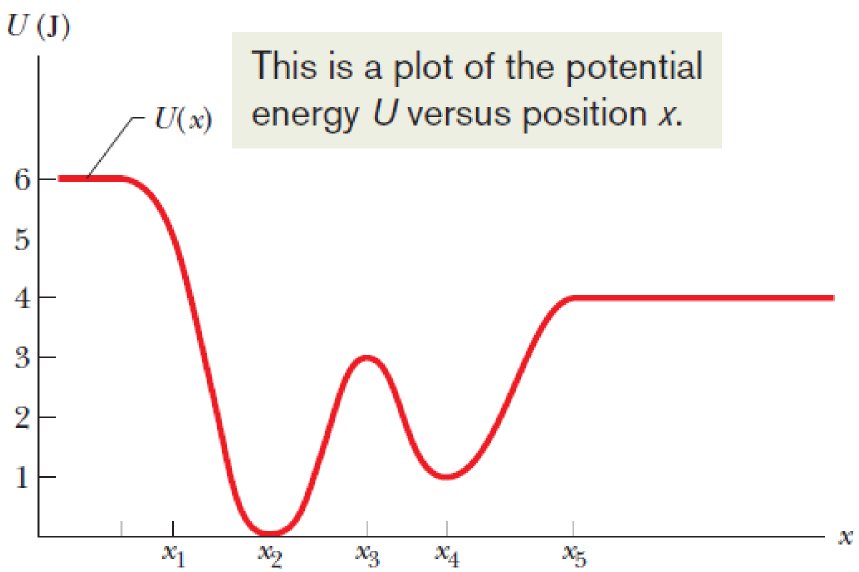
Fig. 4 A pendulum, with its mass concentrated in a bob at the lower end, swings back and forth. One full cycle of the motion is shown. During the cycle the values of the potential and kinetic energies of the pendulum–Earth system vary as the bob rises and falls, but the mechanical energy E_{mec} of the system remains constant. The energy E_{mec} can be described as continuously shifting between the kinetic and potential forms. In stages (a) and (e), all the energy is kinetic energy. The bob then has its greatest speed and is at its lowest point. In stages (c) and (g), all the energy is potential energy. The bob then has zero speed and is at its highest point. In stages (b), (d), (f), and (h), half the energy is kinetic energy and half is potential energy. If the swinging involved a frictional force at the point where the pendulum is attached to the ceiling, or a drag force due to the air, then E_{mec} would not be conserved, and eventually the pendulum would stop.



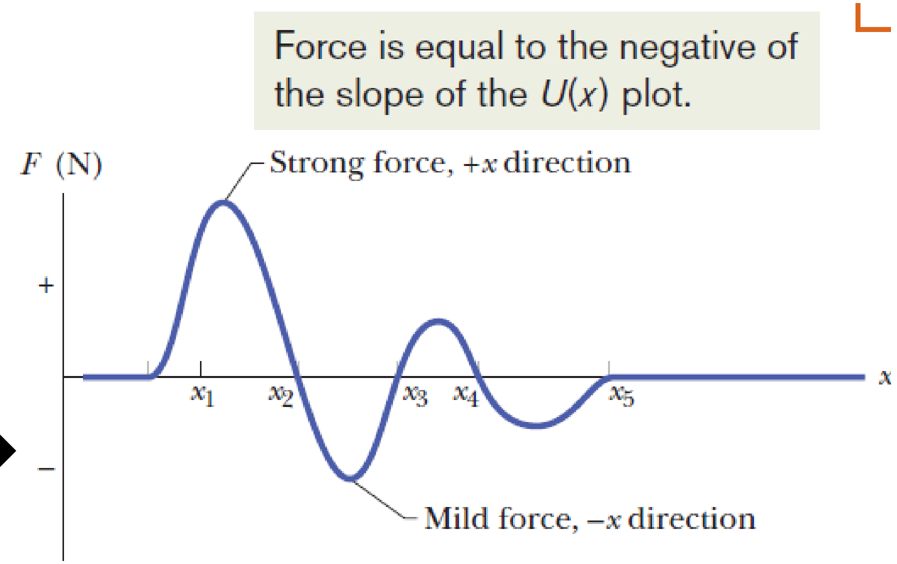
Reading a Potential Energy Curve

$$\Delta U(x) = -W = -F(x)\,\Delta x.$$

$$\rightarrow \quad F(x) = -\frac{dU(x)}{dx} \quad \text{(one-dimensional motion)},$$

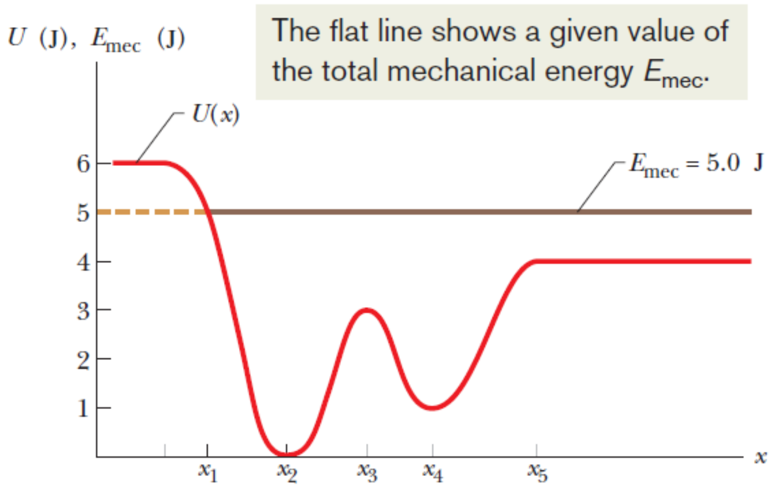


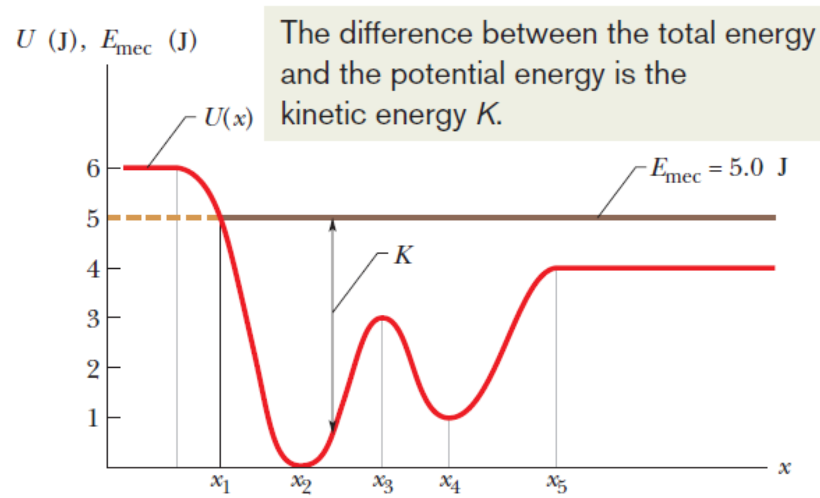
A plot of *U(x)*, the potential energy function of a system containing a particle confined to move along an *x* axis. There is no friction, so mechanical energy is conserved.

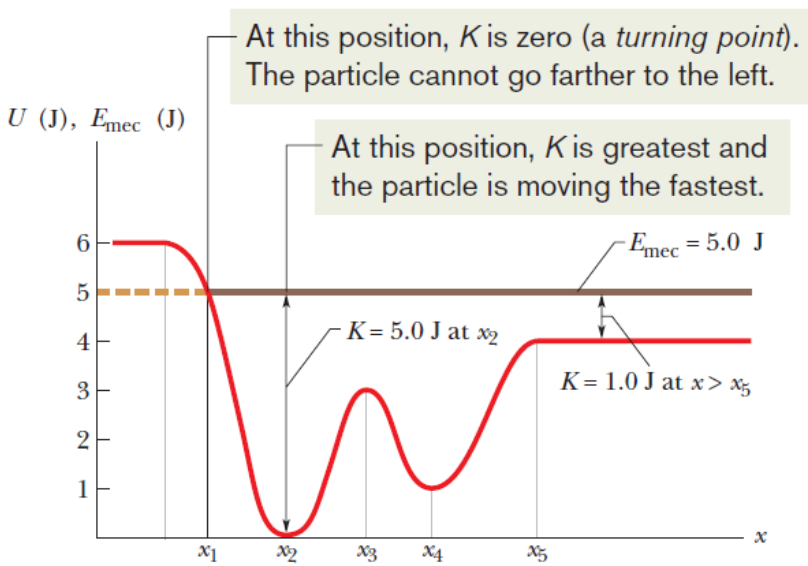


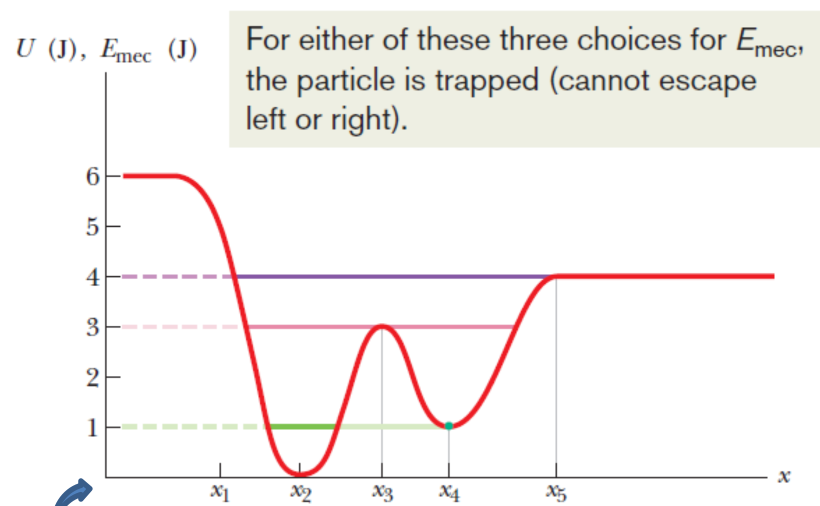
A plot of the force *F(x)* acting on the particle, derived from the potential energy plot by taking its slope at various points.

Reading a Potential Energy Curve



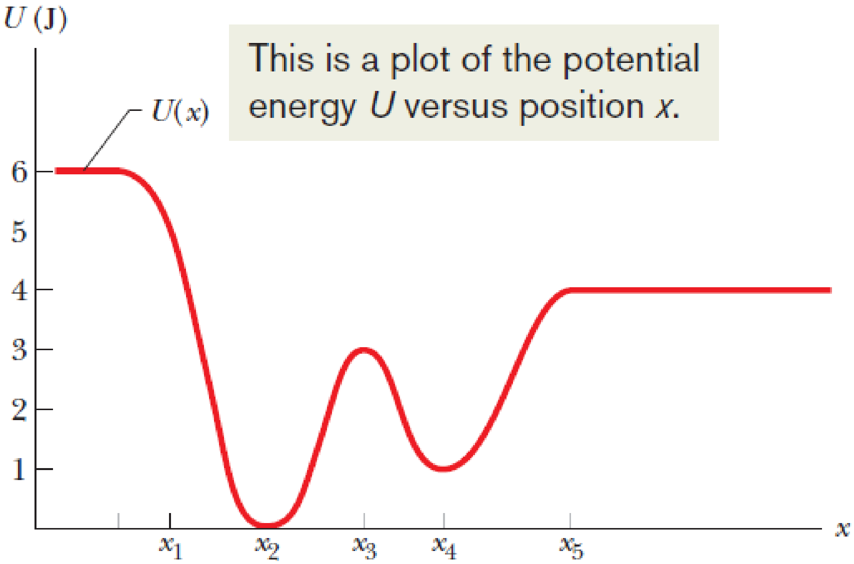






The *$U(x)$ plot with three possible values of E_{mec} shown.*

Potential Energy Curve, Equilibrium Points



- Consider a particle at rest at x_2 or x_4. If we push it slightly left or right, a restoring force appears that moves it back to x_2 or x_4. A particle at such a position is said to be in **stable equilibrium.**

- A particle at rest at any point to the right of x_5 is said to be in **neutral equilibrium.**

- If a particle is located exactly at x_3, the force on it is zero, and the particle remains stationary. However, if it is displaced even slightly in either direction, a nonzero force pushes it farther in the same direction, and the particle continues to move. A particle at such a position is said to be in **unstable equilibrium.**

In Fig. 5 a child of mass m is released from rest at the top of a water slide, at height $h = 8.5$ m above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.

(1) We cannot find her speed at the bottom by using her acceleration along the slide as we might have in earlier chapters because we do not know the slope (angle) of the slide. However, because that speed is related to her kinetic energy, perhaps we can use the principle of conservation of mechanical energy to get the speed. Then we would not need to know the slope. (2) Mechanical energy is conserved in a system *if* the system is isolated and *if* only conservative forces cause energy transfers within it. Let's check.

Forces: Two forces act on the child. The *gravitational force,* a conservative force, does work on her. The *normal force* on her from the slide does no work because its direction at any point during the descent is always perpendicular to the direction in which the child moves.

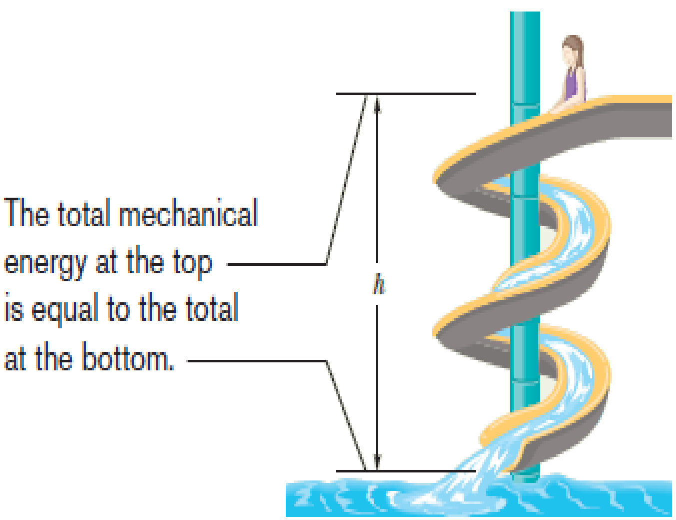


Fig. 5 A child slides down a water slide as she descends a height h.

Thus, we have only a conservative force doing work in an isolated system, so we *can* use the principle of conservation of mechanical energy.

Calculations: Let the mechanical energy be $E_{mec,t}$ when the child is at the top of the slide and $E_{mec,b}$ when she is at the bottom. Then the conservation principle tells us

$$E_{mec,b} = E_{mec,t}.$$

To show both kinds of mechanical energy, we have

$$K_b + U_b = K_t + U_t,$$

or

$$\tfrac{1}{2}mv_b^2 + mgy_b = \tfrac{1}{2}mv_t^2 + mgy_t.$$

Dividing by m and rearranging yield

$$v_b^2 = v_t^2 + 2g(y_t - y_b).$$

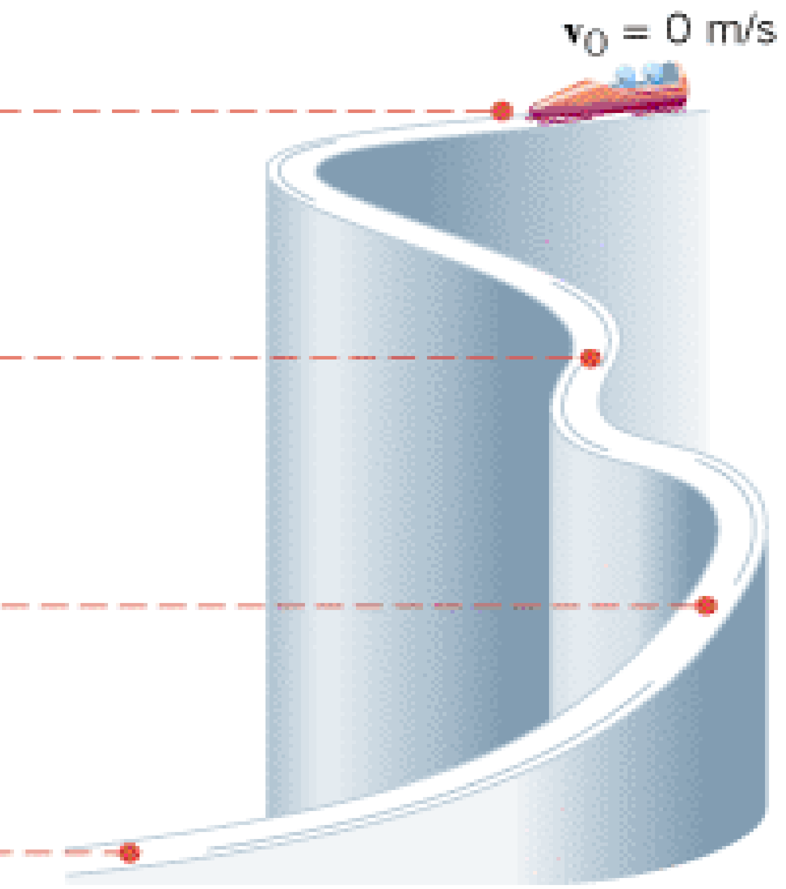
Putting $v_t = 0$ and $y_t - y_b = h$ leads to

$$v_b = \sqrt{2gh} = \sqrt{(2)(9.8\ \text{m/s}^2)(8.5\ \text{m})}$$
$$= 13\ \text{m/s}. \qquad \text{(Answer)}$$

This is the same speed that the child would reach if she fell 8.5 m vertically. On an actual slide, some frictional forces would act and the child would not be moving quite so fast.

Example

KE	PE	E = KE + PE
0 J	600 000 J	600 000 J
200 000 J	400 000 J	600 000 J
400 000 J	200 000 J	600 000 J
600 000 J	0 J	600 000 J



Sample problem: reading a potential energy graph

A 2.00 kg particle moves along an x axis in one-dimensional motion while a conservative force along that axis acts on it. The potential energy $U(x)$ associated with the force is plotted in Fig. 8-10*a*. That is, if the particle were placed at any position between $x = 0$ and $x = 7.00$ m, it would have the plotted value of U. At $x = 6.5$ m, the particle has velocity $\vec{v}_0 = (-4.00 \text{ m/s})\hat{\text{i}}$.

(a) From Fig. 8-10*a*, determine the particle's speed at $x_1 = 4.5$ m.

Calculations: At $x = 6.5$ m, the particle has kinetic energy

$$K_0 = \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}(2.00 \text{ kg})(4.00 \text{ m/s})^2$$
$$= 16.0 \text{ J}.$$

Because the potential energy there is $U = 0$, the mechanical energy is

$$E_{mec} = K_0 + U_0 = 16.0 \text{ J} + 0 = 16.0 \text{ J}.$$

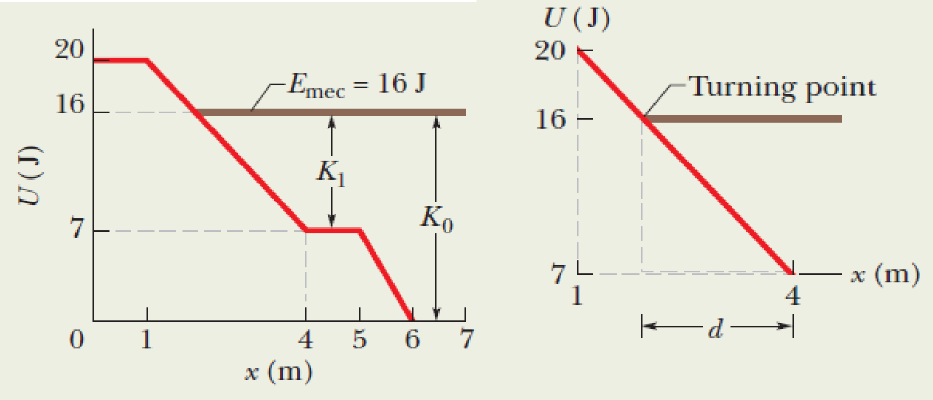
This value for E_{mec} is plotted as a horizontal line in Fig. 8-10*a*. From that figure we see that at $x = 4.5$ m, the potential energy is $U_1 = 7.0$ J. The kinetic energy K_1 is the difference between E_{mec} and U_1:

$$K_1 = E_{mec} - U_1 = 16.0 \text{ J} - 7.0 \text{ J} = 9.0 \text{ J}.$$

Because $K_1 = \frac{1}{2}mv_1^2$, we find

$$v_1 = 3.0 \text{ m/s}. \qquad \text{(Answer)}$$

(b) Where is the particle's turning point located?



(*a*) (*b*)

Calculations: Because K is the difference between E_{mec} and U, we want the point in Fig. 8-10*a* where the plot of U rises to meet the horizontal line of E_{mec}, as shown in Fig. 8-10*b*. Because the plot of U is a straight line in Fig. 8-10*b*, we can draw nested right triangles as shown and then write

$$\frac{16 - 7.0}{d} = \frac{20 - 7.0}{4.0 - 1.0},$$

which gives us $d = 2.08$ m. Thus, the turning point is at

$$x = 4.0 \text{ m} - d = 1.9 \text{ m}. \qquad \text{(Answer)}$$

(c) Evaluate the force acting on the particle when it is in the region $1.9 \text{ m} < x < 4.0$ m.

Calculations: For the graph of Fig. 8-10*b*, we see that for the range $1.0 \text{ m} < x < 4.0$ m the force is

$$F = -\frac{20 \text{ J} - 7.0 \text{ J}}{1.0 \text{ m} - 4.0 \text{ m}} = 4.3 \text{ N}. \qquad \text{(Answer)}$$

Thus, the force has magnitude 4.3 N and is in the positive direction of the x axis. This result is consistent with the fact that the initially leftward-moving particle is stopped by the force and then sent rightward.

Work done on a System by an External Force

Work is energy transferred to or from a system by means of an external force acting on that system.

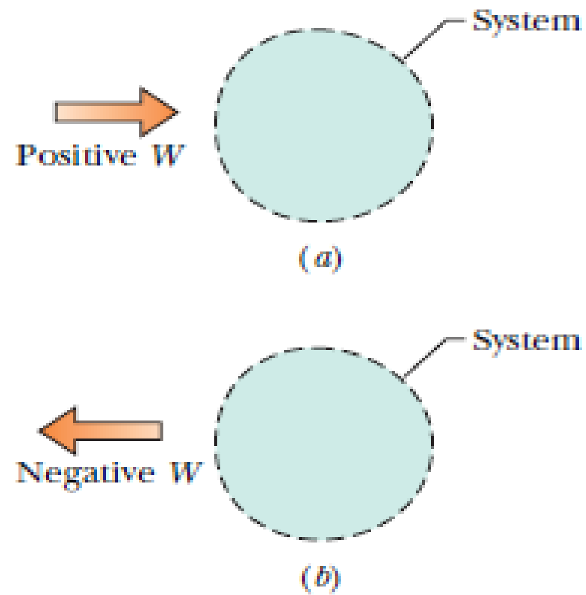


Fig. 6 (*a*) Positive work W done on an arbitrary system means a transfer of energy to the system. (*b*) Negative work W means a transfer of energy from the system.

Work done on a System by an External Force

FRICTION NOT INVOLVED

$$W = \Delta K + \Delta U,$$

$$\Downarrow$$

$$\boxed{W = \Delta E_{\text{mec}}}$$

Your lifting force transfers energy to kinetic energy and potential energy.

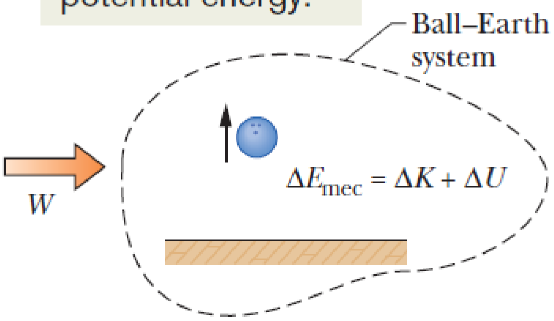


Fig. 8-12 Positive work W is done on a system of a bowling ball and Earth, causing a change ΔE_{mec} in the mechanical energy of the system, a change ΔK in the ball's kinetic energy, and a change ΔU in the system's gravitational potential energy.

FRICTION INVOLVED

$$F - f_k = ma.$$

$$\Downarrow$$

$$Fd = \Delta K + f_k d.$$

$$\Downarrow$$

$$Fd = \Delta E_{\text{mec}} + f_k d.$$

The applied force supplies energy. The frictional force transfers some of it to thermal energy.

So, the work done by the applied force goes into kinetic energy and also thermal energy.

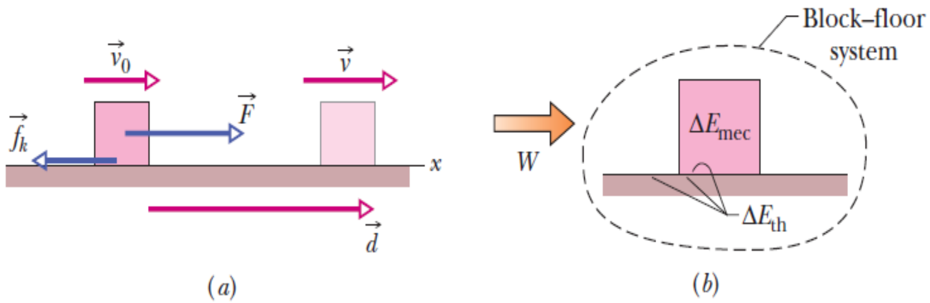


Fig. 8-13 (*a*) A block is pulled across a floor by force $\vec{F}$ while a kinetic frictional force $\vec{f}_k$ opposes the motion. The block has velocity $\vec{v}_0$ at the start of a displacement $\vec{d}$ and velocity $\vec{v}$ at the end of the displacement. (*b*) Positive work W is done on the block–floor system by force $\vec{F}$, resulting in a change ΔE_{mec} in the block's mechanical energy and a change ΔE_{th} in the thermal energy of the block and floor.

Conservation of Energy

Law of Conservation of Energy

The total energy *E* of a system can change only by amounts of energy that are transferred to or from the system.

$$W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}}$$

where E_{mec} is any change in the mechanical energy of the system, E_{th} is any change in the thermal energy of the system, and E_{int} is any change in any other type of internal energy of the system.

The total energy E of an isolated system cannot change.

$$\Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}} = 0 \qquad \text{(isolated system)}$$

An external force can change the kinetic energy or potential energy of an object without doing work on the object—that is, without transferring energy to the object. Instead, the force is responsible for transfers of energy from one type to another inside the object.

Sample problem: change in thermal energy

A food shipper pushes a wood crate of cabbage heads (total mass $m = 14$ kg) across a concrete floor with a constant horizontal force $\vec{F}$ of magnitude 40 N. In a straight-line displacement of magnitude $d = 0.50$ m, the speed of the crate decreases from $v_0 = 0.60$ m/s to $v = 0.20$ m/s.

(a) How much work is done by force $\vec{F}$, and on what system does it do the work?

KEY IDEA

Because the applied force $\vec{F}$ is constant, we can calculate the work it does by using Eq. 7-7 ($W = Fd\cos\phi$).

Calculation: Substituting given data, including the fact that force $\vec{F}$ and displacement $\vec{d}$ are in the same direction, we find

$$W = Fd\cos\phi = (40\text{ N})(0.50\text{ m})\cos 0^\circ$$

$$= 20\text{ J}. \qquad \text{(Answer)}$$

Reasoning: We can determine the system on which the work is done to see which energies change. Because the crate's speed changes, there is certainly a change ΔK in the crate's kinetic energy. Is there friction between the floor and the crate, and thus a change in thermal energy? Note that $\vec{F}$ and the crate's velocity have the same direction. Thus, if there is no friction, then $\vec{F}$ should be accelerating the crate to a *greater* speed. However, the crate is *slowing*, so there must be friction and a change ΔE_{th} in thermal energy of the crate and the floor. Therefore, the system on which the work is done is the crate–floor system, because both energy changes occur in that system.

(b) What is the increase ΔE_{th} in the thermal energy of the crate and floor?

$$W = \Delta E_{mec} + \Delta E_{th}.$$

Calculations: We know the value of W from (a). The change ΔE_{mec} in the crate's mechanical energy is just the change in its kinetic energy because no potential energy changes occur, so we have

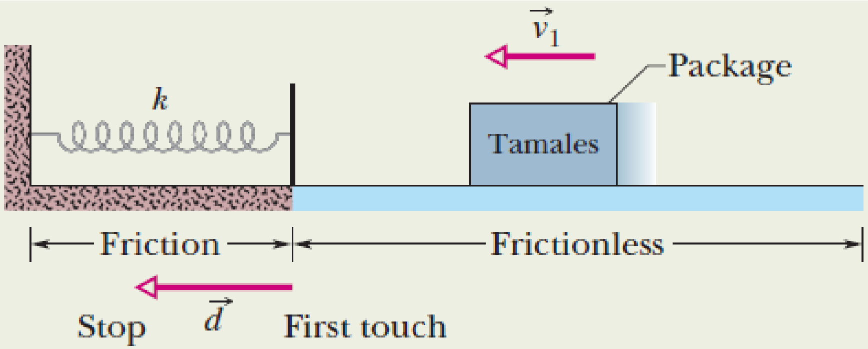
$$\Delta E_{mec} = \Delta K = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2.$$

Substituting this into Eq. 8-34 and solving for ΔE_{th}, we find

$$\begin{aligned}\Delta E_{th} &= W - (\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2) = W - \tfrac{1}{2}m(v^2 - v_0^2)\\ &= 20\text{ J} - \tfrac{1}{2}(14\text{ kg})[(0.20\text{ m/s})^2 - (0.60\text{ m/s})^2]\\ &= 22.2\text{ J} \approx 22\text{ J}. \qquad \text{(Answer)}\end{aligned}$$

Sample problem: energy, friction, spring, and tamales

In Fig. 8-17, a 2.0 kg package of tamales slides along a floor with speed $v_1 = 4.0$ m/s. It then runs into and compresses a spring, until the package momentarily stops. Its path to the initially relaxed spring is frictionless, but as it compresses the spring, a kinetic frictional force from the floor, of magnitude 15 N, acts on the package. If $k = 10\,000$ N/m, by what distance d is the spring compressed when the package stops?



During the rubbing, kinetic energy is transferred to potential energy and thermal energy.

KEY IDEAS

Forces: The normal force on the package from the floor does no work on the package. For the same reason, the gravitational force on the package does no work. As the spring is compressed, a spring force does work on the package. The spring force also pushes against a rigid wall. There is friction between the package and the floor, and the sliding of the package across the floor increases their thermal energies.

System: The package–spring–floor–wall system includes all these forces and energy transfers in one isolated system. From conservation of energy,

$$E_{\text{mec},2} = E_{\text{mec},1} - \Delta E_{\text{th}}. \tag{8-42}$$

Calculations: In Eq. 8-42, let subscript 1 correspond to the initial state of the sliding package and subscript 2 correspond to the state in which the package is momentarily stopped and the spring is compressed by distance d. For both states the mechanical energy of the system is the sum of the package's kinetic energy ($K = \frac{1}{2}mv^2$) and the spring's potential energy ($U = \frac{1}{2}kx^2$). For state 1, $U = 0$ (because the spring is not compressed), and the package's speed is v_1. Thus, we have

$$E_{\text{mec},1} = K_1 + U_1 = \tfrac{1}{2}mv_1^2 + 0.$$

For state 2, $K = 0$ (because the package is stopped), and the compression distance is d. Therefore, we have

$$E_{\text{mec},2} = K_2 + U_2 = 0 + \tfrac{1}{2}kd^2.$$

Finally, by Eq. 8-31, we can substitute $f_k d$ for the change ΔE_{th} in the thermal energy of the package and the floor. We can now rewrite Eq. 8-42 as

$$\tfrac{1}{2}kd^2 = \tfrac{1}{2}mv_1^2 - f_k d.$$

Rearranging and substituting known data give us

$$5000d^2 + 15d - 16 = 0.$$

Solving this quadratic equation yields

$$d = 0.055 \text{ m} = 5.5 \text{ cm}. \qquad \text{(Answer)}$$

Summary

- Conservative forces conserve mechanical energy
- Nonconservative forces convert mechanical energy into other forms
- Conservative force does zero work on any closed path
- Work done by a conservative force is independent of path
- Conservative forces: gravity, spring
- Work done by nonconservative force on closed path is not zero, and depends on the path
- Nonconservative forces: friction, air resistance, tension
- Energy in the form of potential energy can be converted to kinetic or other forms
- Work done by a conservative force is the negative of the change in the potential energy
- Gravity: $U = mgy$
- Spring: $U = \frac{1}{2} kx^2$

- **Mechanical energy is the sum of the kinetic and potential energies; it is conserved only in systems with purely conservative forces**
- **Nonconservative forces change a system’s mechanical energy**
- **Work done by nonconservative forces equals change in a system’s mechanical energy**
- **Potential energy curve: U vs. position**

Lecture (7)

Center of mass and linear momentum

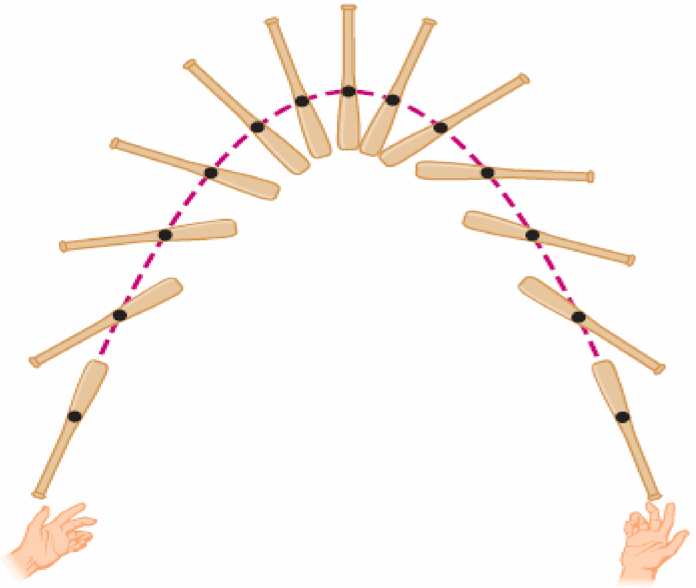
Dr. Hind I. Al-Shaikh

❖The Center of Mass

- **We define the center of mass (com) of a system of particles (such as a person) in order to predict the possible motion of the system.**
- **يعرف مركز الكتلة لنظام الجسيمات هو التنبؤ لاحتمالات الحركة للنظام .**
- **The center of mass of a system of particles is the point that moves as though:**
- **1- all of the system’s mass were concentrated there**
- **2- all external forces were applied there**

The center of mass (black dot) of a baseball bat flipped into the air follows a parabolic path, but all other points of the bat follow more complicated curved paths.

هنا في هذا الشكل مركز الكتلة للمضرب تمثل بنقاط سوداء انقلبت في الهواء يتبع مسار مكافئ, ولكن بقية النقاط تاخذ مسارات منحنية اكثر تعقيدا.

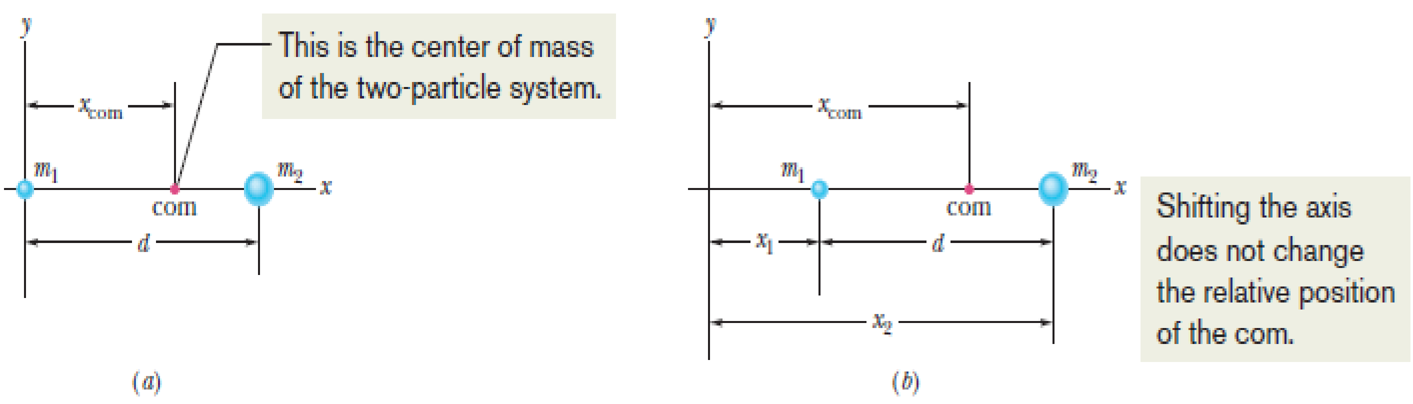


❖The Center of Mass: A system of particles

Consider a situation in which *n* particles are strung out along the *X* axis. Let the mass of the particles are m_1, m_2, ….m_n, and let them be located at x_1, x_2, …x_n respectively. Then if the total mass is $M = m_1 + m_2 + . . . + m_n$, then the location of the center of mass, x_{com}, is

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots + m_nx_n}{M}$$

$$= \frac{1}{M}\sum_{i=1}^{n} m_ix_i.$$



In 3-D, the locations of the center of mass are given by:

$$x_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} m_i x_i, \qquad y_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} m_i y_i, \qquad z_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} m_i z_i.$$

The position of the center of mass can be expressed as:

$$\vec{r}_{\text{com}} = x_{\text{com}}\hat{\mathrm{i}} + y_{\text{com}}\hat{\mathrm{j}} + z_{\text{com}}\hat{\mathrm{k}}.$$

$$\vec{r}_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} m_i \vec{r}_i,$$

➢The Center of Mass: Solid Body

In the case of a solid body, the "particles" become differential mass elements *dm*, the sums become integrals, and the coordinates of the center of mass are defined as

$$x_{com} = \frac{1}{M}\int x\,dm, \qquad y_{com} = \frac{1}{M}\int y\,dm, \qquad z_{com} = \frac{1}{M}\int z\,dm$$

where *M* is the total mass of the object, and the density is mass per unit volume *p*.

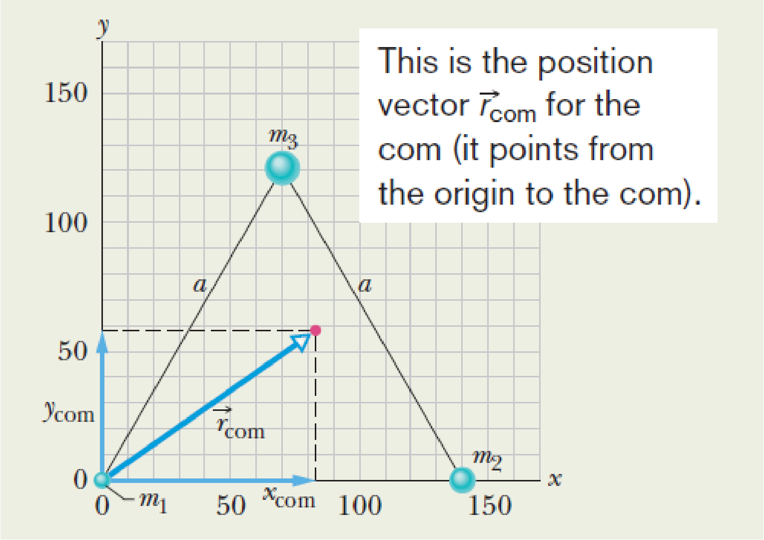
$$\rho = \frac{dm}{dV} = \frac{M}{V}$$

where dV is the volume occupied by a mass element dm, and V is the total volume of the object. Substituting dm= (M/V) dV gives

$$x_{com} = \frac{1}{V}\int x\,dV, \qquad y_{com} = \frac{1}{V}\int y\,dV, \qquad z_{com} = \frac{1}{V}\int z\,dV$$

Sample problem, COM of 3 particles

Three particles of masses $m_1 = 1.2$ kg, $m_2 = 2.5$ kg, and $m_3 = 3.4$ kg form an equilateral triangle of edge length $a = 140$ cm. Where is the center of mass of this system?



We are given the following data:

Particle	Mass (kg)	x (cm)	y (cm)
1	1.2	0	0
2	2.5	140	0
3	3.4	70	120

The total mass M of the system is 7.1 kg.

The coordinates of the center of mass are therefore:

$$x_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i x_i = \frac{m_1x_1 + m_2x_2 + m_3x_3}{M}$$
$$= \frac{(1.2\text{ kg})(0) + (2.5\text{ kg})(140\text{ cm}) + (3.4\text{ kg})(70\text{ cm})}{7.1\text{ kg}}$$
$$= 83\text{ cm} \qquad \text{(Answer)}$$

and $$y_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i y_i = \frac{m_1y_1 + m_2y_2 + m_3y_3}{M}$$
$$= \frac{(1.2\text{ kg})(0) + (2.5\text{ kg})(0) + (3.4\text{ kg})(120\text{ cm})}{7.1\text{ kg}}$$
$$= 58\text{ cm}. \qquad \text{(Answer)}$$

Note that the $z_{com} = 0$.

❖ Newton's 2nd Law for a System of Particles

The vector equation that governs the motion of the center of mass of such a system of particles is:

$$\vec{F}_{net} = M\vec{a}_{com} \qquad \text{(system of particles).}$$

$$\Rightarrow \quad F_{net,x} = Ma_{com,x} \qquad F_{net,y} = Ma_{com,y} \qquad F_{net,z} = Ma_{com,z}$$

Note that:

1. F_{net} is the net force of all external forces that act on the system. Forces on one part of the system from another part of the system (internal forces) are not included

2. M is the total mass of the system. M remains constant, and the system is said to be closed.

3. a_{com} is the acceleration of the center of mass of the system.

The internal forces of the explosion cannot change the path of the com.

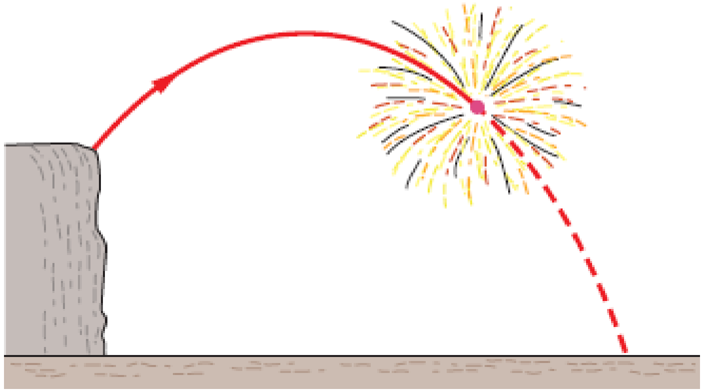


Fig. Shows a fireworks rocket explodes in flight. In the absence of air drag, the center of mass of the fragments would continue to follow the original parabolic path, until fragments began to hit the ground.

Newton's 2^{nd} Law for a System of Particles: Proof

➢For a system of *n* particles, $$M\vec{r}_{com} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots + m_n\vec{r}_n,$$

where M is the total mass, and r_i are the position vectors of the masses m_i.

➢**Differentiating,** $$M\vec{v}_{com} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n.$$

where the *v* vectors are velocity vectors.

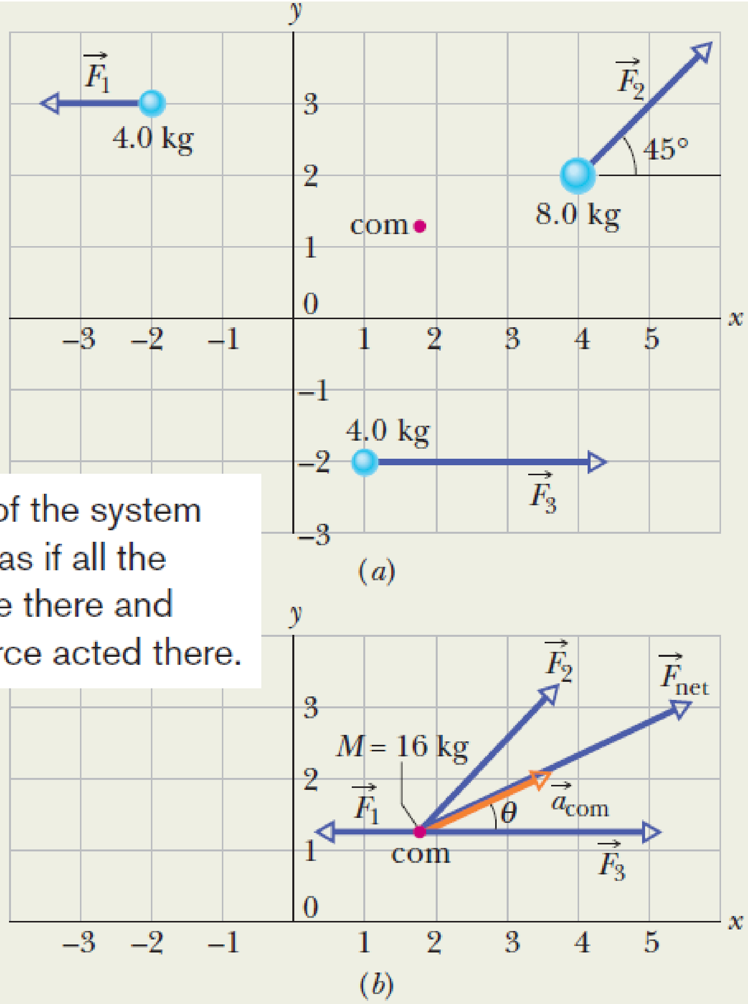
➢**This leads to** $$M\vec{a}_{com} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots + m_n\vec{a}_n.$$

➢**Finally,** $$M\vec{a}_{com} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots + \vec{F}_n.$$

What remains on the right hand side is the vector sum of all the external forces that act on the system, while the internal forces cancel out by Newton's 3^{rd} Law.

Sample problem: motion of the com of 3 particles

The three particles in Fig. 9-7*a* are initially at rest. Each experiences an *external* force due to bodies outside the three-particle system. The directions are indicated, and the magnitudes are $F_1 = 6.0$ N, $F_2 = 12$ N, and $F_3 = 14$ N. What is the acceleration of the center of mass of the system, and in what direction does it move?



The com of the system will move as if all the mass were there and the net force acted there.

Calculations: Applying Newton's second law to the center of mass,

$$\vec{F}_{net} = M\vec{a}_{com}$$

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = M\vec{a}_{com}$$

$$\vec{a}_{com} = \frac{\vec{F}_1 + \vec{F}_2 + \vec{F}_3}{M}.$$

$$a_{com,x} = \frac{F_{1x} + F_{2x} + F_{3x}}{M} = \frac{-6.0\text{ N} + (12\text{ N})\cos 45° + 14\text{ N}}{16\text{ kg}} = 1.03\text{ m/s}^2.$$

Along the y axis, we have

$$a_{com,y} = \frac{F_{1y} + F_{2y} + F_{3y}}{M} = \frac{0 + (12\text{ N})\sin 45° + 0}{16\text{ kg}} = 0.530\text{ m/s}^2.$$

From these components, we find that $\vec{a}_{com}$ has the magnitude

$$a_{com} = \sqrt{(a_{com,x})^2 + (a_{com,y})^2} = 1.16\text{ m/s}^2 \approx 1.2\text{ m/s}^2 \qquad \text{(Answer)}$$

and the angle (from the positive direction of the x axis)

$$\theta = \tan^{-1}\frac{a_{com,y}}{a_{com,x}} = 27°. \qquad \text{(Answer)}$$

❖ Linear momentum

DEFINITION: The linear momentum of a particle is a vector quantity $\vec{P}$ that is defined as

$$\vec{p} = m\vec{v} \qquad \text{(linear momentum of a particle)}$$

in which *m* is the mass of the particle and *v* is its velocity.
(This is a conserved quantity for an isolated system.)
The time rate of change of the momentum of a particle is equal to the net force acting on the particle and is in the direction of that force.

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}.$$

This is actually Newton's 2nd law:

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}.$$

❖ Linear Momentum of a System of Particles

The total linear momentum of a system of particles $\vec{P}$ which is defined to be the vector sum of the individual particles' linear momentum and it is equal to the product of the total mass *M* of the system and the velocity of the center of mass. Thus,

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots + \vec{p}_n$$
$$= m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n$$

$$\vec{P} = M\vec{v}_{com} \quad \text{(linear momentum, system of particles),} \qquad \text{............ 1}$$

If we take the time derivative of Eq. 1 we find

$$\frac{d\vec{P}}{dt} = M\frac{d\vec{v}_{com}}{dt} = M\vec{a}_{com} \qquad \text{...........2}$$

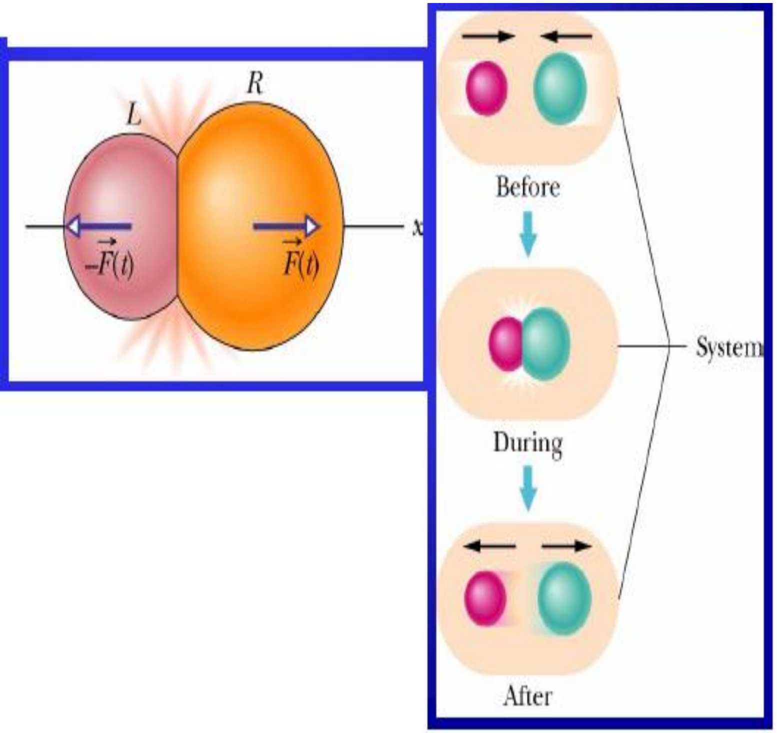
Comparing Eqs. 1 and 2 allows us to write Newton's second law for a system of particles in the equivalent form

$$\vec{F}_{net} = \frac{d\vec{P}}{dt} \quad \text{(system of particles),} \qquad \text{..................3}$$

Collision and Impulse التصادم والاندفاع

Collision: Isolated event in which two or more bodies exert relatively strong forces on each other for a relatively short time.

Impulse: Measures the strength and duration of the collision force and it is vector magnitude.



In this case, the collision is brief, and the ball experiences a force that is great enough to slow, stop, or even reverse its motion.

في هذه الحالة، والاصطدام لفترة وجيزة، والكرة يواجه القوة التي هي كبيرة بما فيه الكفاية لإبطاء، او توقف، أو حتى عكس حركته للكرة.

The figure depicts the collision at one instant. The ball experiences a force *F(t)* that varies during the collision and changes the linear momentum of the ball.

الشكل يصور تصادم في لحظة واحدة. وهنا الكرة تتعرض لقوة خلال الاصطدام وتغيير الزخم الخطي على الكرة.

Third law force pair

$F_R = -F_L \Longrightarrow J_R = -J_L$

- Single collision

The change in linear momentum is related to the force by Newton's 2nd law written in the form:

$$\vec{F} = d\vec{p}/dt.$$

$$d\vec{p} = \vec{F}(t)\,dt. \qquad \text{............4}$$

We can find the net change in the ball's momentum due to the collision if we integrate both sides of Eq. 4 from a time t_i just before the collision to a time t_f just after the collision:

$$\int_{t_i}^{t_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)\,dt. \qquad \text{..........5}$$

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\,dt \qquad \text{(impulse defined)}$$

$$\boxed{\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)dt = \vec{p}_f - \vec{p}_i = \Delta\vec{p}}$$

The right side of the equation is a measure of both the magnitude and the duration of the collision force and is called the *impulse of the collision, J.*

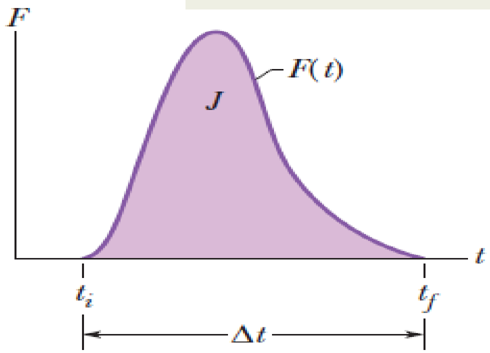
Collision and Impulse theorem

The change in the linear momentum of a body in a collision is equal to the impulse that acts on that body.

$$\boxed{\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \vec{J}}$$

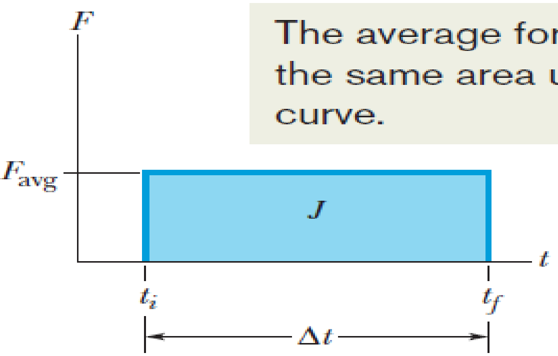
Units: kg m/s

The impulse in the collision is equal to the area under the curve.



(a)

The average force gives the same area under the curve.



(b)

Fig. 9-9 (*a*) The curve shows the magnitude of the time-varying force $F(t)$ that acts on the ball in the collision of Fig. 9-8. The area under the curve is equal to the magnitude of the impulse $\vec{J}$ on the ball in the collision. (*b*) The height of the rectangle represents the average force F_{avg} acting on the ball over the time interval Δt. The area within the rectangle is equal to the area under the curve in (*a*) and thus is also equal to the magnitude of the impulse $\vec{J}$ in the collision.

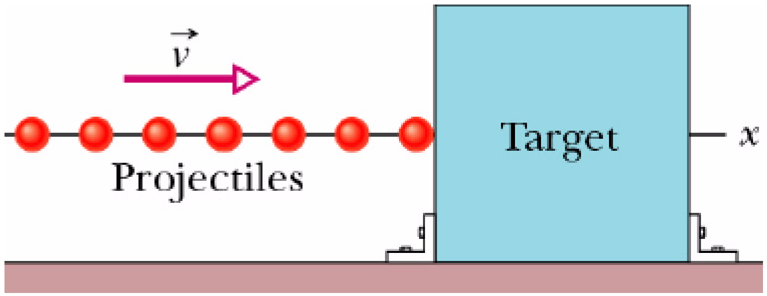
- **Instead of the ball, one can focus on the bat. At any instant, Newton's third law says that the force on the bat has the same magnitude but the opposite direction as the force on the ball.**
- **That means that the impulse on the bat has the same magnitude but the opposite direction as the impulse on the ball.**

F_{avg} such that:

Area under F(t)-Δt curve = Area under F_{avg}- Δt

$$J = F_{avg}\Delta t$$

Collision and Impulse: Series of Collisions



• Let n be the number of projectiles that collide in a time interval Δt.
• Each projectile has initial momentum mv and undergoes a change Δp in linear momentum because of the collision.
• The total change in linear momentum for n projectiles during interval Δt is $n\Delta p$. The resulting impulse on the target during Δt is along the x axis and has the same magnitude of $n\Delta p$ but is in the opposite direction.

Impulse on the target:

$$J_{target} = -J_{projectiles} = -n \cdot \Delta p$$

$$F_{avg} = \frac{J}{\Delta t} = \frac{-n}{\Delta t}\Delta p = \frac{-n}{\Delta t} m\Delta v$$

• In time interval Δt, *an amount of mass* $\Delta m = nm$ *collides with the target.*

$$\Delta m = nm \ in \ \Delta t \rightarrow F_{avg} = -\frac{\Delta m}{\Delta t}\Delta v$$

a) Projectiles stop upon impact:

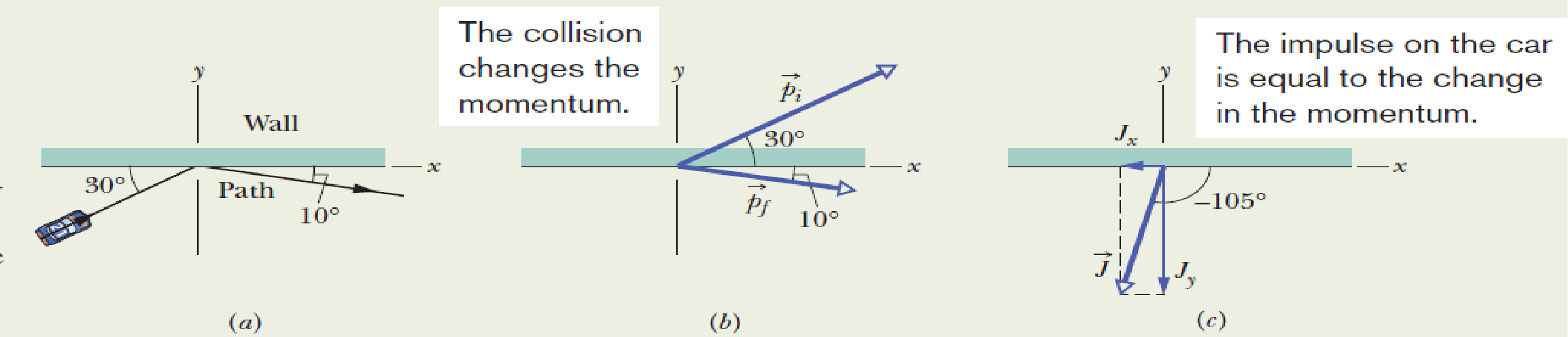
$\Delta v = v_f - v_i = 0 - v = -v$

b) Projectiles bounce:

$\Delta v = v_f - v_i = -v - v = -2v$

Example: 2-D impulse

Race car–wall collision. Figure 9-11*a* is an overhead view of the path taken by a race car driver as his car collides with the racetrack wall. Just before the collision, he is traveling at speed $v_i = 70$ m/s along a straight line at 30° from the wall. Just after the collision, he is traveling at speed $v_f = 50$ m/s along a straight line at 10° from the wall. His mass m is 80 kg.



(a) What is the impulse $\vec{J}$ on the driver due to the collision?

Calculations: Figure 9-11*b* shows the driver's momentum $\vec{p}_i$ before the collision (at angle 30° from the positive x direction) and his momentum $\vec{p}_f$ after the collision (at angle −10°).

x component: Along the x axis we have

$$J_x = m(v_{fx} - v_{ix})$$
$$= (80 \text{ kg})[(50 \text{ m/s})\cos(-10°) - (70 \text{ m/s})\cos 30°]$$
$$= -910 \text{ kg}\cdot\text{m/s}.$$

y component: Along the y axis,

$$J_y = m(v_{fy} - v_{iy})$$
$$= (80 \text{ kg})[(50 \text{ m/s})\sin(-10°) - (70 \text{ m/s})\sin 30°]$$
$$= -3495 \text{ kg}\cdot\text{m/s} \approx -3500 \text{ kg}\cdot\text{m/s}.$$

Impulse: The impulse is then

$$\vec{J} = (-910\hat{\text{i}} - 3500\hat{\text{j}})\ \text{kg}\cdot\text{m/s}, \qquad \text{(Answer)}$$

which means the impulse magnitude is

$$J = \sqrt{J_x^2 + J_y^2} = 3616\ \text{kg}\cdot\text{m/s} \approx 3600\ \text{kg}\cdot\text{m/s}.$$

The angle of $\vec{J}$ is given by

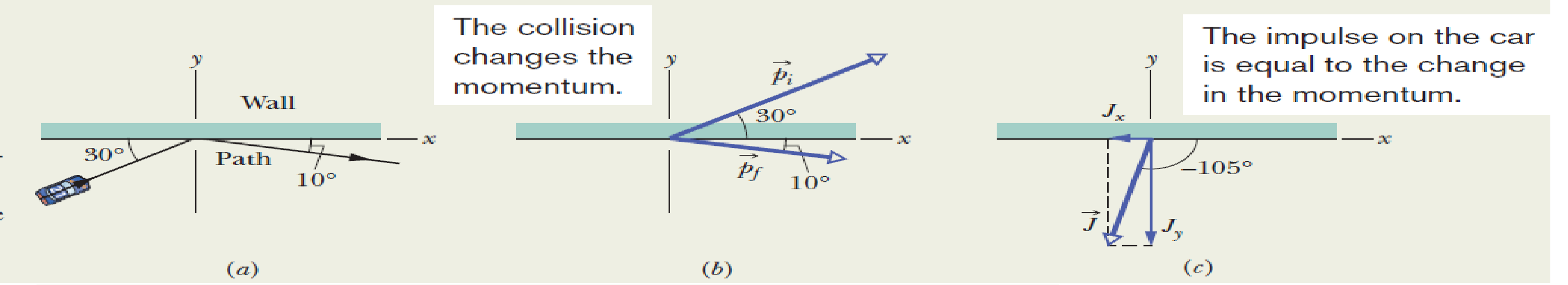
$$\theta = \tan^{-1}\frac{J_y}{J_x}, \qquad \text{(Answer)}$$

which a calculator evaluates as $75.4°$. Recall that the physically correct result of an inverse tangent might be the displayed answer plus $180°$. We can tell which is correct here by drawing the components of $\vec{J}$ (Fig. 9-11*c*). We find that θ is actually $75.4° + 180° = 255.4°$, which we can write as

$$\theta = -105°. \qquad \text{(Answer)}$$

Example: 2-D impulse, cont.

(b) The collision lasts for 14 ms. What is the magnitude of the average force on the driver during the collision?



Calculations: We have

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{3616\ \text{kg}\cdot\text{m/s}}{0.014\ \text{s}}$$

$$= 2.583 \times 10^5\ \text{N} \approx 2.6 \times 10^5\ \text{N}. \qquad \text{(Answer)}$$

Using $F = ma$ with $m = 80$ kg, you can show that the magnitude of the driver's average acceleration during the collision is about $3.22 \times 10^3\ \text{m/s}^2 = 329g$, which is fatal.

Surviving: Mechanical engineers attempt to reduce the chances of a fatality by designing and building racetrack walls with more "give," so that a collision lasts longer. For example, if the collision here lasted 10 times longer and the other data remained the same, the magnitudes of the average force and average acceleration would be 10 times less and probably survivable.

Momentum and Kinetic Energy in Collisions

Assumptions: **Closed systems (no mass enters or leaves them)**
Isolated systems (no external forces act on the bodies within the system)

- **Elastic collision:** **If the total kinetic energy of the system of two colliding bodies is unchanged (conserved) by the collision.**

- **Inelastic collision:** **The kinetic energy of the system is not conserved some goes into thermal energy, sound, etc.**
Example: Superball into hard floor.

- **Completely inelastic collision:** **After the collision the bodies loose energy and stick together.**
Example: Ball of wet putty into floor

Conservation of Linear Momentum

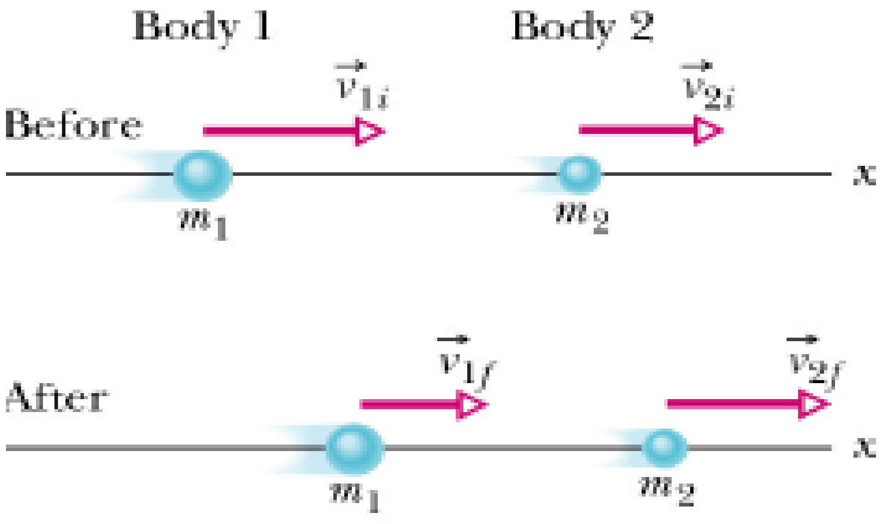
If no net external force acts on a system of particles, the total linear momentum, *P*, of the system cannot change.

$$\vec{P} = \text{constant} \qquad \text{(closed, isolated system).}$$

If the component of the net external force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

 # Newton's 3rd law $\quad \dfrac{d\vec{p}_1}{dt} = -\dfrac{d\vec{p}_2}{dt} \qquad \dfrac{d(\vec{p}_1 + \vec{p}_2)}{dt} = \dfrac{d\vec{P}}{dt} = 0$

9.9: Inelastic collisions in 1-D



Conservation of Linear Momentum

$$\left(\text{Total momentum } \vec{p}_i \text{ before collision}\right) = \left(\text{Total momentum } \vec{p}_f \text{ after collision}\right)$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

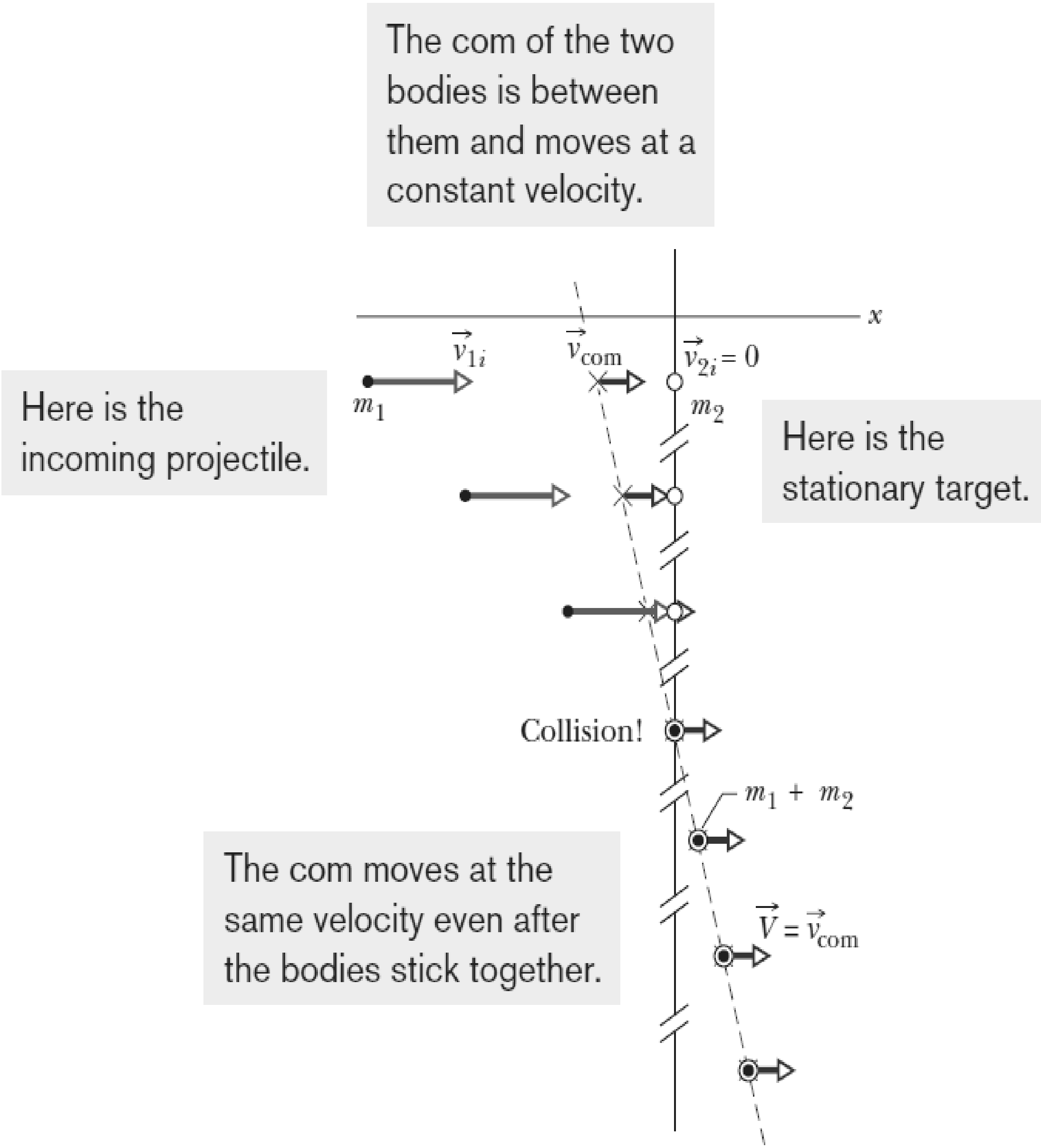
$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}.$$

Completely inelastic collision:

$$m_1 v_{1i} = (m_1 + m_2) V$$

$$V = \frac{m_1}{m_1 + m_2} v_{1i}.$$

❖Velocity of Center of Mass التصادم غير المرن



$$\vec{P} = M\vec{v}_{com} = (m_1 + m_2)\vec{v}_{com}$$

$$\vec{P}\ conserved \rightarrow \vec{P} = \vec{p}_{1i} + \vec{p}_{2i}$$

$$\rightarrow \vec{v}_{com} = \frac{\vec{P}}{m_1 + m_2} = \frac{\vec{p}_{1i} + \vec{p}_{2i}}{m_1 + m_2} = \frac{\vec{p}_{1f} + \vec{p}_{2f}}{m_1 + m_2}$$

Completely inelastic collision ⟹ ***V = vcom***

Fig. Shows some freeze frames of a two-body system, which undergoes a completely inelastic collision. The system's center of mass is shown in each freeze-frame. The velocity v_{com} of the center of mass is unaffected by the collision.

Because the bodies stick together after the collision, their common velocity V must be equal to v_{com}.

Sample problem: conservation of momentum

The *ballistic pendulum* was used to measure the speeds of bullets before electronic timing devices were developed. The version shown in Fig. 9-17 consists of a large block of wood of mass $M = 5.4$ kg, hanging from two long cords. A bullet of mass $m = 9.5$ g is fired into the block, coming quickly to rest. The *block + bullet* then swing upward, their center of mass rising a vertical distance $h = 6.3$ cm before the pendulum comes momentarily to rest at the end of its arc. What is the speed of the bullet just prior to the collision?

There are two events here. The bullet collides with the block. Then the bullet–block system swings upward by height h.

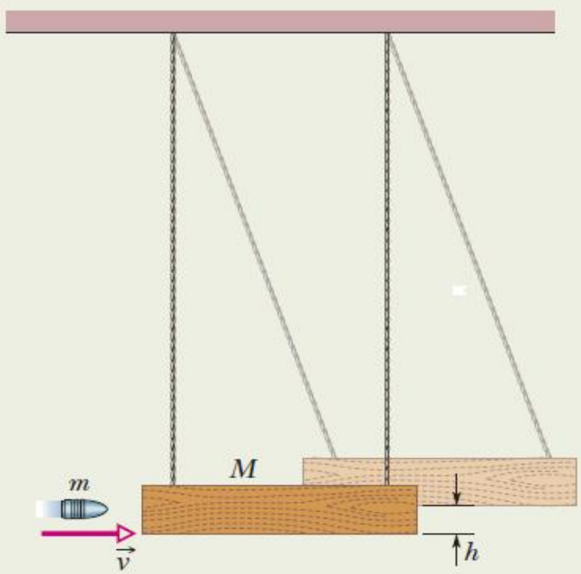


Fig. 9-17 A ballistic pendulum, used to measure the speeds of bullets.

The collision within the bullet– block system is so brief. Therefore:

(1)During the collision, the gravitational force on the block and the force on the block from the cords are still balanced. Thus, during the collision, the net external impulse on the bullet–block system is zero. Therefore, the system is isolated and its total linear momentum is conserved.

(2) The collision is one-dimensional in the sense that the direction of the bullet and block just after the collision is in the bullet's original direction of motion.

$$V = \frac{m}{m + M}v.$$

As the bullet and block now swing up together, the mechanical energy of the bullet–block–Earth system is conserved:

$$\tfrac{1}{2}(m + M)V^2 = (m + M)gh.$$

Combining steps:

$$v = \frac{m + M}{m}\sqrt{2gh}$$

$$= \left(\frac{0.0095 \text{ kg} + 5.4 \text{ kg}}{0.0095 \text{ kg}}\right)\sqrt{(2)(9.8 \text{ m/s}^2)(0.063 \text{ m})}$$

$$= 630 \text{ m/s}. \qquad \text{(Answer)}$$

Elastic collisions in 1-D

$$\left(\text{Total kinetic energy before collision}\right) = \left(\text{Total kinetic energy after collision}\right)$$

Here is the generic setup for an elastic collision with a stationary target.

In an elastic collision, the kinetic energy of each colliding body may change, but the total kinetic energy of the system does not change.

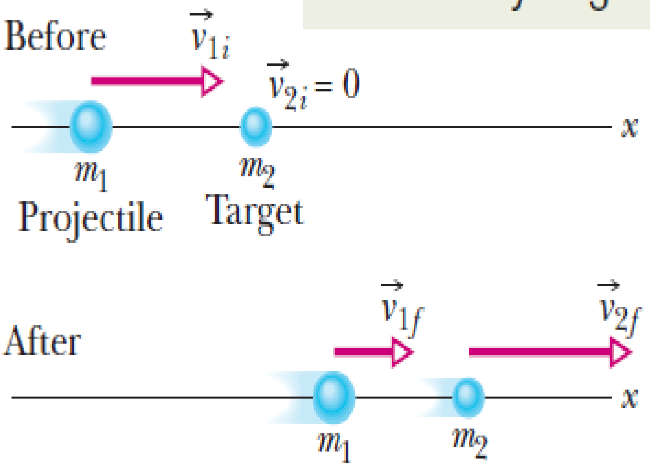


Fig. 9-18 Body 1 moves along an x axis before having an elastic collision with body 2, which is initially at rest. Both bodies move along that axis after the collision.

Stationary target:

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \qquad \text{(linear momentum).}$$

$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 \qquad \text{(kinetic energy).}$$

$v_{2f} > 0$ always

$v_{1f} > 0$ if $m_1 > m_2$ → forward mov.

$v_{1f} < 0$ if $m_1 < m_2$ → rebounds

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}.$$

Elastic collisions in 1-D: Stationary Target

-Equal masses: $m_1=m_2 \longrightarrow v_{1f}=0$ *and* $v_{2f}=v_{1i} \longrightarrow$ **In head-on collisions bodies of equal masses simply exchange velocities.**

- Massive target: $m_2>>m_1 \longrightarrow v_{1f} \approx -v_{1i}$ **and** $v_{2f} \approx (2m_1/m_2)v_{1i} \longrightarrow$ **Body 1 bounces back with approximately same speed. Body 2 moves forward at low speed.**

- Massive projectile: $m_1>>m_2 \longrightarrow v_{1f} \approx v_{1i}$ **and** $v_{2f} \approx 2v_{1i} \longrightarrow$ **Body 1 keeps on going scarcely slowed by the collision. Body 2 charges ahead at twice the initial speed of the projectile.**

Elastic collisions in 1-D: Moving Target

Closed, isolated system

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f},$$ **Linear momentum**

$$\tfrac{1}{2}m_1 v_{1i}^2 + \tfrac{1}{2}m_2 v_{2i}^2 = \tfrac{1}{2}m_1 v_{1f}^2 + \tfrac{1}{2}m_2 v_{2f}^2.$$ **Kinetic energy**

Here is the generic setup for an elastic collision with a moving target.

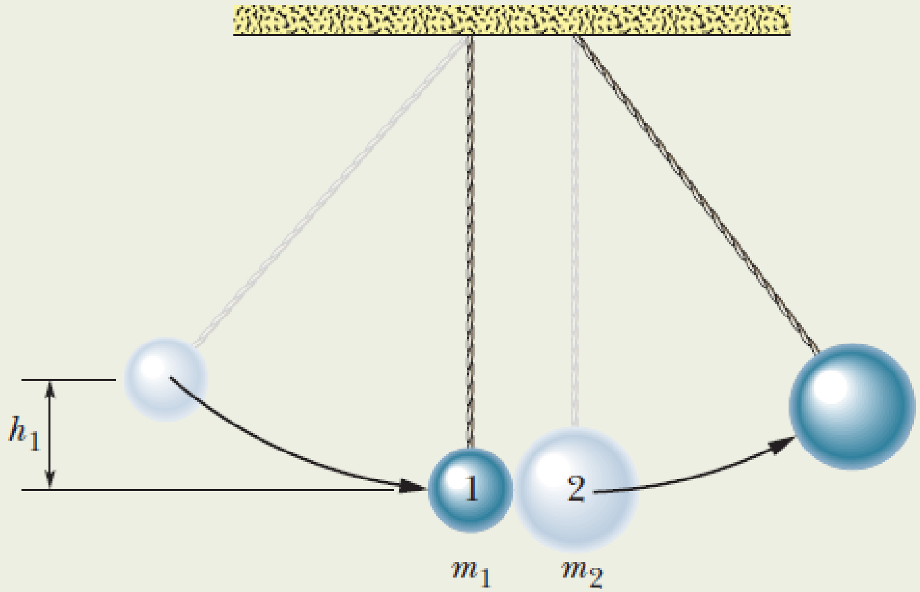
Fig. 9-19 Two bodies headed for a one-dimensional elastic collision.

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}.$$

Sample problem: two pendulums

Two metal spheres, suspended by vertical cords, initially just touch, as shown in Fig. 9-20. Sphere 1, with mass $m_1 = 30$ g, is pulled to the left to height $h_1 = 8.0$ cm, and then released from rest. After swinging down, it undergoes an elastic collision with sphere 2, whose mass $m_2 = 75$ g. What is the velocity v_{1f} of sphere 1 just after the collision?



Step 1: As sphere 1 swings down, the mechanical energy of the sphere–Earth system is conserved. (The mechanical energy is not changed by the force of the cord on sphere 1 because that force is always directed perpendicular to the sphere's direction of travel.)

Calculation: Let's take the lowest level as our reference level of zero gravitational potential energy. Then the kinetic energy of sphere 1 at the lowest level must equal the gravitational potential energy of the system when sphere 1 is at height h_1. Thus,

$$\tfrac{1}{2}m_1 v_{1i}^2 = m_1 g h_1,$$

which we solve for the speed v_{1i} of sphere 1 just before the collision:

$$v_{1i} = \sqrt{2gh_1} = \sqrt{(2)(9.8\ \text{m/s}^2)(0.080\ \text{m})}$$
$$= 1.252\ \text{m/s}.$$

Step 2: Here we can make two assumptions in addition to the assumption that the collision is elastic. First, we can assume that the collision is one-dimensional because the motions of the spheres are approximately horizontal from just before the collision to just after it. Second, because the collision is so

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$
$$= \frac{0.030\ \text{kg} - 0.075\ \text{kg}}{0.030\ \text{kg} + 0.075\ \text{kg}} (1.252\ \text{m/s})$$
$$= -0.537\ \text{m/s} \approx -0.54\ \text{m/s}. \qquad \text{(Answer)}$$

The minus sign tells us that sphere 1 moves to the left just after the collision.

❖Collisions in 2-D

Closed, isolated system → $$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}.$$

Linear momentum conserved

Elastic collision → $$K_{1i} + K_{2i} = K_{1f} + K_{2f}.$$

Kinetic energy conserved

A glancing collision that conserves both momentum and kinetic energy.

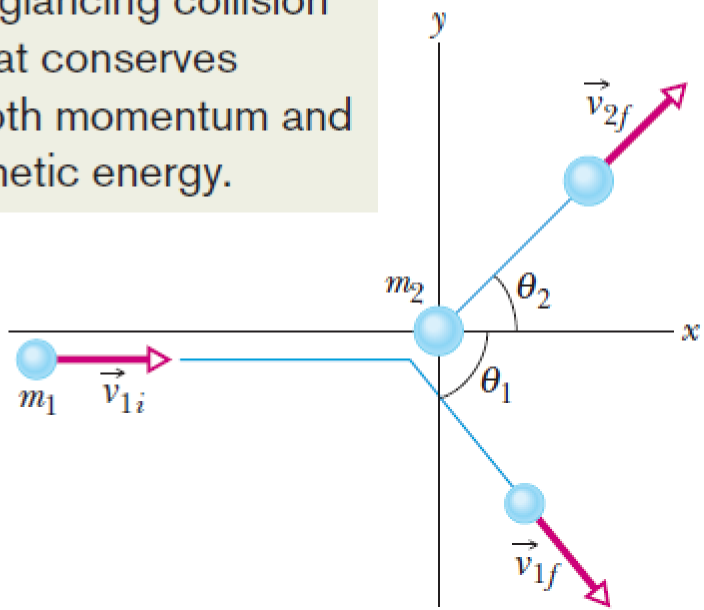


Fig. 9-21 An elastic collision between two bodies in which the collision is not head-on. The body with mass m_2 (the target) is initially at rest.

-For example a stationary target, elastic collision :

$$x - axis \rightarrow m_1 v_{1i} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2$$

$$y - axis \rightarrow 0 = -m_1 v_{1f} \sin\theta_1 + m_2 v_{2f} \sin\theta_2$$

If elastic collision $$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

Systems with Varying Mass: A Rocket

System: rocket + exhaust products

The ejection of mass from the rocket's rear increases the rocket's speed.

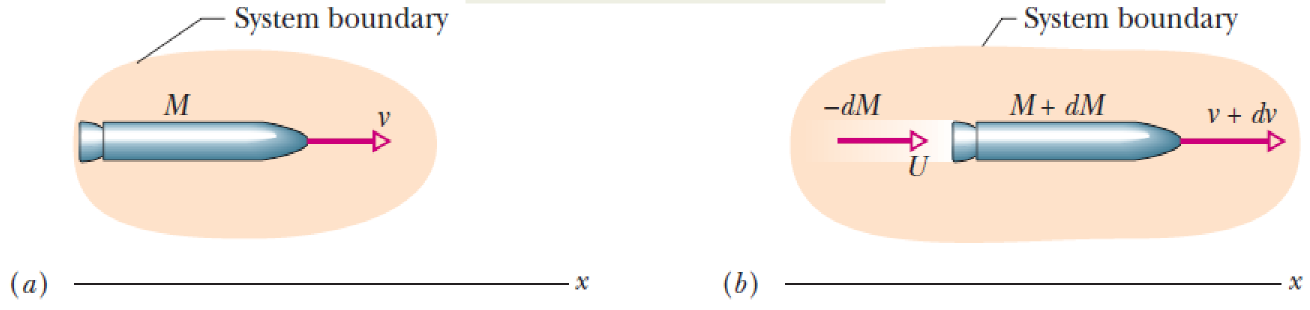


Fig. 9-22 (*a*) An accelerating rocket of mass M at time t, as seen from an inertial reference frame. (*b*) The same but at time $t + dt$. The exhaust products released during interval dt are shown.

The system here consists of the rocket and the exhaust products released during interval *dt*. The system is closed and isolated, so the linear momentum of the system must be conserved during *dt*.

النظام هنا يتكون من الصاروخ و انبعاثات العادم الذي يصدر خلال فترة زمنية

$$P_i = P_f, \Longrightarrow Mv = -dM\,U + (M + dM)(v + dv)$$

Linear momentum of exhaust products released during the interval dt

Linear momentum of rocket at the end of dt

$$(v + dv) = v_{\text{rel}} + U,$$

$$U = v + dv - v_{\text{rel}}.$$

$$\Longrightarrow -\frac{dM}{dt} v_{\text{rel}} = M\frac{dv}{dt}. \Longrightarrow \boxed{Rv_{\text{rel}} = Ma}$$

v_{rel} defined

 # Rv_{rel} is called the thrust of the engine.

Systems with Varying Mass: Finding the velocity

$$dv = -v_{\text{rel}} \frac{dM}{M}.$$

$$\int_{v_i}^{v_f} dv = -v_{\text{rel}} \int_{M_i}^{M_f} \frac{dM}{M},$$

in which M_i is the initial mass of the rocket and M_f its final mass. Evaluating the integrals then gives

$$v_f - v_i = v_{\text{rel}} \ln \frac{M_i}{M_f}$$

for the increase in the speed of the rocket during the change in mass from M_i to M_f.

Sample problem: rocket engine, thrust, acceleration

A rocket whose initial mass M_i is 850 kg consumes fuel at the rate $R = 2.3$ kg/s. The speed v_{rel} of the exhaust gases relative to the rocket engine is 2800 m/s. What thrust does the rocket engine provide?

KEY IDEA

Thrust T is equal to the product of the fuel consumption rate R and the relative speed v_{rel} at which exhaust gases are expelled, as given by Eq. 9-87.

Calculation: Here we find

$$T = Rv_{rel} = (2.3\ \text{kg/s})(2800\ \text{m/s})$$
$$= 6440\ \text{N} \approx 6400\ \text{N}. \qquad \text{(Answer)}$$

(b) What is the initial acceleration of the rocket?

KEY IDEA

We can relate the thrust T of a rocket to the magnitude a of the resulting acceleration with $T = Ma$, where M is the rocket's mass. However, M decreases and a increases as fuel is consumed. Because we want the initial value of a here, we must use the intial value M_i of the mass.

Calculation: We find

$$a = \frac{T}{M_i} = \frac{6440\ \text{N}}{850\ \text{kg}} = 7.6\ \text{m/s}^2. \qquad \text{(Answer)}$$

To be launched from Earth's surface, a rocket must have an initial acceleration greater than $g = 9.8$ m/s². That is, it must be greater than the gravitational acceleration at the surface. Put another way, the thrust T of the rocket engine must exceed the initial gravitational force on the rocket, which here has the magnitude $M_i g$, which gives us

$$(850\ \text{kg})(9.8\ \text{m/s}^2) = 8330\ \text{N}.$$

Because the acceleration or thrust requirement is not met (here $T = 6400$ N), our rocket could not be launched from Earth's surface by itself; it would require another, more powerful, rocket.

Sample problem: 1-D explosion

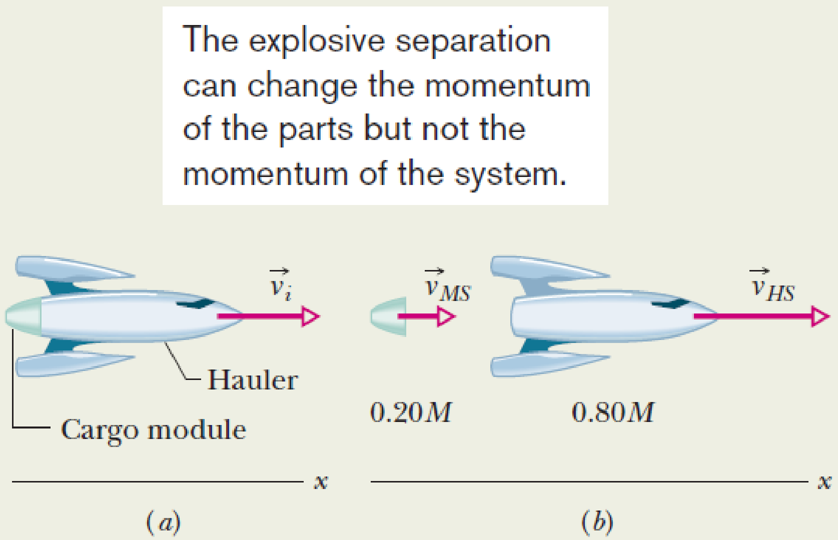
One-dimensional explosion: Figure 9-12*a* shows a space hauler and cargo module, of total mass M, traveling along an x axis in deep space. They have an initial velocity $\vec{v}_i$ of magnitude 2100 km/h relative to the Sun. With a small explosion, the hauler ejects the cargo module, of mass $0.20M$ (Fig. 9-12*b*). The hauler then travels 500 km/h faster than the module along the x axis; that is, the relative speed v_{rel} between the hauler and the module is 500 km/h. What then is the velocity $\vec{v}_{HS}$ of the hauler relative to the Sun?

KEY IDEA

Because the hauler–module system is closed and isolated, its total linear momentum is conserved; that is,

$$\vec{P}_i = \vec{P}_f, \tag{9-44}$$

Fig. 9-12



where the subscripts i and f refer to values before and after the ejection, respectively.

Calculations: Because the motion is along a single axis, we can write momenta and velocities in terms of their x components, using a sign to indicate direction. Before the ejection, we have

$$P_i = Mv_i. \tag{9-45}$$

Let v_{MS} be the velocity of the ejected module relative to the Sun. The total linear momentum of the system after the ejection is then

$$P_f = (0.20M)v_{MS} + (0.80M)v_{HS}, \tag{9-46}$$

where the first term on the right is the linear momentum of the module and the second term is that of the hauler.

We do not know the velocity v_{MS} of the module relative to the Sun, but we can relate it to the known velocities with

$$\begin{pmatrix}\text{velocity of}\\ \text{hauler relative}\\ \text{to Sun}\end{pmatrix} = \begin{pmatrix}\text{velocity of}\\ \text{hauler relative}\\ \text{to module}\end{pmatrix} + \begin{pmatrix}\text{velocity of}\\ \text{module relative}\\ \text{to Sun}\end{pmatrix}.$$

In symbols, this gives us

$$v_{HS} = v_{rel} + v_{MS} \tag{9-47}$$

or

$$v_{MS} = v_{HS} - v_{rel}.$$

Substituting this expression for v_{MS} into Eq. 9-46, and then substituting Eqs. 9-45 and 9-46 into Eq. 9-44, we find

$$Mv_i = 0.20M(v_{HS} - v_{rel}) + 0.80Mv_{HS},$$

which gives us

$$v_{HS} = v_i + 0.20v_{rel},$$

or

$$v_{HS} = 2100 \text{ km/h} + (0.20)(500 \text{ km/h})$$
$$= 2200 \text{ km/h}. \qquad \text{(Answer)}$$