AL-Karkh University of Science

College of Geophysics and Remote Sensing Department of Geophysics

ELECTROSTATICE

For Geophysics / First Semester

2018-2019

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ELECTROSTATICS

Outlines of studies:

- **Electric Charge**
- **Coulum's Law**
- **Electric Fields**
- **Gauss' Law**
- **Electric Potential**

CHAPTER 1

E LECT R IC CHARGE

Electrostatics

- **Electrostatics is the physics term for static charge.**
- **Electro means charge, and of course static means stationary or not moving.**

- **Charge**

  2 Types of Charge:

Positive (+) and Negative (-)

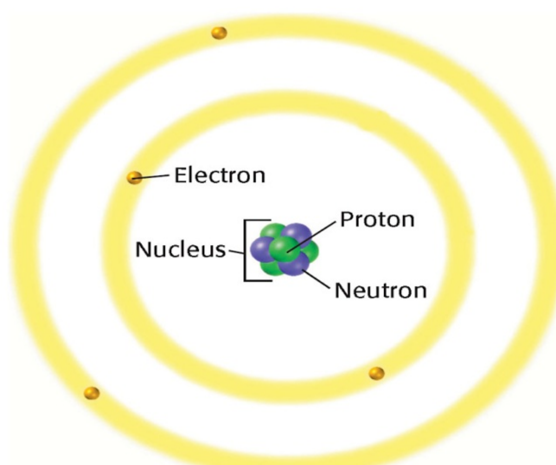
- **Electric Charge**

All matter is made up of atoms

Atoms contain

-Protons (+)

-Neutrons (0)



-Electrons (-)

-Law of Electric Charges

- **Charges with the same electrical sign repel each other, and charges with the opposite electrical signs attract each other.**
- **Protons are positively charged and electrons are negatively charged, so they are attracted to each other.**
- **Without this attraction, electrons would not be held in atoms.**
- **Unlike a gravitational force which always attracts, <u>electrostatic force may repel or attract depending on the type charge</u>.**

Ben's Rule and Paula Abdul - Opposites attract and likes repel.

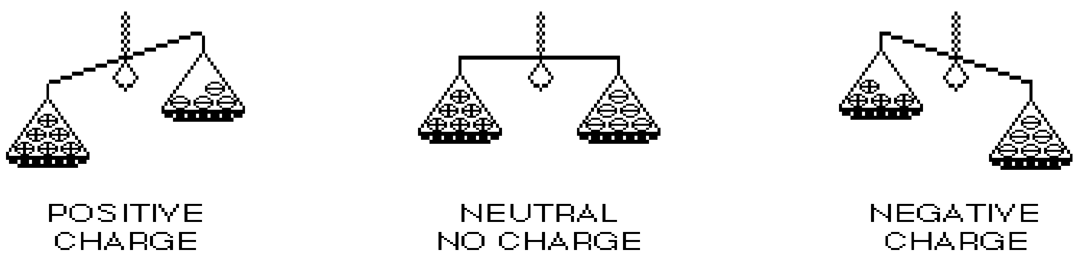
(+) (-) = attract
(+) (+) = repel
(-) (-) = repel

▪ Conductors and Insulators

-The properties of conductors and insulators are due to the structure and electrical nature of atoms.

-Atoms consist of positively charged *protons*, negatively charged *electrons*, and electrically neutral *neutrons*. The protons and neutrons are packed tightly together in a central nucleus.

- When certain types of objects are rubbed together, electrons from one object may be transferred to an object with a greater affinity for the electrons. When this happens, the object that gave up the electrons is positive, whereas the object that collected the electrons is negative.



- **Conductors are materials through which charge can move freely; examples include**

***metals (such as copper in common lamp wire), the human body, and tap water*.**

- **Nonconductors also called insulators are materials through which charge cannot move freely; examples include *rubber, plastic, glass, and chemically pure water*.**

- **Semiconductors are materials that are intermediate between conductors and insulators; examples include *silicon and germanium in computer chips.***

- **Superconductors are materials that are perfect conductors, allowing charge to move without any hindrance.**

How Can You Charge Objects?

There are 3 ways objects can be charged:

- **Friction**
- **Conduction**

o **Induction**

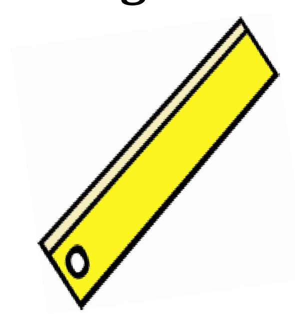
****In each of these, only the electrons move. The protons stay in the nucleus****

- **Friction**

Charging by friction occurs when electrons are "wiped" from one object onto another.

Example

If you use a cloth to rub a plastic ruler, electrons move from the cloth to the ruler. The ruler gains electrons and the cloth loses electrons.



- **Conduction**

Charging by conduction happens when electrons move from one object to another through direct contact (touching).

Example: Suppose you touch an uncharged piece of metal with a positively charged glass rod. Electrons from the metal will move to the glass rod. The metal loses electrons and becomes positively charged.

- **Induction**

Charging by induction happens when charges in an uncharged object are rearranged without direct contact with a charged object.

Ex. If you charge up a balloon through friction and place the balloon near pieces of paper, the charges of the paper will be rearranged and the paper will be attracted to the balloon.

Coulomb's Law

All bodies are able to take a charge of electricity and this is termed static electricity. The charge on a body is measure by means of the force between the charges. The Coulomb force law, which only applies to charged points, is stated below..

The force of attraction or repulsion between two charged points is directly proportional to the charges and inversely proportional to the square of the distance between them.

In vector form, it is stated thus,

$$\boxed{\mathrm{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 |\mathrm{R}_{12}|^2}\mathrm{a}_{12}}$$

Where;

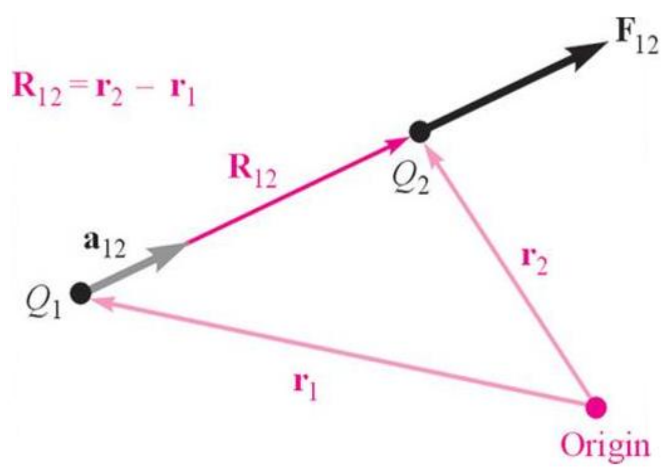
F = Force between points (N)

Q_1, Q_2 = Charges on point 1 and point 2 (Coulomb)

R = radial separation on points/distance (m)

a_{12} **= the unit vector in the direction from Q_1 to Q_2**

ε_o **= Permittivity of the free space (vacuum)**



Charge Q_1 exerts a vector force $\mathbf{F}_{12}$ in Newton's (N) on charge Q_2,

$$\boxed{\mathrm{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 |\mathrm{R}_{12}|^2}\mathrm{a}_{12}}$$

The equation giving the **electrostatic force** for charged particles is called **Coulomb's law:**

$$\vec{F} = k \frac{q_1 q_2}{r^2} \hat{r} \qquad \text{(Coulomb's law)},$$

The SI unit of charge is the ***coulomb.***

The **electrostatic constant** is

$$k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \, \mathrm{N \cdot m^2/C^2}.$$

The quantity ε_0 is called the ***permittivity constant***

$$\varepsilon_0 = 8.85 \times 10^{-12} \, \mathrm{C^2/N \cdot m^2}.$$

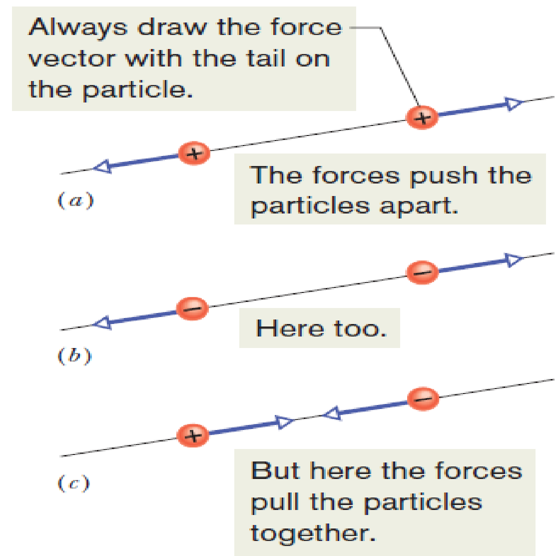


Fig. 21-6 Two charged particles repel each other if they have the same sign of charge, either (*a*) both positive or (*b*) both negative. (*c*) They attract each other if they have opposite signs of charge.

Note: a negative force results if the points have opposite charges and a positive force results if the points have the same polarity.

The above equation can be simplified as follows:

$$F = k \frac{Q_1 Q_2}{R^2}$$

The proportional constant, *k* is:

$$k = \frac{1}{4\pi\varepsilon} \quad , \qquad \varepsilon = \varepsilon_r \varepsilon_0$$

If the space between the charges is another material or air, the law may be written:

$$\boxed{\mathbf{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r \left|\mathbf{R}_{12}\right|^2}\mathbf{a}_{12}}$$

Where ε_r **is the relative permittivity of material.**

The unit *coulomb* (C) is derived from the SI unit *ampere* (A) of the electric current. Current is the rate *dq/dt* at which charge moves past a point or through a region

$$i = \frac{dq}{dt} \qquad \text{(electric current),}$$

Therefore,

$$1\ \text{C} = (1\ \text{A})(1\ \text{s}).$$

If there are n charged particles, they interact independently in pairs, and the force on any one of them, say particle 1, is given by the vector sum

$$\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \cdots + \vec{F}_{1n}$$

As with gravitational force law, the shell theorem has analogs in electrostatics:

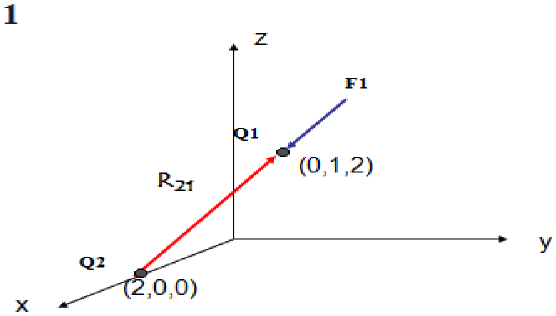
- A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at its center.

- If a charged particle is located inside a shell of uniform charge, there is no net electrostatic force on the particle from the shell.

Example 1

Find the force on charge Q1, 20 μC, due to charge Q2, -300 μC, where Q1 is at (0,1,2)m and Q2 at (2,0,0)m.

Solution 1



- Because 1C is a rather large unit, charges are often given in microcoulombs (μC), nanocoulombs (nC) and picocoulombs (pC).

  Referring to figure,

  $R_{21}=-2a_x+a_y+2a_z$

  $\therefore R=3$

  $a_{21}=1/3(-2a_x+a_y+2a_z)$

$\therefore$ using Coulmb's Law equation; $F_{12} = \dfrac{Q_1Q_2}{4\pi\varepsilon_0 |R_{12}|^2} a_{12}$

$$F_{21} = \frac{(20\times10^{-6})(-300\times10^{-6})}{4\pi(8.854\times10^{-12})(3)^2}\left(\frac{-2a_x+a_y+2a_z}{3}\right)$$

$$= 6\left(\frac{2a_x-a_y-2a_z}{3}\right)N$$

The force magnitude is 6N and the direction is such that Q_1 is attracted to Q_2 (unlike charges attract)

Example 2

(c) Figure 21-8*e* is identical to Fig. 21-8*a* except that particle 4 is now included. It has charge $q_4 = -3.20 \times 10^{-19}$ C, is at a distance $\frac{3}{4}R$ from particle 1, and lies on a line that makes an angle $\theta = 60°$ with the x axis. What is the net electrostatic force $\vec{F}_{1,\text{net}}$ on particle 1 due to particles 2 and 4?

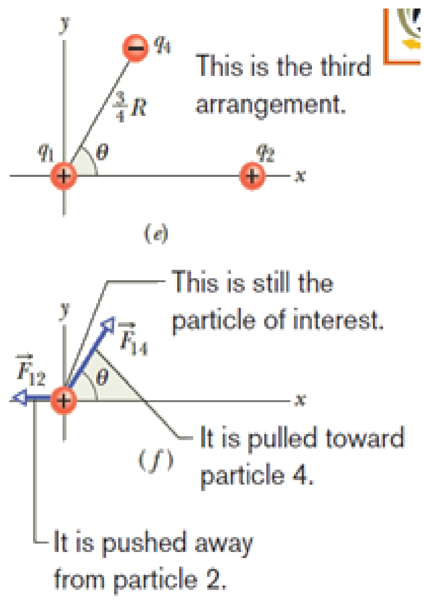


Fig. 21-8 (*e*) Particle 4 included. (*f*) Freebody diagram for particle 1.

$$\begin{aligned} F_{14} &= \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_4|}{(\frac{3}{4}R)^2} \\ &= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \\ &\quad \times \frac{(1.60 \times 10^{-19}\ \text{C})(3.20 \times 10^{-19}\ \text{C})}{(\frac{3}{4})^2(0.0200\ \text{m})^2} \\ &= 2.05 \times 10^{-24}\ \text{N}. \end{aligned}$$

$$\begin{aligned} F_{1,\text{net},x} &= F_{12,x} + F_{14,x} = F_{12} + F_{14}\cos 60° \\ &= -1.15 \times 10^{-24}\ \text{N} + (2.05 \times 10^{-24}\ \text{N})(\cos 60°) \\ &= -1.25 \times 10^{-25}\ \text{N}. \end{aligned}$$

The sum of the y components gives us

$$\begin{aligned} F_{1,\text{net},y} &= F_{12,y} + F_{14,y} = 0 + F_{14}\sin 60° \\ &= (2.05 \times 10^{-24}\ \text{N})(\sin 60°) \\ &= 1.78 \times 10^{-24}\ \text{N}. \end{aligned}$$

The net force $\vec{F}_{1,\text{net}}$ has the magnitude

$$F_{1,\text{net}} = \sqrt{F^2_{1,\text{net},x} + F^2_{1,\text{net},y}} = 1.78 \times 10^{-24}\ \text{N}. \quad \text{(Answer)}$$

To find the direction of $\vec{F}_{1,\text{net}}$, we take

$$\theta = \tan^{-1}\frac{F_{1,\text{net},y}}{F_{1,\text{net},x}} = -86.0°.$$

However, this is an unreasonable result because $\vec{F}_{1,\text{net}}$ must have a direction between the directions of $\vec{F}_{12}$ and $\vec{F}_{14}$. To correct θ, we add 180°, obtaining

$$-86.0° + 180° = 94.0°. \quad \text{(Answer)}$$

❖Charge is Quantized

Since the days of Benjamin Franklin, our understanding of of the nature of electricity has changed from being a type of 'continuous fluid' to a collection of smaller charged particles. The total charge was found to always be a multiple of a certain **elementary charge**, "e" :

$$q = ne, \qquad n = \pm 1, \pm 2, \pm 3, \ldots,$$

The value of this elementary charge is one of the fundamental constants of nature, and it is the magnitude of the charge of both the proton and the electron. The value of "e" is:

$$e = 1.602 \times 10^{-19}\ \text{C}.$$

Table 21-1

The Charges of Three Particles

Particle	Symbol	Charge
Electron	e or e^-	$-e$
Proton	p	$+e$
Neutron	n	0

Quarks have charge of ± ⅓ e or ± ⅔ e, but they never appear individually (color confinement).

Example, Mutual Electric Repulsion in a Nucleus:

The nucleus in an iron atom has a radius of about 4.0×10^{-15} m and contains 26 protons.

(a) What is the magnitude of the repulsive electrostatic force between two of the protons that are separated by 4.0×10^{-15} m?

KEY IDEA

The protons can be treated as charged particles, so the magnitude of the electrostatic force on one from the other is given by Coulomb's law.

Calculation: Table 21-1 tells us that the charge of a proton is $+e$. Thus, Eq. 21-4 gives us

$$F = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$= \frac{(8.99 \times 10^9\ \mathrm{N\cdot m^2/C^2})(1.602 \times 10^{-19}\ \mathrm{C})^2}{(4.0 \times 10^{-15}\ \mathrm{m})^2}$$

$$= 14\ \mathrm{N}. \qquad \text{(Answer)}$$

No explosion: This is a small force to be acting on a macroscopic object like a cantaloupe, but an enormous force to be acting on a proton. Such forces should explode the nucleus of any element but hydrogen (which has only one proton in its nucleus). However, they don't, not even in nuclei with a great many protons. Therefore, there must be some enormous attractive force to counter this enormous repulsive electrostatic force.

(b) What is the magnitude of the gravitational force between those same two protons?

KEY IDEA

Because the protons are particles, the magnitude of the gravitational force on one from the other is given by Newton's equation for the gravitational force (Eq. 21-2).

Calculation: With m_p ($= 1.67 \times 10^{-27}$ kg) representing the mass of a proton, Eq. 21-2 gives us

$$F = G\frac{m_p^2}{r^2}$$

$$= \frac{(6.67 \times 10^{-11}\ \mathrm{N\cdot m^2/kg^2})(1.67 \times 10^{-27}\ \mathrm{kg})^2}{(4.0 \times 10^{-15}\ \mathrm{m})^2}$$

$$= 1.2 \times 10^{-35}\ \mathrm{N}. \qquad \text{(Answer)}$$

❖Charge is Conserved

The hypothesis of **conservation of charge** has stood up under close examination, both for large-scale charged bodies and for atoms, nuclei, and elementary particles.

Example 1: ***Radioactive decay of nuclei,*** in which a nucleus transforms into (becomes) a different type of nucleus.
A uranium-238 nucleus (^{238}U) transforms into a thorium- 234 nucleus (^{234}Th) by emitting an *alpha particle.* An alpha particle has the same makeup as a helium-4 nucleus, it has the symbol ^{4}He. Here the net charge is 92➔90+2.

$$^{238}\mathrm{U} \rightarrow {}^{234}\mathrm{Th} + {}^{4}\mathrm{He},$$

Example 2: An electron e^- (charge *-e)* and its antiparticle, the positron e^+ (charge *+e),* undergo an annihilation process, transforming into two gamma rays (high-energy light):. Here the net charge is 0➔0.

$$e^- + e^+ \rightarrow \gamma + \gamma \qquad \text{(annihilation).}$$

Example 3: A gamma ray (in a certain environment) transforms into an electron and a positron. Here the net charge is again 0➔0.

$$\gamma \rightarrow e^- + e^+ \qquad \text{(pair production).}$$

Homework:

1 Figure 21-12 shows four situations in which five charged particles are evenly spaced along an axis. The charge values are indicated except for the central particle, which has the same charge in all four situations. Rank the situations according to the magnitude of the net electrostatic force on the central particle, greatest first.

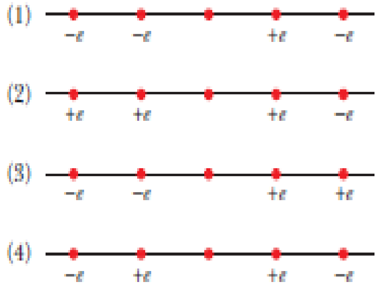


Fig. 21-12 Question 1.

2 Figure 21-13 shows three pairs of identical spheres that are to be touched together and then separated. The initial charges on them are indicated. Rank the pairs according to (a) the magnitude of the charge transferred during touching and (b) the charge left on the positively charged sphere, greatest first.

Fig. 21-13 Question 2.

3 Figure 21-14 shows four situations in which charged particles are fixed in place on an axis. In which situations is there a point to the left of the particles where an electron will be in equilibrium?

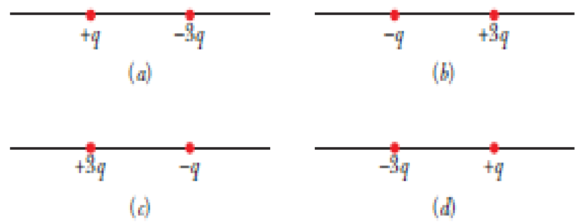


Fig. 21-14 Question 3.

4 Figure 21-15 shows two charged particles on an axis. The charges are free to move. However, a third charged particle can be placed at a certain point such that all three particles are then in equilibrium. (a) Is that point to the left of the first two particles, to their right, or between them? (b) Should the third particle be positively or negatively charged? (c) Is the equilibrium stable or unstable?

Fig. 21-15 Question 4.

5 In Fig. 21-25, particle 1 of charge $+1.0\ \mu C$ and particle 2 of charge $-3.0\ \mu C$ are held at separation $L = 10.0$ cm on an x axis. If particle 3 of unknown charge q_3 is to be located such that the net electrostatic force on it from particles 1 and 2 is zero, what must be the (a) x and (b) y coordinates of particle 3?

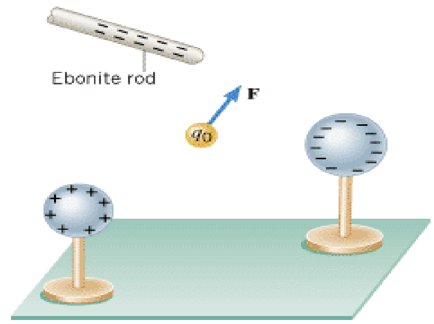
Fig. 21-25 Problems 13, 19, 30, 58, and 67.

CHAPTER 2

ELECTRIC FIELDS

1-The Electric Field

The electric field E that exists at a point is the electrostatic force F experienced by a small test charge q_0 placed at that point divided by the charge itself:



$$\vec{E} = \frac{\vec{F}}{q_0} \quad \text{(electric field)}.$$

- **The electric field is a vector, and its direction is the same as the direction of the force F on a positive test charge.**
- ***SI Unit of Electric Field:*** **Newton per coulomb (N/C).**

❖Important about electric field:

- **It is the surrounding charges that create an electric field at a given point.**
- **Any charge *q* placed at the point with the electric field *E* will experiences a force, *F=qE. For a* positive charge, the force points in the same direction as the electric**

field; for a negative charge, the force points in the opposite direction as the electric field.

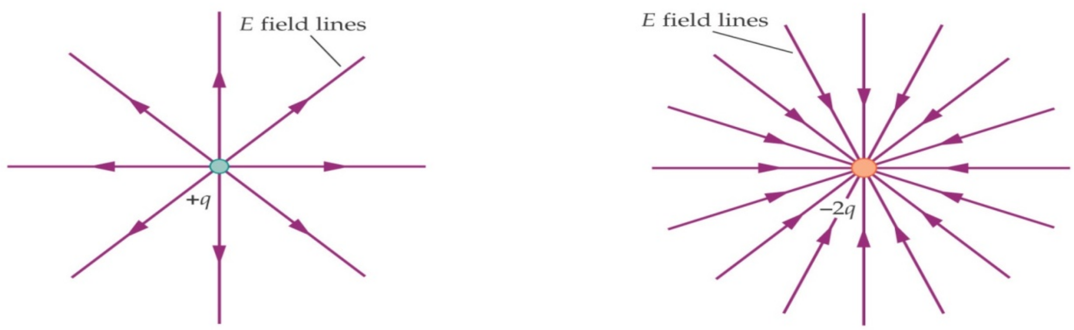
- At a particular point in space, each of the surrounding charges contributes to the net electric field that exists there.

2- The Electric Field Lines

The electric charges create an electric field in the space surrounding them. It is useful to have a kind of "map" that gives the direction and indicates the strength of the field at various places. This can be done by drawing the electric field lines.

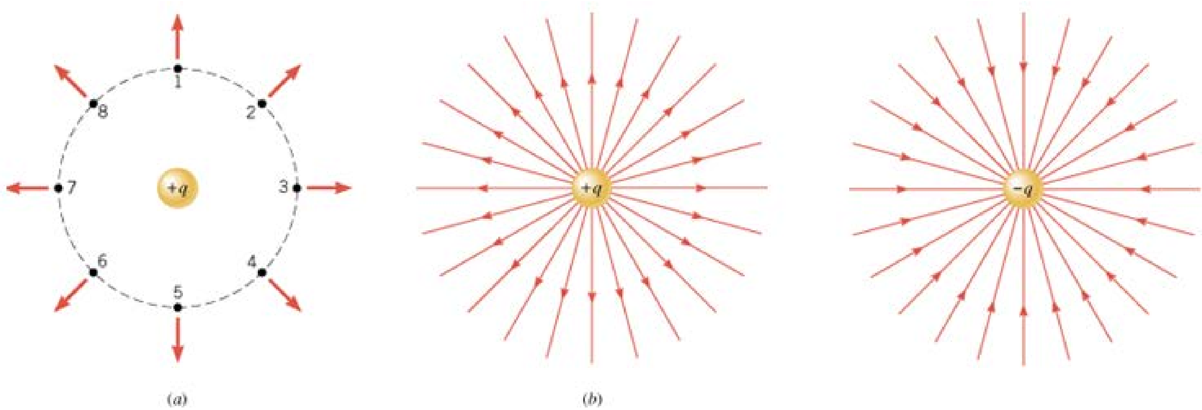
- ### The properties of the electric field lines

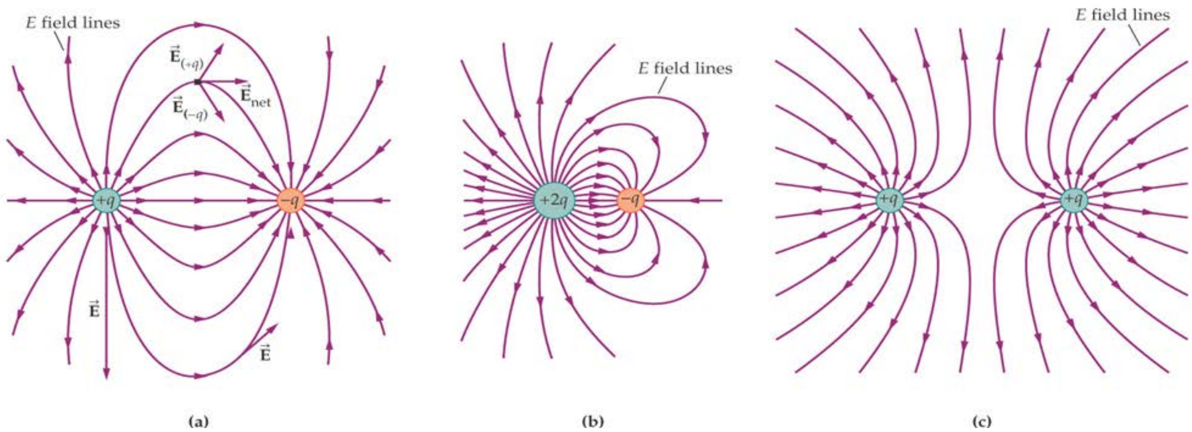
Electric field lines (lines of force) are continuous lines whose direction is everywhere that of the electric field



(a) (b)

- At any point, the tangent direction of the electric line is the direction of electric field.
- The density of the electric field lines provides information about the magnitude of the field. The lines are closer together where the electric field is stronger, the lines are closer together. The lines are more spread out where the electric field is weaker.
- The electric field lines always begin on a positive charge and end on a negative charge and do not start or stop in mid space.





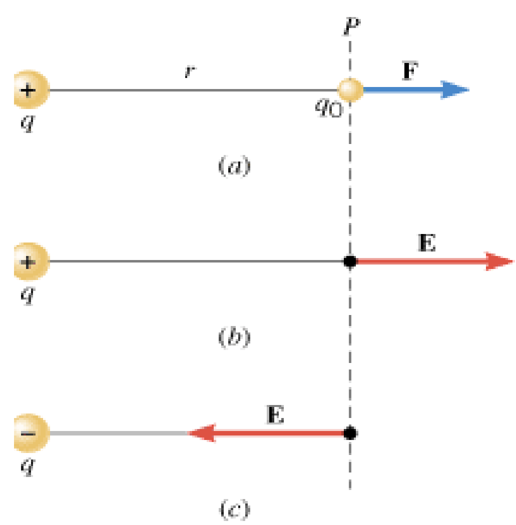
The electric charges create an electric field in the space surrounding them. It is useful to have a kind of "map" that gives the direction and indicates the strength of the field at various places. This can be done by drawing the electric field lines.

3-The Electric Field Due to a Point Charge

To find the electric field due to a point charge *q* (or charged particle) at any point a distance *r* from the point charge, we put a positive test charge q_o at that point. From Coulomb's law, the electrostatic force acting on q_o is:

$$\vec{\mathbf{F}} = \frac{1}{4\pi\varepsilon_0}\frac{Qq_\circ}{r^2}\hat{\mathbf{r}}$$

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \quad \text{(point charge)}.$$



(1) The magnitude of electric field by a point charge is given by:

Point charge q $$E = \frac{k|q|}{r^2}$$

(2) If *q* is positive, then *E* is directed away from *q*, as in Figure *b*. On the other hand, if *q* is negative, then *E* is directed toward *q*.

We can quickly find the net, or resultant, electric field due to more than one point charge. If we place a positive test charge q_0 near n point charges $q_1, q_2, \ldots, q_n$, then, from Eq. 21-7, the net force $\vec{F}_0$ from the n point charges acting on the test charge is

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \cdots + \vec{F}_{0n}.$$

Therefore, from Eq. 22-1, the net electric field at the position of the test charge is

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \cdots + \frac{\vec{F}_{0n}}{q_0}$$
$$= \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n.$$

Sample Problem

Net electric field due to three charged particles

Figure 22-7*a* shows three particles with charges $q_1 = +2Q$, $q_2 = -2Q$, and $q_3 = -4Q$, each a distance d from the origin. What net electric field $\vec{E}$ is produced at the origin?

KEY IDEA

Charges q_1, q_2, and q_3 produce electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, respectively, at the origin, and the net electric field is the vector sum $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$. To find this sum, we first must find the magnitudes and orientations of the three field vectors.

Magnitudes and directions: To find the magnitude of $\vec{E}_1$, which is due to q_1, we use Eq. 22-3, substituting d for r and $2Q$ for q and obtaining

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}.$$

Similarly, we find the magnitudes of $\vec{E}_2$ and $\vec{E}_3$ to be

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} \quad \text{and} \quad E_3 = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}.$$

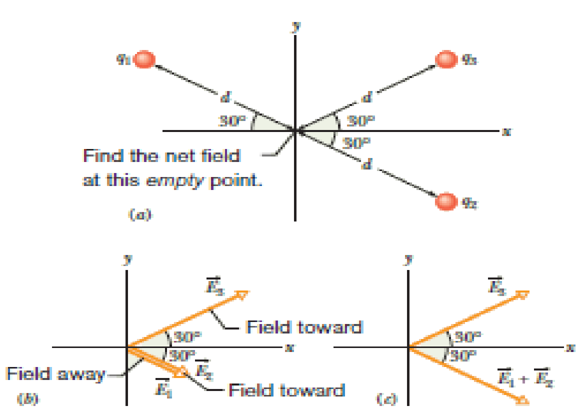


Fig. 22-7 (*a*) Three particles with charges q_1, q_2, and q_3 are at the same distance d from the origin. (*b*) The electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, at the origin due to the three particles. (*c*) The electric field vector $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$ at the origin.

We next must find the orientations of the three electric field vectors at the origin. Because q_1 is a positive charge, the field vector it produces points directly *away* from it, and because q_2 and q_3 are both negative, the field vectors they produce point directly *toward* each of them. Thus, the three electric fields produced at the origin by the three charged particles are oriented as in Fig. 22-7*b*. (*Caution:* Note that we have placed the tails of the vectors at the point where the fields are to be evaluated; doing so decreases the chance of error. Error becomes very probable if the tails of the field vectors are placed on the particles creating the fields.)

Adding the fields: We can now add the fields vectorially just as we added force vectors in Chapter 21. However, here we can use symmetry to simplify the procedure. From Fig. 22-7*b*, we see that electric fields $\vec{E}_1$ and $\vec{E}_2$ have the same direction. Hence, their vector sum has that direction and has the magnitude

$$E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} + \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2},$$

which happens to equal the magnitude of field $\vec{E}_3$.

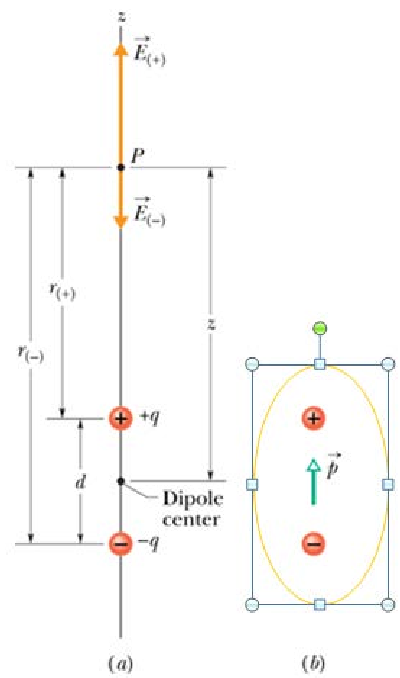
We must now combine two vectors, $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$, that have the same magnitude and that are oriented symmetrically about the x axis, as shown in Fig. 22-7*c*. From the symmetry of Fig. 22-7*c*, we realize that the equal y components of our two vectors cancel (one is upward and the other is downward) and the equal x components add (both are rightward). Thus, the net electric field $\vec{E}$ at the origin is in the positive direction of the x axis and has the magnitude

$$E = 2E_{3x} = 2E_3 \cos 30^\circ$$
$$= (2)\frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}(0.866) = \frac{6.93Q}{4\pi\varepsilon_0 d^2}. \qquad \text{(Answer)}$$

4-The Electric Field Due to an Electric Dipole

- **Two charged particles of magnitude *q* but of opposite sign, separated by a distance *d*. We call this configuration an *electric dipole.***
- **The product *qd*, which involves the two intrinsic properties *q* and *d* of the dipole, is the magnitude *p* of a vector quantity known as the electric dipole moment of the dipole. The direction of P is taken to be from the negative to the positive end of the dipole.**

The Electric Dipole



From symmetry, the electric field $\vec{E}$ at point P—and also the fields $\vec{E}_{(+)}$ and $\vec{E}_{(-)}$ due to the separate charges that make up the dipole—must lie along the dipole axis, which we have taken to be a z axis. Applying the superposition principle for electric fields, we find that the magnitude E of the electric field at P is

$$E = E_{(+)} - E_{(-)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2}$$

$$= \frac{q}{4\pi\varepsilon_0(z - \frac{1}{2}d)^2} - \frac{q}{4\pi\varepsilon_0(z + \frac{1}{2}d)^2}. \qquad (1)$$

After a little algebra, we can rewrite this equation as

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left(\frac{1}{\left(1 - \frac{d}{2z}\right)^2} - \frac{1}{\left(1 + \frac{d}{2z}\right)^2}\right). \qquad (2)$$

After forming a common denominator and multiplying its terms, we come to

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\frac{2d/z}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2} = \frac{q}{2\pi\varepsilon_0 z^3}\frac{d}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2}. \qquad (3)$$

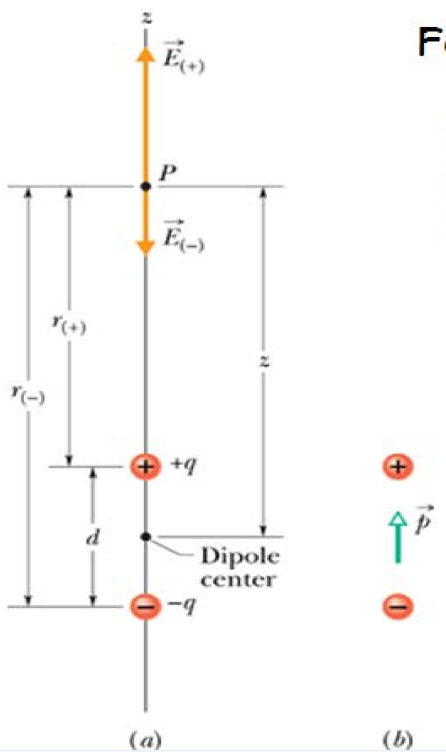
We are usually interested in the electrical effect of a dipole only at distances that are large compared with the dimensions of the dipole—that is, at distances such that $z \gg d$. At such large distances, we have $d/2z \ll 1$ in Eq (3). Thus, in our approximation, we can neglect the $d/2z$ term in the denominator, which leaves us with

$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}. \qquad (4)$$

The product qd, which involves the two intrinsic properties q and d of the dipole, is the magnitude p of a vector quantity known as the **electric dipole moment** $\vec{p}$ of the dipole. (The unit of $\vec{p}$ is the coulomb-meter.) Thus, we can write Eq. 22-8 as

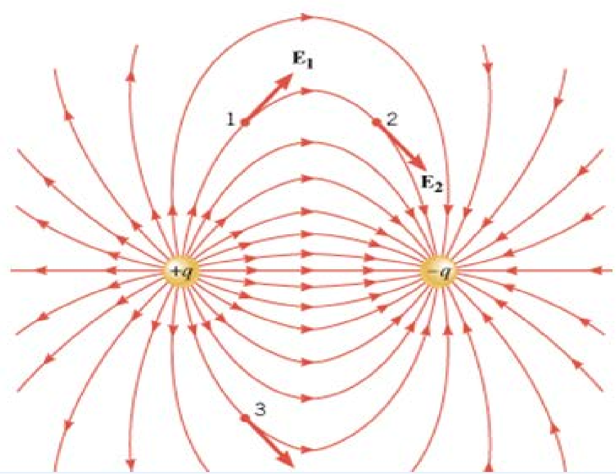
$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3} \quad \text{(electric dipole).} \tag{5}$$

The Electric Field Due to an Electric Dipole



For z>>d,

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3} \quad \text{(electric dipole)} .$$

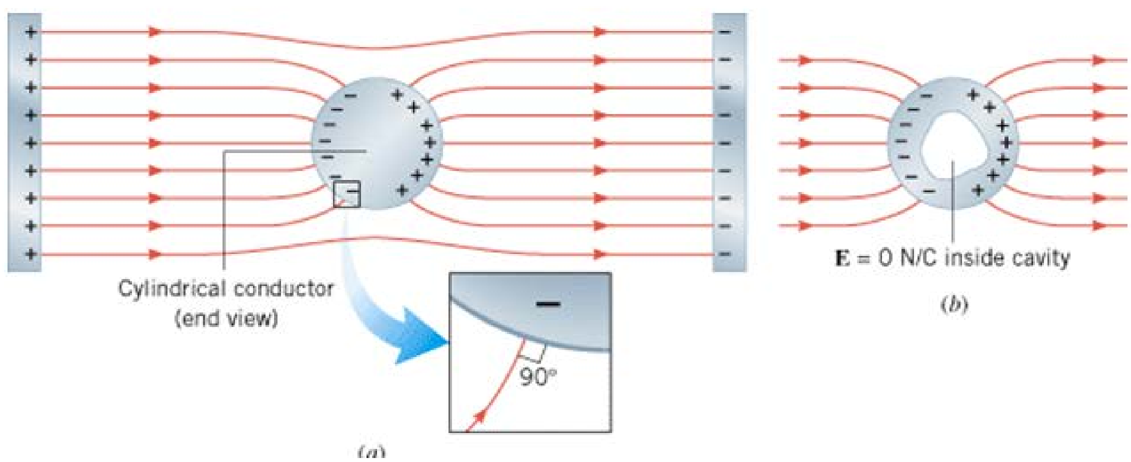


5-The Electric Field Inside a Conductor: Shielding

(1) At equilibrium under electrostatic conditions, any excess charge resides on the surface of a conductor.

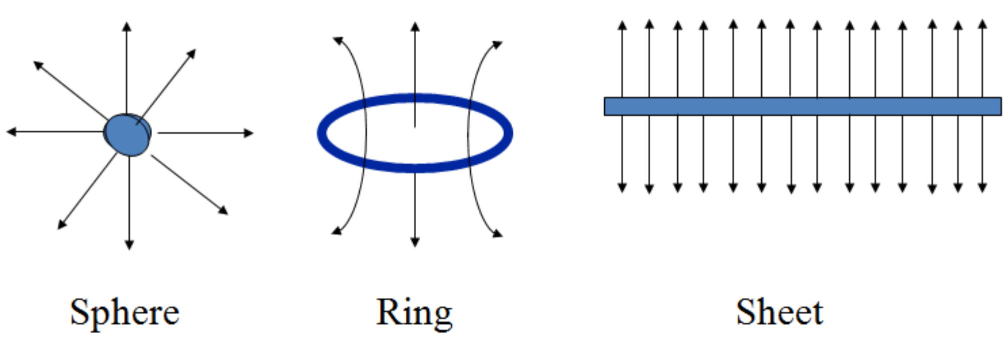
(2) At equilibrium under electrostatic conditions, the electric field is zero at any point within a conducting material.

(3) The electric field just outside the surface of a conductor is perpendicular to the surface at equilibrium under electrostatic conditions.



5-The Electric Field Due to a Line of Charge

Up to now we have only considered the electric field of point charges. Now let's look at continuous distributions of charge lines - surfaces - volumes of charge and determine the resulting electric fields.



Name	Symbol	SI Unit
Charge	***q***	**C**
Linear charge density	**λ**	**C/m**
Surface charge density	**σ**	**C/m^2**
Volume charge density	**ρ**	**C/m^3**

❖ Example: Electric field of a ring of uniform positive charge

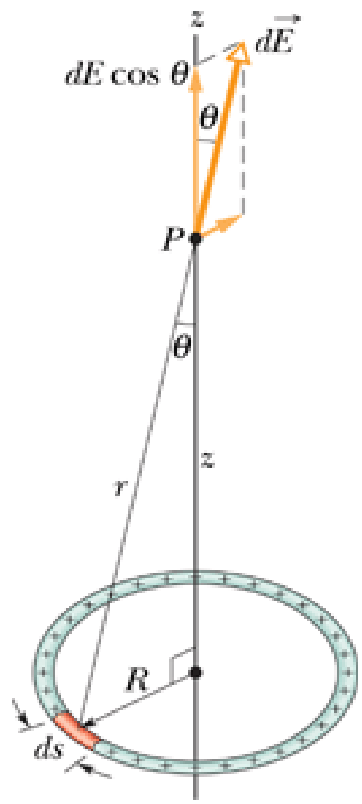


Figure 1 shows a thin ring of radius *R* with a uniform positive linear charge density λ around its circumference. We may imagine the ring to be made of plastic or some other insulator, so that the charges can be regarded as fixed in place. What is the electric field $\vec{E}$ point *P*, a distance *z* from the plane of the ring along its central axis?

If:

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,\hat{\mathrm{r}} \qquad \text{(point charge).} \qquad (1)$$

To answer, we cannot just apply Eq. (1) , which gives the electric field set up by a point charge, because the ring is obviously not a point charge. However, we can mentally divide the ring into differential elements of charge that are so small that they are like point charges, and then we can apply Eq. (1) to each of them. Next, we can add the electric fields set up at P by all the differential elements. The vector sum of the fields gives us the field set up at P by the ring.

Let ds be the (arc) length of any differential element of the ring. Since λ is the charge per unit (arc) length, the element has a charge of magnitude

$$dq = \lambda\, ds. \tag{2}$$

This differential charge sets up a differential electric field $d\vec{E}$ at point P, which is a distance r from the element. Treating the element as a point charge and using Eq. (2) , we can rewrite Eq. (1) to express the magnitude of $d\vec{E}$ as

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}. \tag{3}$$

From Fig. 1 , we can rewrite Eq. (3) as

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{(z^2 + R^2)}. \tag{4}$$

Figure 1 shows that $d\vec{E}$ is at angle θ to the central axis (which we have taken to be a z axis) and has components perpendicular to and parallel to that axis.

Every charge element in the ring sets up a differential field $d\vec{E}$ at P, with magnitude given by Eq. 4 . All the $d\vec{E}$ vectors have identical components parallel to the central axis, in both magnitude and direction. All these $d\vec{E}$ vectors have components perpendicular to the central axis as well; these perpendicular components are identical in magnitude but point in different directions. In fact, for any perpendicular component that points in a given direction, there is another one that points in the opposite direction. The sum of this pair of components, like the sum of all other pairs of oppositely directed components, is zero.

Thus, the perpendicular components cancel and we need not consider them further. This leaves the parallel components; they all have the same direction, so the net electric field at P is their sum.

The parallel component of $d\vec{E}$ shown in Fig. 2 has magnitude $dE \cos\theta$. The figure also shows us that

$$\cos\theta = \frac{z}{r} = \frac{z}{(z^2 + R^2)^{1/2}}. \tag{5}$$

Then multiplying Eq. 4 by Eq. 5 gives us, for the parallel component of $d\vec{E}$,

$$dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\, ds. \tag{6}$$

To add the parallel components $dE \cos\theta$ produced by all the elements, we integrate Eq. (6) around the circumference of the ring, from $s = 0$ to $s = 2\pi R$. Since the only quantity in Eq. (6) that varies during the integration is s, the other quantities can be moved outside the integral sign. The integration then gives us

$$E = \int dE \cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} \int_0^{2\pi R} ds$$

$$= \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}. \tag{7}$$

Since λ is the charge per length of the ring, the term $\lambda(2\pi R)$ in Eq. (7) is q, the total charge on the ring. We then can rewrite Eq. (7) as

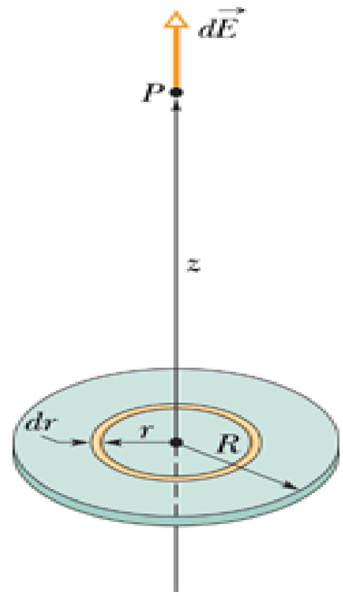
$$E = \frac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} \quad \text{(charged ring).} \tag{8}$$

If the charge on the ring is negative, instead of positive as we have assumed, the magnitude of the field at P is still given by Eq. (8). However, the electric field vector then points toward the ring instead of away from it.

Let us check Eq. (8) for a point on the central axis that is so far away that $z \gg R$. For such a point, the expression $z^2 + R^2$ in Eq. (8) can be approximated as z^2, and Eq. 22-16 becomes

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2} \quad \text{(charged ring at large distance).} \tag{9}$$

2- The Electric Field Due to a Charged Disk



$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right) \quad \text{(charged disk)}$$

For infinite sheet, R→∞,

$$E = \frac{\sigma}{2\varepsilon_0} \quad \text{(infinite sheet)} .$$

Sample Problem: A Point Charge in an Electric Field

Figure shows the deflecting plates of an ink-jet printer, with superimposed coordinate axes. An ink drop with a mass *m* of $1.3\text{x}10^{-10}$ kg and a negative charge of magnitude $Q=1.5\text{x}10^{-13}$ C enters the region between the plates, initially moving along the *x* axis with speed $v_x=18$ m/s. The length *L* of each plate is 1.6 cm. The plates are charged and thus produce an electric field at all points between them. Assume that field E is downward directed, is uniform, and has a magnitude of $1.4\text{x}10^6$ N/C. What is the vertical deflection of the drop at the far edge of the plates? (The gravitational force on the drop is small relative to the electrostatic force acting on the drop and can be neglected.)

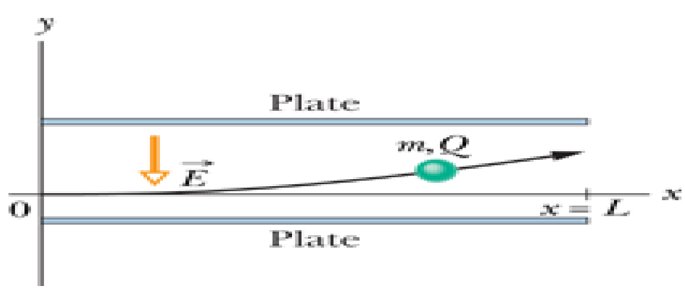


Fig. 1 An ink drop of mass m and charge magnitude Q is deflected in the electric field of an ink-jet printer.

Calculations: **Applying Newton's second law (*F*= *ma*) for components along the *y* axis, we find that**

$$a_y = \frac{F}{m} = \frac{QE}{m}. \qquad (1)$$

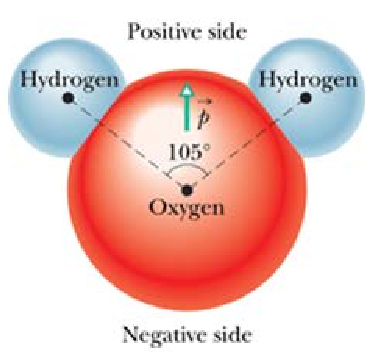
Let t represent the time required for the drop to pass through the region between the plates. During t the vertical and horizontal displacements of the drop are

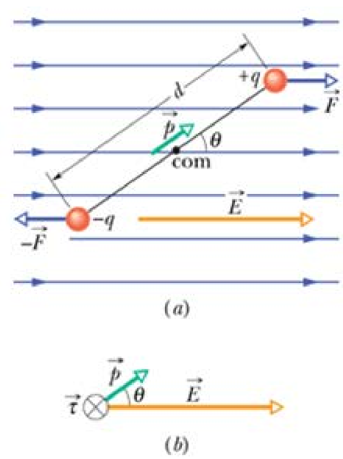
$$y = \tfrac{1}{2} a_y t^2 \quad \text{and} \quad L = v_x t, \qquad (2)$$

respectively. Eliminating t between these two equations and substituting Eq. (1) for a_y, we find

$$\begin{aligned} y &= \frac{QEL^2}{2mv_x^2} \\ &= \frac{(1.5 \times 10^{-13}\ \text{C})(1.4 \times 10^6\ \text{N/C})(1.6 \times 10^{-2}\ \text{m})^2}{(2)(1.3 \times 10^{-10}\ \text{kg})(18\ \text{m/s})^2} \\ &= 6.4 \times 10^{-4}\ \text{m} \\ &= 0.64\ \text{mm.} \end{aligned} \qquad \text{(Answer)}$$

A Dipole in an Electric Field





but we have defined : **p = q d**
and the direction of **p** is from -q to +q

Then, the **torque** can be written as:

$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$ $\qquad$ $\boldsymbol{\tau} = \mathbf{p\ E\ \sin\theta}$

with an associated **potential en**

U = - p.E $\qquad$ $U = -pE\cos\theta.$

When a dipole rotates from an initial orientation θ_i to another orientation θ_f, the work W done on the dipole by the electric field is

$$W = -\Delta U = -(U_f - U_i),$$

Sample Problem

Torque and energy of an electric dipole in an electric field

A neutral water molecule (H_2O) in its vapor state has an electric dipole moment of magnitude 6.2×10^{-30} C·m.

(a) How far apart are the molecule's centers of positive and negative charge?

KEY IDEA

A molecule's dipole moment depends on the magnitude q of the molecule's positive or negative charge and the charge separation d.

Calculations: There are 10 electrons and 10 protons in a neutral water molecule; so the magnitude of its dipole moment is

$$p = qd = (10e)(d),$$

in which d is the separation we are seeking and e is the elementary charge. Thus,

$$d = \frac{p}{10e} = \frac{6.2 \times 10^{-30}\ \text{C}\cdot\text{m}}{(10)(1.60 \times 10^{-19}\ \text{C})}$$
$$= 3.9 \times 10^{-12}\ \text{m} = 3.9\ \text{pm}. \qquad \text{(Answer)}$$

This distance is not only small, but it is also actually smaller than the radius of a hydrogen atom.

(b) If the molecule is placed in an electric field of 1.5×10^4 N/C, what maximum torque can the field exert on it? (Such a field can easily be set up in the laboratory.)

KEY IDEA

The torque on a dipole is maximum when the angle θ between $\vec{p}$ and $\vec{E}$ is 90°.

Calculation: Substituting $\theta = 90°$

$$\tau = pE\sin\theta$$
$$= (6.2 \times 10^{-30}\ \text{C}\cdot\text{m})(1.5 \times 10^4\ \text{N/C})(\sin 90°)$$
$$= 9.3 \times 10^{-26}\ \text{N}\cdot\text{m}. \qquad \text{(Answer)}$$

(c) How much work must an *external agent* do to rotate this molecule by 180° in this field, starting from its fully aligned position, for which $\theta = 0$?

KEY IDEA

The work done by an external agent (by means of a torque applied to the molecule) is equal to the change in the molecule's potential energy due to the change in orientation.

Calculation: From Eq. 22-40, we find

$$W_a = U_{180°} - U_0$$
$$= (-pE\cos 180°) - (-pE\cos 0)$$
$$= 2pE = (2)(6.2 \times 10^{-30}\ \text{C}\cdot\text{m})(1.5 \times 10^4\ \text{N/C})$$
$$= 1.9 \times 10^{-25}\ \text{J}. \qquad \text{(Answer)}$$

QUESTIONS

1 Figure (1) shows three arrangements of electric field lines. In each arrangement, a proton is released from rest at point A and is then accelerated through point B by the electric field. Points A and B have equal separations in the three arrangements. Rank the arrangements according to the linear momentum of the proton at point B, greatest first.

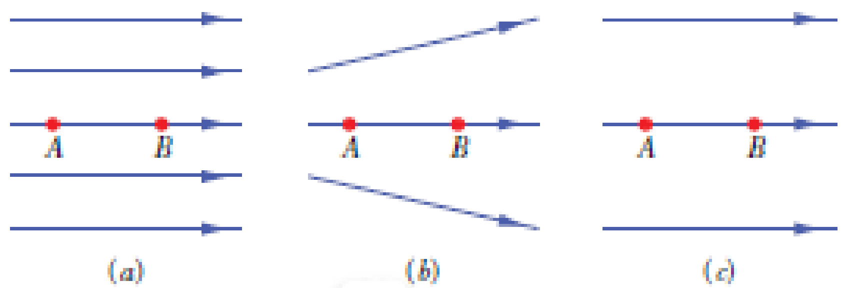


Fig. (1)

2 In Fig. 2 two particles of charge $-q$ are arranged symmetrically about the y axis; each produces an electric field at point P on that axis. (a) Are the magnitudes of the fields at P equal? (b) Is each electric field directed toward or away from the charge producing it? (c) Is the magnitude of the net electric field at P equal to the sum of the magnitudes E of the two field vectors (is it equal to $2E$)? (d) Do the x components of those two field vectors add or cancel? (e) Do their y components add or cancel? (f) Is the direction of the net field at P that of the canceling components or the adding components? (g) What is the direction of the net field?

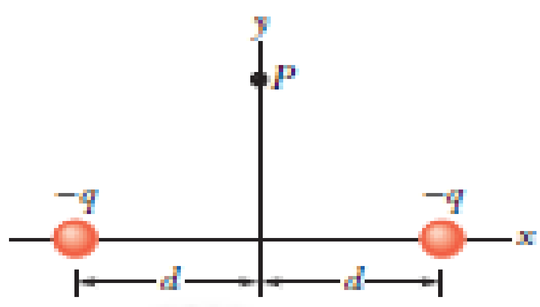


Fig. (1)

3 In Fig. 3 the electric field lines on the left have twice the separation of those on the right. (a) If the magnitude of the field at A is 40 N/C, what is the magnitude of the force on a proton at A? (b) What is the magnitude of the field at B?

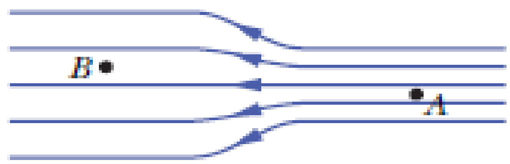


Fig. (3)

•**4** Two particles are attached to an x axis: particle 1 of charge -2.00×10^{-7} C at $x = 6.00$ cm, particle 2 of charge $+2.00 \times 10^{-7}$ C at $x = 21.0$ cm. Midway between the particles, what is their net electric field in unit-vector notation?

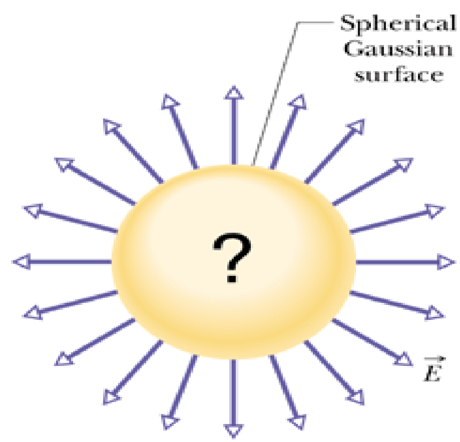
•**5** SSM What is the magnitude of a point charge whose electric field 50 cm away has the magnitude 2.0 N/C?

CHAPTER 3

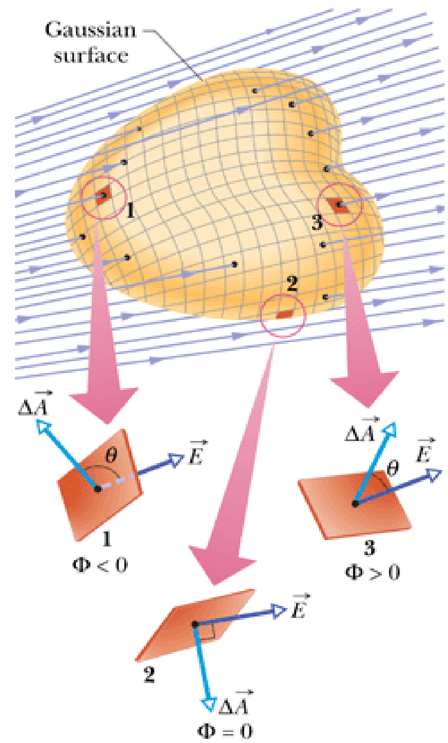
GAUS S ' L AW

1- Gaussian surface

- **Gaussian surface is a hypothetical (any imaginary shape) closed surface enclosing the charge distribution.**
- **Gauss' law relates the electric fields at points on a (closed) Gaussian surface to the net charge enclosed by that surface.**



Let us divide the surface into small squares of area ΔA, each square being small enough to permit us to neglect any curvature and to consider the individual square to be flat. We represent each such element of area with an area vector $\Delta \vec{A}$

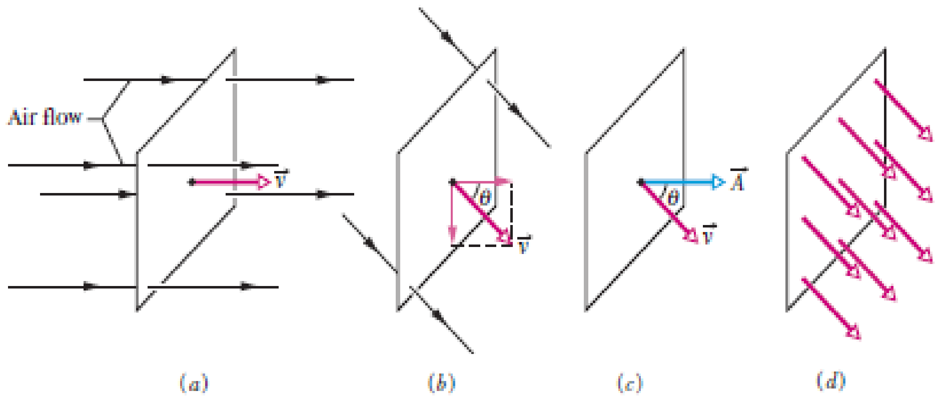


- **Magnitude is the area ΔA.**

- **Direction is perpendicular to the Gaussian surface and directed away from the interior of the surface.**

2- Flux of an Electric Field

Fig. 23-2 (*a*) A uniform airstream of velocity $\vec{v}$ is perpendicular to the plane of a square loop of area A. (*b*) The component of $\vec{v}$ perpendicular to the plane of the loop is $v\cos\theta$, where θ is the angle between $\vec{v}$ and a normal to the plane. (*c*) The area vector $\vec{A}$ is perpendicular to the plane of the loop and makes an angle θ with $\vec{v}$. (*d*) The velocity field intercepted by the area of the loop.



The rate of volume flow through the loop is :

$$\Phi = vA\cos\theta = \vec{v}\cdot\vec{A},$$

$$\Phi = (v\cos\theta)A.$$

- **The electric field for a surface is:**

$$\Phi = \sum \overrightarrow{E} \cdot \Delta\overrightarrow{A} .$$

- **The electric field for a gaussian surface is:**

$$\Phi = \oint \overrightarrow{E} \cdot d\overrightarrow{A} \qquad \text{(electric flux through a Gaussian surface).}$$

The electric flux Φ through a Gaussian surface is proportional to the net number of electric field lines passing through that surface.

SI Unit of Electric Flux: N·m^2/C

$$\Phi = \oint \vec{E} \cdot d\vec{A} \quad \text{(electric flux through a Gaussian surface).}$$

Flux through a closed cylinder, uniform field

Figure 23-4 shows a Gaussian surface in the form of a cylinder of radius R immersed in a uniform electric field $\vec{E}$, with the cylinder axis parallel to the field. What is the flux Φ of the electric field through this closed surface?

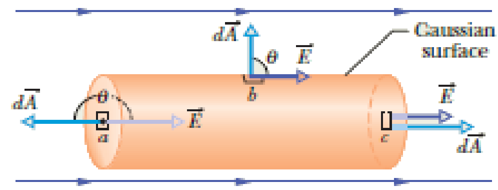


Fig. 23-4 A cylindrical Gaussian surface, closed by end caps, is immersed in a uniform electric field. The cylinder axis is parallel to the field direction.

KEY IDEA

We can find the flux Φ through the Gaussian surface by integrating the scalar product $\vec{E}\cdot d\vec{A}$ over that surface.

Calculations: We can do the integration by writing the flux as the sum of three terms: integrals over the left cylinder cap a, the cylindrical surface b, and the right cap c. Thus, from Eq. 23-4,

$$\Phi = \oint \vec{E}\cdot d\vec{A}$$

$$= \int_a \vec{E}\cdot d\vec{A} + \int_b \vec{E}\cdot d\vec{A} + \int_c \vec{E}\cdot d\vec{A}. \qquad (23\text{-}5)$$

For all points on the left cap, the angle θ between $\vec{E}$ and $d\vec{A}$ is 180° and the magnitude E of the field is uniform. Thus,

$$\int_a \vec{E}\cdot d\vec{A} = \int E(\cos 180°)\, dA = -E\int dA = -EA,$$

where $\int dA$ gives the cap's area $A\ (= \pi R^2)$. Similarly, for the right cap, where $\theta = 0$ for all points,

$$\int_c \vec{E}\cdot d\vec{A} = \int E(\cos 0)\, dA = EA.$$

Finally, for the cylindrical surface, where the angle θ is 90° at all points,

$$\int_b \vec{E}\cdot d\vec{A} = \int E(\cos 90°)\, dA = 0.$$

Substituting these results into Eq. 23-5 leads us to

$$\Phi = -EA + 0 + EA = 0. \qquad \text{(Answer)}$$

The net flux is zero because the field lines that represent the electric field all pass entirely through the Gaussian surface, from the left to the right.

Sample Problem

Flux through a closed cube, nonuniform field

A *nonuniform* electric field given by $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$ pierces the Gaussian cube shown in Fig. 23-5*a*. (E is in newtons per coulomb and x is in meters.) What is the electric flux through the right face, the left face, and the top face? (We consider the other faces in another sample problem.)

KEY IDEA

We can find the flux Φ through the surface by integrating the scalar product $\vec{E}\cdot d\vec{A}$ over each face.

Right face: An area vector $\vec{A}$ is always perpendicular to its surface and always points away from the interior of a Gaussian surface. Thus, the vector $d\vec{A}$ for any area element (small section) on the right face of the cube must point in the positive direction of the x axis. An example of such an element is shown in Figs. 23-5*b* and *c*, but we would have an identical vector for any other choice of an area element on that face. The most convenient way to express the vector is in unit-vector notation,

$$d\vec{A} = dA\hat{i}.$$

From Eq. 23-4, the flux Φ_r through the right face is then

$$\Phi_r = \int \vec{E}\cdot d\vec{A} = \int (3.0x\hat{i} + 4.0\hat{j})\cdot(dA\hat{i})$$

$$= \int [(3.0x)(dA)\hat{i}\cdot\hat{i} + (4.0)(dA)\hat{j}\cdot\hat{i}]$$

$$= \int (3.0x\, dA + 0) = 3.0\int x\, dA.$$

We are about to integrate over the right face, but we note that x has the same value everywhere on that face—namely, $x = 3.0$ m. This means we can substitute that constant value for x. This can be a confusing argument. Although x is certainly a variable as we move left to right across the figure, because the right face is perpendicular to the x axis, every point on the face has the same x coordinate. (The y and z coordinates do not matter in our integral.) Thus, we have

$$\Phi_r = 3.0\int (3.0)\, dA = 9.0\int dA.$$

The integral $\int dA$ merely gives us the area $A = 4.0\ \text{m}^2$ of the right face; so

$$\Phi_r = (9.0\ \text{N/C})(4.0\ \text{m}^2) = 36\ \text{N}\cdot\text{m}^2/\text{C}. \qquad \text{(Answer)}$$

Left face: The procedure for finding the flux through the left face is the same as that for the right face. However, two factors change. (1) The differential area vector $d\vec{A}$ points in the negative direction of the x axis, and thus $d\vec{A} = -dA\hat{i}$ (Fig. 23-5*d*). (2) The term x again appears in our integration, and it is again constant over the face being considered. However, on the left face, $x = 1.0$ m. With these two changes, we find that the flux Φ_l through the left face is

$$\Phi_l = -12\ \text{N}\cdot\text{m}^2/\text{C}. \qquad \text{(Answer)}$$

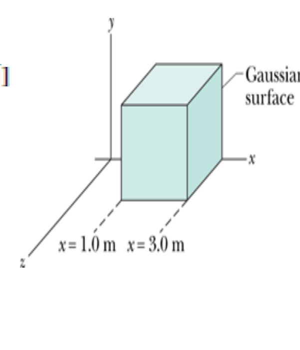
Top face: The differential area vector $d\vec{A}$ points in the positive direction of the y axis, and thus $d\vec{A} = dA\hat{j}$ (Fig. 23-5*e*). The flux Φ_t through the top face is then

$$\Phi_t = \int (3.0x\hat{i} + 4.0\hat{j})\cdot(dA\hat{j})$$

$$= \int [(3.0x)(dA)\hat{i}\cdot\hat{j} + (4.0)(dA)\hat{j}\cdot\hat{j}]$$

$$= \int (0 + 4.0\, dA) = 4.0\int dA$$

$$= 16\ \text{N}\cdot\text{m}^2/\text{C}.$$



3- Gauss' Law

Gauss' law relates the net flux Φ of an electric field through a closed surface (a Gaussian surface) to the *net* charge q_{enc} that is *enclosed* by that surface. It tells us that:

$$\varepsilon_0 \Phi = q_{enc} \qquad \text{(Gauss' law).}$$

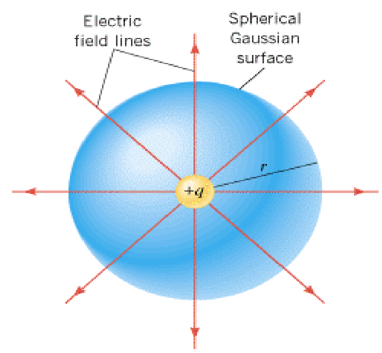
From Eq. 1

$$\Phi = \oint \vec{E} \cdot d\vec{A} \qquad \text{(electric flux through a Gaussian surface).} \qquad \ldots\ldots\ldots (1)$$

Then, we can also write Gauss' law as:

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc} \qquad \text{(Gauss' law).}$$

For a point charge:



$$E = k\frac{q}{r^2}, k = \frac{1}{4\pi\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{q}{(4\pi r^2)\varepsilon_0} = \frac{q}{A\varepsilon_0}$$

Gauss' law for a point charge

$$\underbrace{EA}_{\text{Electric flux, } \Phi_E} = \frac{q}{\epsilon_0}$$

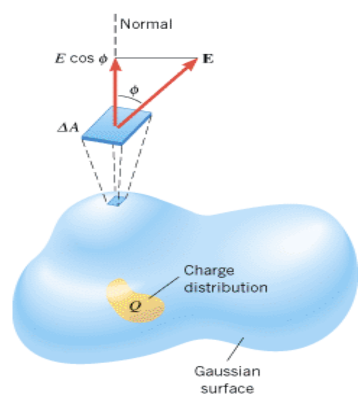
For charge distribution Q:

The electric flux through a Gaussian surface times by ε_0 (the permittivity of free space) is equal to the net charge *Q* enclosed :

$$\varepsilon_0 \Phi = q_{enc} \qquad \text{(Gauss' law)},$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc} \qquad \text{(Gauss' law)}.$$

- **The net charge q_{enc} is the algebraic sum of all the *enclosed* charges.**
- **Charge outside the surface, no matter how large or how close it may be, is not included in the term q_{enc}.**



Sample Problem

Relating the net enclosed charge and the net flux

Figure 23-7 shows five charged lumps of plastic and an electrically neutral coin. The cross section of a Gaussian surface S is indicated. What is the net electric flux through the surface if $q_1 = q_4 = +3.1$ nC, $q_2 = q_5 = -5.9$ nC, and $q_3 = -3.1$ nC?

KEY IDEA

The *net* flux Φ through the surface depends on the *net* charge q_{enc} enclosed by surface S.

Calculation: The coin does not contribute to Φ because it is neutral and thus contains equal amounts of positive and negative charge. We could include those equal amounts, but they would simply sum to be zero when we calculate the *net* charge enclosed by the surface. So, let's not bother. Charges q_4 and q_5 do not contribute because they are outside surface S. They certainly send electric field lines through the surface, but as much enters as leaves and no net flux is contributed. Thus, q_{enc} is only the sum $q_1 + q_2 + q_3$ and Eq. 23-6 gives us

$$\Phi = \frac{q_{enc}}{\varepsilon_0} = \frac{q_1 + q_2 + q_3}{\varepsilon_0}$$

$$= \frac{+3.1 \times 10^{-9}\,\text{C} - 5.9 \times 10^{-9}\,\text{C} - 3.1 \times 10^{-9}\,\text{C}}{8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2}$$

$$= -670\ \text{N}\cdot\text{m}^2/\text{C}. \qquad \text{(Answer)}$$

The minus sign shows that the net flux through the surface is inward and thus that the net charge within the surface is negative.

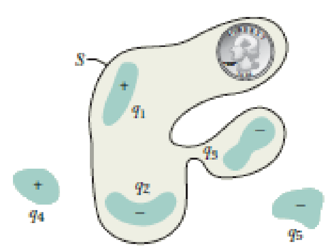
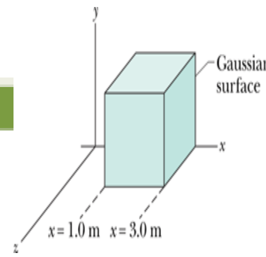


Fig. 23-7 Five plastic objects, each with an electric charge, and a coin, which has no net charge. A Gaussian surface, shown in cross section, encloses three of the plastic objects and the coin.

Sample Problem

Enclosed charge in a nonuniform field



What is the net charge enclosed by the Gaussian cube of Fig. 23-5, which lies in the electric field $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$? ($E$ is in newtons per coulomb and x is in meters.)

KEY IDEA

The net charge enclosed by a (real or mathematical) closed surface is related to the total electric flux through the surface by Gauss' law as given by Eq. 23-6 ($\varepsilon_0\Phi = q_{enc}$).

Flux: To use Eq. 23-6, we need to know the flux through all six faces of the cube. We already know the flux through the right face ($\Phi_r = 36\ \mathrm{N\cdot m^2/C}$), the left face ($\Phi_l = -12\ \mathrm{N\cdot m^2/C}$), and the top face ($\Phi_t = 16\ \mathrm{N\cdot m^2/C}$).

For the bottom face, our calculation is just like that for the top face *except* that the differential area vector $d\vec{A}$ is now directed downward along the y axis (recall, it must be *outward* from the Gaussian enclosure). Thus, we have $d\vec{A} = -dA\hat{j}$, and we find

$$\Phi_b = -16\ \mathrm{N\cdot m^2/C}.$$

For the front face we have $d\vec{A} = dA\hat{k}$, and for the back face, $d\vec{A} = -dA\hat{k}$. When we take the dot product of the given electric field $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$ with either of these expressions for $d\vec{A}$, we get 0 and thus there is no flux through those faces. We can now find the total flux through the six sides of the cube:

$$\Phi = (36 - 12 + 16 - 16 + 0 + 0)\ \mathrm{N\cdot m^2/C} = 24\ \mathrm{N\cdot m^2/C}.$$

Enclosed charge: Next, we use Gauss' law to find the charge q_{enc} enclosed by the cube:

$$q_{enc} = \varepsilon_0\Phi = (8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2})(24\ \mathrm{N\cdot m^2/C}) = 2.1 \times 10^{-10}\ \mathrm{C}. \quad \text{(Answer)}$$

Thus, the cube encloses a *net* positive charge.

4- A Charged Isolated Conductor

Gauss' law permits us to prove an important theorem about conductors:

If an excess charge is placed on an isolated conductor that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.

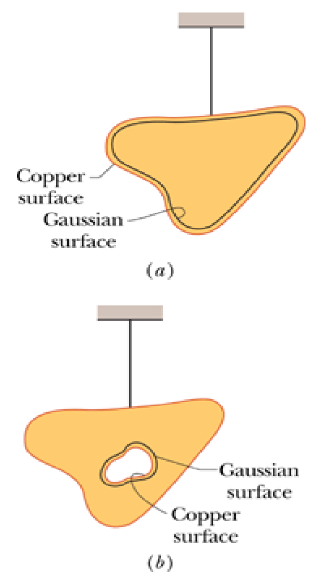


Figure 23-9*a* shows, in cross section, an isolated lump of copper hanging from an insulating thread and having an

excess charge q. We place a Gaussian surface just inside the actual surface of the conductor.

If E is zero everywhere inside our copper conductor, it must be zero for all points on the Gaussian surface because that surface, though close to the surface of the conductor, is definitely inside the conductor. This means that the flux through the Gaussian surface must be zero. Gauss' law then tells us that the net charge inside the Gaussian surface must also be zero.

Also, for an Isolated Conductor with a Cavity as shown in fig b, there is no net charge on the cavity walls; all the excess charge remains on the outer surface of the conductor.

5- The External Electric Field of a Conductor

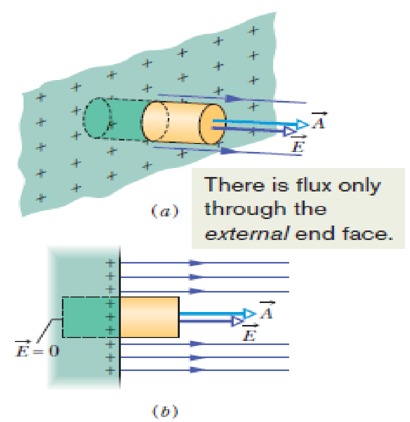


Fig. 23-10 (*a*) Perspective view and (*b*) side view of a tiny portion of a large, isolated conductor with excess positive charge on its surface. A (closed) cylindrical Gaussian surface, embedded perpendicularly in the conductor, encloses some of the charge. Electric field lines pierce the external end cap of the cylinder, but not the internal end cap. The external end cap has area A and area vector $\vec{A}$.

If σ is the charge per unit area,

According to Gauss' law

$$\varepsilon_0 EA = \sigma A,$$

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{(conducting surface).}$$

7-Applying Gauss' Law: Cylindrical Symmetry

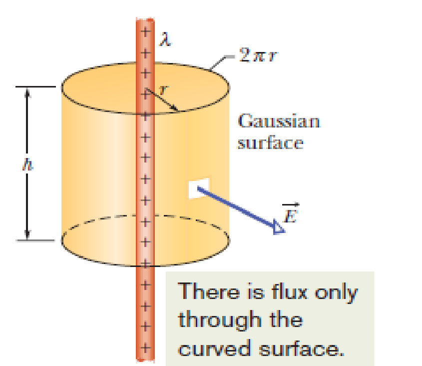


Fig. 23-12 A Gaussian surface in the form of a closed cylinder surrounds a section of a very long, uniformly charged, cylindrical plastic rod.

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \quad \text{(line of charge).}$$

8-Applying Gauss' Law: Planar Symmetry

Nonconducting Sheet

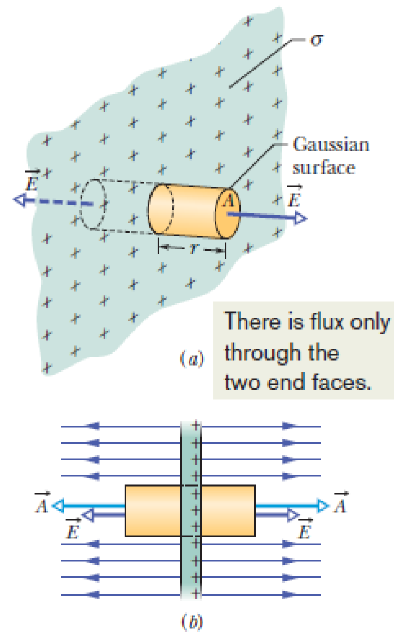


Fig. 23-15 (*a*) Perspective view and (*b*) side view of a portion of a very large, thin plastic sheet, uniformly charged on one side to surface charge density σ. A closed cylindrical Gaussian surface passes through the sheet and is perpendicular to it.

$$E = \frac{\sigma}{2\varepsilon_0} \quad \text{(sheet of charge).}$$

9-Two Conducting Plates

Figure 23-16*b* shows an identical plate with excess negative charge having the same magnitude of surface charge density σ_1. The only difference is that now the electric field is directed toward the plate.

Suppose we arrange for the plates of Figs. 23-16*a* and *b* to be close to each other and parallel (Fig. 23-16*c*). Since the plates are conductors, when we bring them into this arrangement, the excess charge on one plate attracts the excess charge on the other plate, and all the excess charge moves onto the inner faces of the plates as in Fig. 23-16*c*. With twice as much charge now on each inner face, the new surface charge density (call it σ) on each inner face is twice σ_1. Thus, the electric field at any point between the plates has the magnitude

$$E = \frac{2\sigma_1}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}.$$

This field is directed away from the positively charged plate and toward the negatively charged plate. Since no excess charge is left on the outer faces, the electric field to the left and right of the plates is zero.

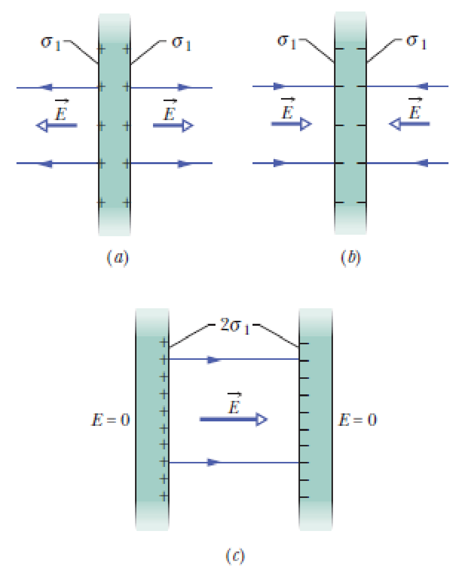
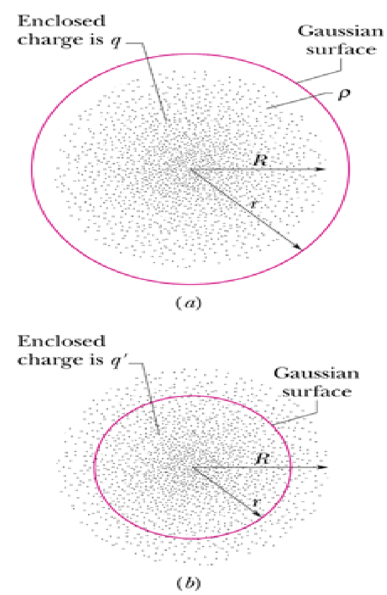


Fig. 23-16 (*a*) A thin, very large conducting plate with excess positive charge. (*b*) An identical plate with excess negative charge. (*c*) The two plates arranged so they are parallel and close.

10- Applying Gauss' Law: Spherical Symmetry

- **A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at the center of the shell.**
- **If a charged particle is located inside a shell of uniform charge, there is no electrostatic force on the particle from the shell.**

Any spherically symmetric charge distribution with the volume charge density *ρ*



- **For r>R, the charge produces an electric field on the Gaussian surface as if the charge were a point charge located at the center,**

For r<R, the electric field is

$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r \qquad \text{(uniform charge, field at } r \leq R\text{)} .$$

Sample Problem

Electric field near two parallel charged metal plates

Figure 23-17*a* shows portions of two large, parallel, nonconducting sheets, each with a fixed uniform charge on one side. The magnitudes of the surface charge densities are $\sigma_{(+)} = 6.8\ \mu\text{C/m}^2$ for the positively charged sheet and $\sigma_{(-)} = 4.3\ \mu\text{C/m}^2$ for the negatively charged sheet.

Find the electric field $\vec{E}$ (a) to the left of the sheets, (b) between the sheets, and (c) to the right of the sheets.

KEY IDEA

With the charges fixed in place (they are on nonconductors), we can find the electric field of the sheets in Fig. 23-17*a* by (1) finding the field of each sheet as if that sheet were isolated and (2) algebraically adding the fields of the isolated sheets via the superposition principle. (We can add the fields algebraically because they are parallel to each other.)

Calculations: At any point, the electric field $\vec{E}_{(+)}$ due to the positive sheet is directed *away* from the sheet and, from Eq. 23-13, has the magnitude

$$E_{(+)} = \frac{\sigma_{(+)}}{2\varepsilon_0} = \frac{6.8 \times 10^{-6}\ \text{C/m}^2}{(2)(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 3.84 \times 10^5\ \text{N/C}.$$

Similarly, at any point, the electric field $\vec{E}_{(-)}$ due to the negative sheet is directed *toward* that sheet and has the magnitude

$$E_{(-)} = \frac{\sigma_{(-)}}{2\varepsilon_0} = \frac{4.3 \times 10^{-6}\ \text{C/m}^2}{(2)(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 2.43 \times 10^5\ \text{N/C}.$$

Figure 23-17*b* shows the fields set up by the sheets to the left of the sheets (L), between them (B), and to their right (R).

The resultant fields in these three regions follow from the superposition principle. To the left, the field magnitude is

$$\begin{aligned} E_L &= E_{(+)} - E_{(-)} \\ &= 3.84 \times 10^5\ \text{N/C} - 2.43 \times 10^5\ \text{N/C} \\ &= 1.4 \times 10^5\ \text{N/C}. \qquad \text{(Answer)} \end{aligned}$$

Because $E_{(+)}$ is larger than $E_{(-)}$, the net electric field $\vec{E}_L$ in this region is directed to the left, as Fig. 23-17*c* shows. To the right of the sheets, the electric field has the same magnitude but is directed to the right, as Fig. 23-17*c* shows.

Between the sheets, the two fields add and we have

$$\begin{aligned} E_B &= E_{(+)} + E_{(-)} \\ &= 3.84 \times 10^5\ \text{N/C} + 2.43 \times 10^5\ \text{N/C} \\ &= 6.3 \times 10^5\ \text{N/C}. \qquad \text{(Answer)} \end{aligned}$$

The electric field $\vec{E}_B$ is directed to the right.

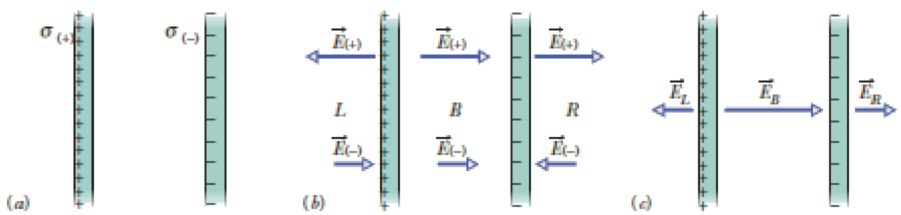


Fig. 23-17 (*a*) Two large, parallel sheets, uniformly charged on one side. (*b*) The individual electric fields resulting from the two charged sheets. (*c*) The net field due to both charged sheets, found by superposition.

REVIEW & SUMMARY

Gauss' Law *Gauss' law* and Coulomb's law are different ways of describing the relation between charge and electric field in static situations. Gauss' law is

$$\varepsilon_0 \Phi = q_{enc} \quad \text{(Gauss' law)}, \tag{23-6}$$

in which q_{enc} is the net charge inside an imaginary closed surface (a *Gaussian surface*) and Φ is the net *flux* of the electric field through the surface:

$$\Phi = \oint \vec{E} \cdot d\vec{A} \quad \text{(electric flux through a Gaussian surface)}. \tag{23-4}$$

Coulomb's law can be derived from Gauss' law.

Applications of Gauss' Law Using Gauss' law and, in some cases, symmetry arguments, we can derive several important results in electrostatic situations. Among these are:

1. An excess charge on an isolated *conductor* is located entirely on the outer surface of the conductor.
2. The external electric field near the *surface of a charged conductor* is perpendicular to the surface and has magnitude

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{(conducting surface)}. \tag{23-11}$$

Within the conductor, $E = 0$.

3. The electric field at any point due to an infinite *line of charge* with uniform linear charge density λ is perpendicular to the line of charge and has magnitude

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \quad \text{(line of charge)}, \tag{23-12}$$

where r is the perpendicular distance from the line of charge to the point.

4. The electric field due to an *infinite nonconducting sheet* with uniform surface charge density σ is perpendicular to the plane of the sheet and has magnitude

$$E = \frac{\sigma}{2\varepsilon_0} \quad \text{(sheet of charge)}. \tag{23-13}$$

5. The electric field *outside a spherical shell of charge* with radius R and total charge q is directed radially and has magnitude

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \text{(spherical shell, for } r \geq R). \tag{23-15}$$

Here r is the distance from the center of the shell to the point at which E is measured. (The charge behaves, for external points, as if it were all located at the center of the sphere.) The field *inside* a uniform spherical shell of charge is exactly zero:

$$E = 0 \quad \text{(spherical shell, for } r < R). \tag{23-16}$$

6. The electric field *inside a uniform sphere of charge* is directed radially and has magnitude

$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r. \tag{23-20}$$

QUESTIONS

1 A surface has the area vector $\vec{A} = (2\hat{i} + 3\hat{j})\ m^2$. What is the flux of a uniform electric field through the area if the field is (a) $\vec{E} = 4\hat{i}$ N/C and (b) $\vec{E} = 4\hat{k}$ N/C?

2 Figure 23-20 shows, in cross section, three solid cylinders, each of length L and uniform charge Q. Concentric with each cylinder is a cylindrical Gaussian surface, with all three surfaces having the same radius. Rank the Gaussian surfaces according to the electric field at any point on the surface, greatest first.

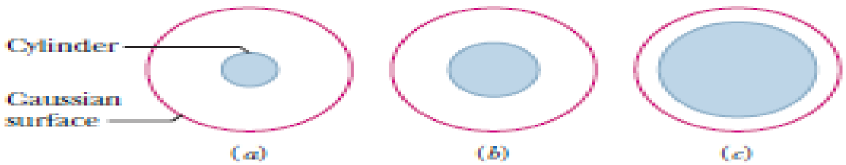


Fig. 23-20 Question 2.

3 Figure 23-21 shows, in cross section, a central metal ball, two spherical metal shells, and three spherical Gaussian surfaces of radii R, $2R$, and $3R$, all with the same center. The uniform charges on the three objects are: ball, Q; smaller shell, $3Q$; larger shell, $5Q$. Rank the Gaussian surfaces according to the magnitude of the electric field at any point on the surface, greatest first.

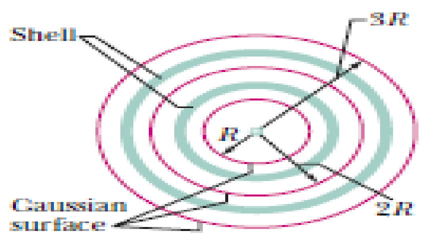


Fig. 23-21 Question 3.

sec. 23-3 Flux of an Electric Field

•**1** SSM The square surface shown in Fig. 23-26 measures 3.2 mm on each side. It is immersed in a uniform electric field with magnitude $E = 1800$ N/C and with field lines at an angle of $\theta = 35°$ with a normal to the surface, as shown. Take that normal to be directed "outward," as though the surface were one face of a box. Calculate the electric flux through the surface.

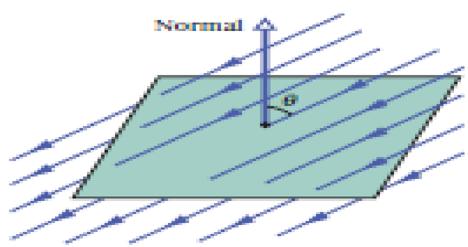
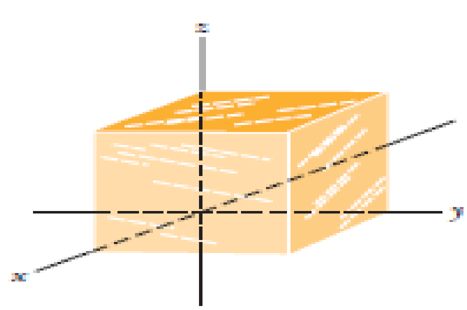


Fig. 23-26 Problem 1.

••**2** An electric field given by $\vec{E} = 4.0\hat{i} - 3.0(y^2 + 2.0)\hat{j}$ pierces a Gaussian cube of edge length 2.0 m and positioned as shown in Fig. 23-5. (The magnitude E is in newtons per coulomb and the position x is in meters.) What is the electric flux through the (a) top face, (b) bottom face, (c) left face, and (d) back face? (e) What is the net electric flux through the cube?

••**3** The cube in Fig. 23-27 has edge length 1.40 m and is oriented as shown in a region of uniform electric field. Find the electric flux through the right face if the electric field, in newtons per coulomb, is given by (a) $6.00\hat{i}$, (b) $-2.00\hat{j}$, and (c) $-3.00\hat{i} + 4.00\hat{k}$. (d) What is the total flux through the cube for each field?



CHAPTER4

E LECT R IC P OT E N T I A L

1- Electric Potential Energy

When an electrostatic force acts between two or more charged particles within a system of particles, we can assign an electric potential energy *U* to the system.

If the system changes its configuration from an initial state *i* to a different final state *f*, the electrostatic force does work *W* on the particles. If the resulting change is Δ*U*, then

$$\Delta U = U_f - U_i = -W.$$

As with other conservative forces, the work done by the electrostatic force is *path independent.*Usually the reference configuration of a system of charged particles is taken to be that in which the particles are all infinitely separated from one another. The corresponding reference potential energy is usually set be zero. Therefore

$$U = -W_{\infty}.$$

Example, Work and potential energy in an electric field:

Electrons are continually being knocked out of air molecules in the atmosphere by cosmic-ray particles coming in from space. Once released, each electron experiences an electrostatic force $\vec{F}$ due to the electric field $\vec{E}$ that is produced in the atmosphere by charged particles already on Earth. Near Earth's surface the electric field has the magnitude $E = 150$ N/C and is directed downward. What is the change ΔU in the electric potential energy of a released electron when the electrostatic force causes it to move vertically upward through a distance $d = 520$ m (Fig. 24-1)?

KEY IDEAS

(1) The change ΔU in the electric potential energy of the electron is related to the work W done on the electron by the electric field. Equation 24-1 ($\Delta U = -W$) gives the relation.

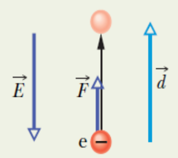


Fig. 24-1 An electron in the atmosphere is moved upward through displacement $\vec{d}$ by an electrostatic force $\vec{F}$ due to an electric field $\vec{E}$.

(2) The work done by a constant force $\vec{F}$ on a particle undergoing a displacement $\vec{d}$ is

$$W = \vec{F} \cdot \vec{d}. \qquad (24\text{-}3)$$

(3) The electrostatic force and the electric field are related by the force equation $\vec{F} = q\vec{E}$, where here q is the charge of an electron $(= -1.6 \times 10^{-19}$ C).

Calculations: Substituting for $\vec{F}$ in Eq. 24-3 and taking the dot product yield

$$W = q\vec{E} \cdot \vec{d} = qEd \cos\theta, \qquad (24\text{-}4)$$

where θ is the angle between the directions of $\vec{E}$ and $\vec{d}$. The field $\vec{E}$ is directed downward and the displacement $\vec{d}$ is directed upward; so $\theta = 180°$. Substituting this and other data into Eq. 24-4, we find

$$W = (-1.6 \times 10^{-19}\ \text{C})(150\ \text{N/C})(520\ \text{m}) \cos 180°$$
$$= 1.2 \times 10^{-14}\ \text{J}.$$

Equation 24-1 then yields

$$\Delta U = -W = -1.2 \times 10^{-14}\ \text{J}. \qquad \text{(Answer)}$$

This result tells us that during the 520 m ascent, the electric potential energy of the electron *decreases* by 1.2×10^{-14} J.

2- Electric Potential

The potential energy per unit charge at a point in an electric field is called the electric potential V (or simply the potential) at that point. This is a scalar quantity. Thus,

$$V = \frac{U}{q}.$$

The electric potential difference V between any two points i and f in an electric field is equal to the difference in potential energy per unit charge between the two points. Thus,

$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q}. = -\frac{W}{q} \qquad \text{(potential difference defined).}$$

The potential difference between two points is thus the negative of the work done by the electrostatic force to move a unit charge from one point to the other.

If we set $U_i = 0$ at infinity as our reference potential energy, then the electric potential V must also be zero there. Therefore, the electric potential at any point in an electric field can be defined to be

$$V = -\frac{W_\infty}{q} \qquad \text{(potential defined)}$$

Here W_∞ is the work done by the electric field on a charged particle as that particle moves in from infinity to point *f*.

The SI unit for potential is the joule per coulomb. This combination is called the *volt (abbreviated V)*.

$$1 \text{ volt} = 1 \text{ joule per coulomb.}$$

3-Electric Potential: Units

This unit of volt allows us to adopt a more conventional unit for the electric field, E, which is expressed in newtons per coulomb.

$$1 \text{ N/C} = \left(1\frac{\text{N}}{\text{C}}\right)\left(\frac{1 \text{ V}\cdot\text{C}}{1 \text{ J}}\right)\left(\frac{1 \text{ J}}{1 \text{ N}\cdot\text{m}}\right) = 1 \text{ V/m}.$$

We can now define an energy unit that is a convenient one for energy measurements in the atomic/subatomic domain: One *electron-volt (eV)* is the energy equal to the work required to move a single elementary charge *e*, such as that of the electron or the proton, through a potential difference of exactly one volt. The magnitude of this work is $q\Delta V$, and

$$1 \text{ eV} = e(1 \text{ V}) = (1.60 \times 10^{-19} \text{ C})(1 \text{ J/C}) = 1.60 \times 10^{-19} \text{ J.}$$

4- Electric Potential: Work done by an Applied Force

If a particle of charge *q* is moved from point *i* to point *f* in an electric field by applying a force to it, the applied force does work W_{app} on the charge while the electric field does work *W* on it. The change *K* in the kinetic energy of the particle is:

$$\Delta K = K_f - K_i = W_{\text{app}} + W.$$

If the particle is stationary before and after the move, Then K_f and K_i are both zero.

$$W_{\text{app}} = -W.$$

Relating the work done by our applied force to the change in the potential energy of the particle during the move, one has:

$$\Delta U = U_f - U_i = W_{\text{app}}.$$

We can also relate W_{app} to the electric potential difference ΔV between the initial and final locations of the particle:

$$W_{\text{app}} = q\,\Delta V.$$

5- Equipotential Surfaces

Adjacent points that have the same electric potential form an equipotential surface, which can be either an imaginary surface or a real, physical surface.

No net work *W* is done on a charged particle by an electric field when the particle moves between two points *i* and *f* on the same equipotential surface.

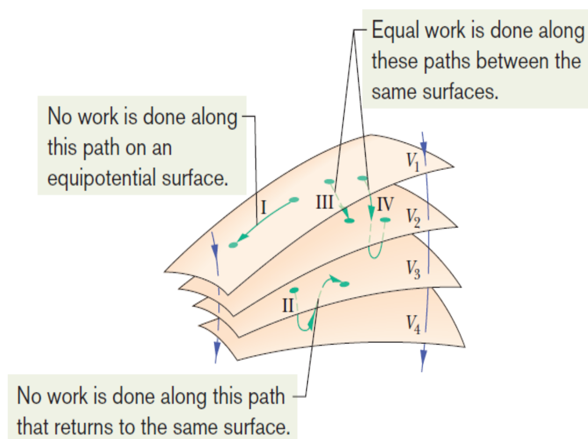


Fig. 24-2 Portions of four equipotential surfaces at electric potentials $V_1=100$ V, $V_2=80$ V, $V_3=60$ V, and $V_4=40$ V. Four paths along which a test charge may move are shown. Two electric field lines are also indicated.

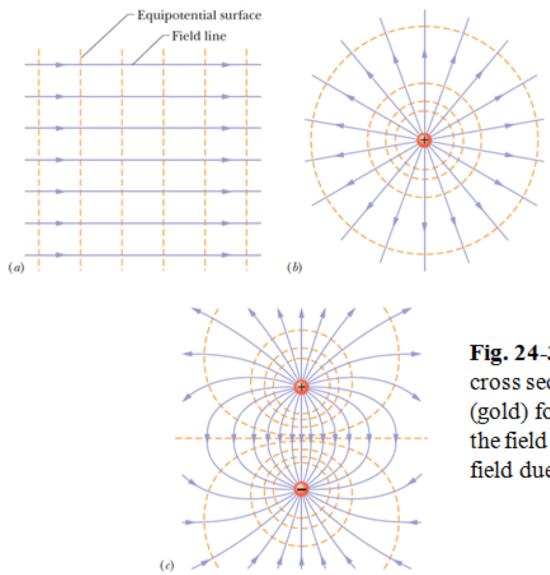


Fig. 24-3 Electric field lines (purple) and cross sections of equipotential surfaces (gold) for *(a)* a uniform electric field, *(b)* the field due to a point charge, and *(c)* the field due to an electric dipole.

6-Calculating the Potential from the Field

$$dW = \vec{F} \cdot d\vec{s}.$$

For the situation of Fig. 24-4, $dW = q_0 \vec{E} \cdot d\vec{s}.$

Total work: $$W = q_0 \int_i^f \vec{E} \cdot d\vec{s}.$$

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}.$$

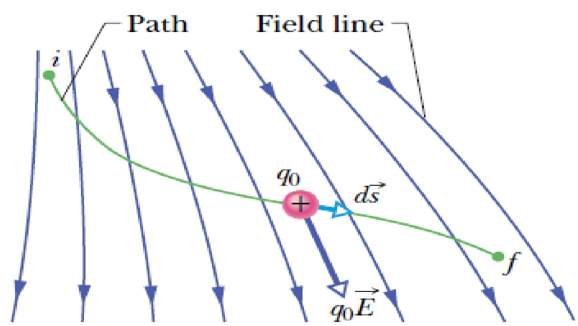


Fig. 24-4 A test charge q_0 moves from point i to point f along the path shown in a nonuniform electric field. During a displacement $d\vec{s}$, an electrostatic force $q_0\vec{E}$ acts on the test charge. This force points in the direction of the field line at the location of the test charge.

Thus, the potential difference $V_f - V_i$ between any two points i and f in an electric field is equal to the negative of the line integral from i to f. Since the electrostatic force is conservative, all paths yield the same result.
If we set potential $V_i = 0$, then

$$V = -\int_i^f \vec{E} \cdot d\vec{s},$$

This is the potential V at any point f in the electric field relative to the zero potential at point i. If point i is at infinity, then this is the potential V at any point f relative to the zero potential at infinity.

Sample Problem

Finding the potential change from the electric field

(a) Figure 24-5*a* shows two points *i* and *f* in a uniform electric field $\vec{E}$. The points lie on the same electric field line (not shown) and are separated by a distance *d*. Find the potential difference $V_f - V_i$ by moving a positive test charge q_0 from *i* to *f* along the path shown, which is parallel to the field direction.

KEY IDEA

We can find the potential difference between any two points in an electric field by integrating $\vec{E}\cdot d\vec{s}$ along a path connecting those two points according to Eq. 24-18.

Calculations: We begin by mentally moving a test charge q_0 along that path, from initial point *i* to final point *f*. As we move such a test charge along the path in Fig. 24-5*a*, its differential displacement $d\vec{s}$ always has the same direction as $\vec{E}$. Thus, the angle θ between $\vec{E}$ and $d\vec{s}$ is zero and the dot product in Eq. 24-18 is

$$\vec{E}\cdot d\vec{s} = E\,ds\cos\theta = E\,ds. \tag{24-20}$$

Equations 24-18 and 24-20 then give us

$$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s} = -\int_i^f E\,ds. \tag{24-21}$$

Since the field is uniform, *E* is constant over the path and can be moved outside the integral, giving us

$$V_f - V_i = -E\int_i^f ds = -Ed, \qquad \text{(Answer)}$$

in which the integral is simply the length *d* of the path. The minus sign in the result shows that the potential at point *f* in Fig. 24-5*a* is lower than the potential at point *i*. This is a general result: The potential always decreases along a path that extends in the direction of the electric field lines.

(b) Now find the potential difference $V_f - V_i$ by moving the positive test charge q_0 from *i* to *f* along the path *icf* shown in Fig. 24-5*b*.

Calculations: The Key Idea of (a) applies here too, except now we move the test charge along a path that consists of two lines: *ic* and *cf*. At all points along line *ic*, the displacement $d\vec{s}$ of the test charge is perpendicular to $\vec{E}$. Thus, the angle θ between $\vec{E}$ and $d\vec{s}$ is 90°, and the dot product $\vec{E}\cdot d\vec{s}$ is 0. Equation 24-18 then tells us that points *i* and *c* are at the same potential: $V_c - V_i = 0$.

For line *cf* we have $\theta = 45°$ and, from Eq. 24-18,

$$V_f - V_i = -\int_c^f \vec{E}\cdot d\vec{s} = -\int_c^f E(\cos 45°)\,ds$$

$$= -E(\cos 45°)\int_c^f ds.$$

The integral in this equation is just the length of line *cf*; from Fig. 24-5*b*, that length is $d/\cos 45°$. Thus,

$$V_f - V_i = -E(\cos 45°)\frac{d}{\cos 45°} = -Ed. \qquad \text{(Answer)}$$

This is the same result we obtained in (a), as it must be; the potential difference between two points does not depend on the path connecting them. Moral: When you want to find the potential difference between two points by moving a test charge between them, you can save time and work by choosing a path that simplifies the use of Eq. 24-18.

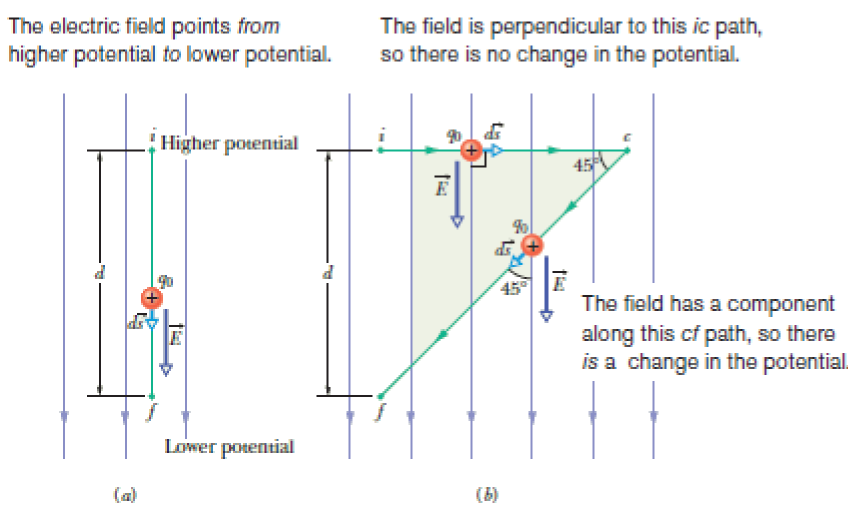


Fig. 24-5 (*a*) A test charge q_0 moves in a straight line from point *i* to point *f*, along the direction of a uniform external electric field. (*b*) Charge q_0 moves along path *icf* in the same electric field.